Time characteristics of Lévy flights in a steep potential well

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The analytical investigation of time characteristics of Lévy flights remains an open problem because meets with some difficulties. For instance, the theory of mean first-passage times assumes the presence of some boundary conditions which are not so evident for Markovian process having a long jumps. As a consequence, a lot of results in this area were obtained by numerical simulations.

At the same time, as has been shown (A.V. Chechkin et al, J. Stat. Phys. 2000. V.115, 1505), starting from the quartic potential we observe a confinement of Lévy flights, i.e. the variance of particle displacement is finite. As a result, one can find, in principle, the correlation function and the power spectral density of Lévy motion in a steady state. But, unlike the stationary probability distributions being bimodal (A.A. Dubkov et al, Acta Phys. Pol. B 2007. V.38, 1745; see also review A.A. Dubkov et al, Int. J. Bifurc. Chaos 2008. V.18, 2649), exact analytical results can be only obtained for some time characteristics of Lévy flights in the steep potential well $U(x) = ax^{2m}/(2m)$.

Using the procedure for usual Brownian diffusion based on the Laplace transform of transient probability density (A.A. Dubkov et al, Radiophys. Quant. Electr. 2000. V.43, 335), we derive new equation which is convenient to calculate the correlation time of stationary Lévy flights. This equation, involving a fractional derivative and the steady-state probability density function, can be written using the Fourier transform in differential form, and then solved for the case of Cauchy-stable noise producing the Lévy flights with index $\alpha = 1$.

For the quartic potential ($m = 2$) we find the correlation time as a function of noise intensity and steepness of potential well. As follows from the exact relation, the correlation time of stationary Lévy flights decreases with increasing noise intensity or steepness a of potential. There is the possibility of calculating the correlation time for steeper potential wells ($m \geq 3$).