To hydrodynamics in the presence of external potential field

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It is discussed hydrodynamics of an one-component liquid in the presence of external potential field $U(x)$ with small gradients $\frac{\partial^s U(x)}{\partial_{n_1}\ldots\partial_{n_s}} \sim g^s$. The same estimation is assumed to be valid for gradients of usual hydrodynamic variables: mass density $\rho(x,t)$, velocity $v_n(x,t)$ and temperature $T(x,t)$. So, in this approach we consider weak and weak nonuniform fields. Hydrodynamics is based on the local equilibrium assumption. Therefore, one needs first thermodynamics in the presence of the field. It is obviously from the literature (see, for example, [1,2])that this theory is a weakly developed one. Our consideration of thermodynamics is based on the formula for average value of a local translation invariant microscopic quantity $\hat{a}(x)$ calculated with the grand canonical distribution. It was shown that this average $a(x)$ can be expanded in a series over gradients of potential $U(x)$ near space point x: $a(x) = a^{(0)}(x) + a^{(1)}(x) + O(g^2)$. It follows from our consideration that $a^{(0)}(x) = a(T, \mu - U(x))$ where function $a(T,\mu)$ gives equilibrium thermodynamic dependence of value a on temperature T and chemical potential μ in the absence of external field. This relation is very important one because it describes thermodynamics in the zero order in gradients of the field approximation. It is obviously that standard thermodynamics in the presence of external field [1] is a theory of this approximation.

Derivation of hydrodynamic equations needs also expressions for microscopic quantities of energy $\hat{q}_n(x)$ and momentum $\hat{t}_{nl}(x)$ fluxes. Using expressions for this values obtained in [3] we show that they can be expanded in a series in gradients of the field $U(x)$ too. On this basis hydrodynamic equations in the presence of external field have been construction and compared with the standard theory [2].

[1] L.D. Landau and E.M. Lifshitz, Statistical physics.– Oxford: Pergamon Press, Part 1, 1982.

[2] L.D. Landau and E.M. Lifshitz, Fluid mechanics.– Oxford: Pergamon Press, 1984.

[3] S.V. Peletminsky, A.I. Sokolovsky, Theoretical and Mathematical Physics, 18, 85-91 (1974).