Low-frequency electromagnetic field in Wigner crystal

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Lets consider low-frequency long-wave electromagnetic field in three-dimension Wigner (electron) crystal. It satisfies Maxwell equations with hydrodynamic currents. Equation for electron subsystem in continuum approximation in isotropic case is $\rho_e \dot{v}_{e\alpha} = (A+B)\,\partial^2 u_\chi/\partial x_\alpha \partial x_\chi + B\partial^2 u_\alpha/\partial x_\chi \partial x_\chi - en_e E_\alpha$, here u_α is deformation vector. Ion subsystem has been considered in jelly approximation $\rho_i \dot{v}_{i\alpha} = Zen_i E_\alpha$. We have such linearized Fourier-transformed equation system for wave-vector projections like $\mathbf{E}\mathbf{k}/k = E^\parallel$ in dimensionless variables $\left(E^\parallel, u^\parallel, v_e{}^\parallel, v_i^\parallel\right) \leftrightarrow \left(\frac{eE^\parallel}{ms^2k}, u^\parallel k, \frac{v_e^\parallel}{s}, \frac{v_i^\parallel M}{smZ}\right)$, $t \leftrightarrow tks$, where $s^2 = (A+2B)/\rho_{e0}$: $\dot{E}^\parallel = \Omega_e^2 v_e^\parallel - \Omega_i^2 v_i^\parallel$, $\dot{u}^\parallel = v_e^\parallel$, $\dot{v}_i^\parallel = E^\parallel$, $\dot{v}_e^\parallel = -u^\parallel - E^\parallel$. Here Ω_e and Ω_i are electron and ion plasma frequencies. Low-frequency oscillation branch is $\Lambda = \pm\frac{i}{2}\sqrt{\Omega_e^2 + \Omega_i^2 + 1 - \sqrt{\left(\Omega_e^2+\Omega_i^2+1\right)^2 - 4\Omega_i^2}}$ that gives solution like ordinary ion sound in limit $\Omega_e^2 \to \infty$: $\Lambda \approx \pm i\Omega_i/\Omega_e$. In the case of infinity heavy ions the sound disappears. For transversal oscillations like $\left(\delta_{\alpha\beta} - k_\alpha k_\beta/k^2\right) E_\beta = E_\alpha^\perp$ we denote $\mathbf{B} \to [\mathbf{k}/k, \mathbf{B}] = \mathbf{Z}$ and have $\left(\mathbf{E}^\perp, \mathbf{Z}, \mathbf{u}^\perp, \mathbf{v}_e^\perp, \mathbf{v}_i^\perp\right) \leftrightarrow \left(\frac{e\mathbf{E}^\perp}{ms^2k}, \frac{e\mathbf{Z}}{ms^2k}, \mathbf{u}^\perp k, \frac{\mathbf{v}_e^\perp}{s}, \frac{\mathbf{v}_i^\perp M}{smZ}\right)$, $t \to tks$, $c \leftrightarrow c/s$ where $s^2 = B/\rho_{e0}$. In this variables we have $\dot{\mathbf{E}}^\perp = ic\mathbf{Z} + \Omega_e^2 \mathbf{v}_e^\perp - \Omega_i^2 \mathbf{v}_i^\perp$, $\dot{\mathbf{Z}} = ic\mathbf{E}^\perp$, $\dot{\mathbf{u}}^\perp = \mathbf{v}_e^\perp$, $\dot{\mathbf{v}}_i^\perp = \mathbf{E}^\perp$, $\dot{\mathbf{v}}_e^\perp = -\mathbf{u}^\perp - \mathbf{E}^\perp$. Solution branch $\Lambda = \pm\frac{i}{2}\sqrt{\Omega_e^2 + \Omega_i^2 + 1 + c^2 - \sqrt{\left(\Omega_e^2+\Omega_i^2+1+c^2\right)^2 - 4\left(\Omega_i^2 + c^2\right)}}$ gives low-frequency oscillations that gives transversal sound in limit $\Omega_e^2 \to \infty$ and condition $\Omega_i \gg kc$: $\Lambda \approx \pm i\Omega_i/\Omega_e$. But in the case of infinity heavy ions a new low-frequency quadratic dispersion law appears $\Lambda \approx \pm ic/\Omega_e$. So resiliency modules of electron subsystem that are determined by short-acting potential enter in long wavelength oscillation frequencies: longitudinal $\omega^2 = k^2(A+2B)/\rho_{i0}$ and transversal $\omega^2 = k^2 B/\rho_{i0} + k^4 c^2 B/\rho_{e0}\Omega_e^2$. This consideration can be applied to covalent bounded isotropic crystals also.