

Fokker-Planck equation with memory: the cross over from ballistic to diffusive processes in many particle systems and incompressible media

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The unified description of diffusion processes that crosses over from a ballistic behavior at short times to a fractional diffusion (sub- or superdiffusion), as well as to the ordinary diffusion at longer times, is proposed on the basis of a non-Markovian generalization of the Fokker-Planck equation. The relations between the non-Markovian kinetic coefficients and observable quantities (mean and mean square displacements) are established. The problem of calculations of the kinetic coefficients using the Langevin equations is discussed. Solutions of the non-Markovian equation describing diffusive processes in the real (coordinate) space are obtained. For long times, such a solution agrees with results obtained within the continuous random walk theory but is much superior to this solution at shorter times, where the effect of the ballistic region is crucial.