On derivation of quantum kinetic equations and nonlinear Schrödinger equation

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We develop a new approach to the problem of the rigorous derivation of quantum kinetic equations from underlying many-particle dynamics.

At first we construct a solution of the Cauchy problem of the Bogolyubov chain of equations (BBGKY hierarchy) for marginal density operators of quantum many-particle systems in the space of sequences of trace class operators. Then we discuss the relationships between infinite-particle dynamics and quantum kinetic equations. A conventional approach to the problem of the rigorous derivation of kinetic equations from microscopic equations of the von Neumann or the corresponding quantum BBGKY hierarchy consists of the construction of a suitable scaling limit of the solution of the BBGKY hierarchy of equations with initial data satisfying the chaos property. As a result of the scaling limit a solution preserves the chaos property if the limit one-particle density operator satisfies the corresponding kinetic equation.

Our main result consists of the proof that, if initial data are completely defined only by the trace class one-particle density operator, the Cauchy problem of the BBGKY hierarchy is equal to the corresponding initial value problem of certain generalized quantum kinetic equation and an infinite sequence of explicitly defined functionals of a solution of the generalized kinetic equation. For initial data from the space of trace class operators we prove the existence theorem for the generalized kinetic equation.

The specific quantum kinetic equations such as the quantum Boltzmann equation and other ones, can be derived from constructed generalized quantum kinetic equation in the appropriate scaling limits. As a consequence of this result we derive the nonlinear Schrödinger equation for the Bose gas in the mean field limit. In conclusion we also discuss the problem of the derivation of quantum kinetic equations in the condensate state.