An aggregate model for the particle size distribution in planetary rings

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Numerous measurements of the particle size distribution in Saturn's rings reveal, that particles with radii r, ranging from a few centimeters to about a few meters obey a power law distribution, $n(r) \sim r^{-q}$ with $q \approx 3$, while for larger radii, $n(r)$ steeply drops down with increasing r. Although it is widely accepted, that the observed particle size distribution is a result of a subtle balance between two competing processes – aggregation and fragmentation, neither the power-law dependence, nor the upper-size cutoff have been explained and quantified within a unique model. Here we present a new model which accurately predicts the exponent q of the power-law of the particle size distribution for small particles and the exponential cutoff for large particles.

We consider a system of granular particles with discrete size distribution, which undergoe pairwise collisions. The model is characterized by two energy thresholds, E_{agg} and E_{frag}, which demarcate different types of impacts: If the kinetic energy of the relative motion of a colliding pair is smaller than E_{agg} or larger than E_{frag}, particles respectively merge or break; otherwise they rebound. We start from the Boltzmann equation for the mass-velocity distribution function and derive Smoluchowski-like equations for concentrations of particles of different mass. We analyze these equations analytically and solve them numerically. We show, that the obtained stationary solution coincides with size distribution of particles, observed in the Saturn's rings.