Goldstone mode singularities in $O(n)$ models

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Monte Carlo (MC) analysis of the Goldstone mode singularities for the transverse and the longitudinal correlation functions, behaving as $G_{\perp}(\mathbf{k}) \simeq ak^{-\lambda_{\perp}}$ and $G_{\parallel}(\mathbf{k}) \simeq bk^{-\lambda_{\parallel}}$ in the ordered phase at $k \to 0$, is performed in the three–dimensional $O(n)$ models with $n = 2, 4, 10$. Our aim is to test the predictions of [1], according to which the exponents $\lambda_{\perp}$ and $\lambda_{\parallel}$ are non-trivial ($3/2 < \lambda_{\perp} < 2$ and $0 < \lambda_{\parallel} < 1$ in three dimensions) and the ratio bM^2/a^2 (where M is the spontaneous magnetization) is universal. The trivial standard–theoretical values are $\lambda_{\perp} = 2$ and $\lambda_{\parallel} = 1$. The MC analysis of [2] gives $\lambda_{\perp} = 1.955 \pm 0.020$ for the $O(4)$ model. The MC estimation of $\lambda_{\parallel}$, assuming corrections to scaling of the standard theory, yields $\lambda_{\parallel} = 0.69 \pm 0.10$ for the $O(2)$ model [3]. This result clearly disagrees with $\lambda_{\parallel} = 1$. Currently, we have performed a similar MC estimation for the $O(10)$ model, yielding $\lambda_{\perp} = 1.9723(90)$ and $\lambda_{\parallel} = 0.85 \pm 0.06$. We have observed that the plot of the effective transverse exponent for the $O(4)$ model is systematically shifted down with respect to the same plot for the $O(10)$ model by $\Delta\lambda_{\perp} = 0.0121(52)$. It is consistent with the idea that $2 - \lambda_{\perp}$ decreases for large n and tends to zero at $n \to \infty$. We have also verified and confirmed the expected universality of bM^2/a^2 for the $O(4)$ model, where simulations at two different temperatures (couplings) have been performed.

References

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