On new vortex invariants in three-dimensional magneto-hydrodynamics

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A modern symplectic and symmetry analysis for studying MHD superfluid flows is devised, a new version of the helicity theorem based on differential-geometric and group-theoretical methods is derived. Having reanalyzed the helicity theorem within the modern symplectic theory of differential-geometric structures on manifolds, a new unified proof and a new generalization of this theorem for the case of compressible MHD superfluid flow are proposed. As a by-product, a sequence of nontrivial helicity type local and global conservation laws for the case of incompressible superfluid flow, playing a crucial role [3,4] for studying the stability problem under suitable boundary conditions, is constructed. The symplectic and symmetry analysis of compressible MHD superfluid, appeared to be effective for constructing the related helicity type conservation laws, important for practical applications. In particular, the conservative quantities play a decisive role [1,3], when studying the stability of MHD superfluid flows under special boundary conditions. Some of the results in this direction can be eventually obtained making use of group–theoretical and topological tools developed in [2,4], where the importance of the basic group of diffeomorphisms $\mathrm{Diff}(M)$ of a manifold $M \subset \mathbb{R}^3$ and its differential–geometric characteristics were stated.

[1] Blackmore D., Prykarpatsky A.K. and Samoylenko V.H. Nonlinear Dynamical Systems of Mathematical Physics. World Scientific Publ., NJ USA, 2011.

[2] Slawianowski J.J. The geometry of phase spaces, Wiley, 1991

[3] Peradzynski Z. Helicity theorem and vertex lines in superfluid 4-He. Int. J. Theor. Phys., 29,11 (1990), pp. 1277–1284.

[4] Arnold V.I. and Khesin B.A. Topological methods in hydrodynamics. Springer, NY, 1998.