Hydrodynamics of superfluid Bose liquid as a one-component system

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Standard hydrodynamics of superfluid Bose liquid was elaborated by Landau and Khalatnikov [1]. Domain of applicability of the Landau-Khalatnikov hydrodynamics (LKH) is discussed in the literature (see, for example, [2]). Idea that the superfluid liquid consist of two subsystems with different properties is very attractive. However, this liquid consists of identical particles. Here hydrodynamics of the superfluid Bose liquid is formulated as a hydrodynamics of an one-component liquid with additional variable (order parameter) which describes its broken symmetry. We introduce mass velocity by usual formula $\pi_n = \sigma v_n$ where π_n, σ are total momentum and mass density of the liquid. This relation and standard definition allow expressing of velocity of syperfluid v_{sl} and normal v_{nl} components by formulae $v_{sl} = v_l - \sigma_n \omega_l/\sigma$, $v_{nl} = v_l + \sigma_s \omega_l/\sigma$ ($\sigma \equiv \sigma_s + \sigma_n$, $\omega_l \equiv v_{nl} - v_{sl}$). Now we can write down all transformation laws from laboratory reference system (RS) to local RS which moves with the velocity v_l: $\varepsilon = \varepsilon^0 + \sigma v^2/2$, $q_l = q_l^0 + t_{lm}^0 v_m + (\varepsilon^0 + \sigma v^2/2) v_l$, $t_{lm} = t_{lm}^0 + \sigma v_l v_m$ where q_l, t_{lm} are flux densities of energy and momentum, ε is density of energy. In the LKH [1] RS of the local rest of superfluid component is used. The basic thermodynamic relation of the LKH can be rewritten in the form $d\varepsilon^0 = T ds_0 + \mu d\sigma + \alpha \pi_{l0} d\pi_{l0}$ ($\mu \equiv \mu_0 + \pi_0^2/2\sigma^2$, $\pi_{l0} \equiv \sigma_n \omega_l$, $\alpha \equiv \sigma_s/\sigma\sigma_n$). Pressure and entropy are connected by relation $\varepsilon^0 + p_0 = s_0 T + \mu\sigma + \alpha \pi_0^2$ (μ_0,s_0,p_0 are chemical potential, entropy density, pressure and are denoted in the LKH with μ,S,p). Zero order contributions in gradients to energy and momentum fluxes are given by expressions $q_l^{0(0)} = (s_0 T + \alpha \pi_0^2) \alpha \pi_{l0}$, $t_{lm}^{0(0)} = p_0 \delta_{lm} + \alpha \pi_{l0} \pi_{m0}$. Obtained formulas show that deviation of hydrodynamics of superfluid liquid from hydrodynamics of an one-component normal liquid is defined by parameter α. For the considered system density of momentum π_{l0} can be treated as an order parameter.

[1] I. M. Khalatnikov, Theory of superfluidity.– Moscow: Nauka, 1971 (in Russian).

[2] S. J. Putterman, Superfluid hydrodynamics.– New York: Elsevier, 1974.