Microscopic representation of nonextensive systems within the Ising model

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In recent years many systems, which are characterized by nonadditivity (nonextensity), are found. These are ferromagnetics, two-dimensional electron plasma in the turbulent regime, systems with anomalous Levy diffusion, granular systems, solids subjected to ion bombardment, gravitational system, the elementary particles, colliding with high energy, and many others. Formally, the theory of nonextensive system is based on the deformation of logarithmic and exponential functions, which modifies the Boltzmann-Gibbs entropy in such a way that the distribution function either takes long-range power-law asymptotic, or is cut off at finite energies. These presentations are based on the formalism of quantum groups, which reduces to the so-called q-calculus.

In this work a microscopic presentation of the transition between the paramagnetic and ferromagnetic was developed within the q-calculus by modifying the Ising Hamiltonian.

$$\mathcal{H} = -\frac{1}{2}\sum_{i,j}^{N} J_{i,j} s_i^q s_j^q - h\sum_i s_i^q. \qquad (1)$$

The latter was carried out by replacing the spin variable s_i on the deformed s_i^q.

Since the Ising model has a discrete symmetry that is invariant to the transformation of the spin $s_i \to -s_i$, then our presentation makes sense at the condition $q = \frac{2m+1}{2n+1}$, where m and n are integers. Based on the Gibbs distribution the partition function of the nonextensive system was obtained. Then density of the free energy per node of a regular lattice was found. Using the equilibrium condition and the Landau approximation (closeness to the critical temperature), the equilibrium value of the order parameter, that characterizes the average spin $\eta \equiv \langle s \rangle$, was obtained. Analyzing the dependence of the free energy and the equilibrium value of the order parameter we can say that the value of parameter q has a strong influence on the state of the system.