Phase transitions and quantum stabilization in quantum anharmonic crystals

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A unified theory of phase transitions and quantum effects in quantum anharmonic crystals is presented. The theory is based on the representation of the Gibbs states of the model in terms of path measures. It covers the case of crystals without translation invariance, as well as the case of asymmetric anharmonic potentials. In the simplest case where the lattice is $\mathbf{Z}^d$, the model is translation invariant, and the interaction is of nearest neighbor type with strength $J > 0$, it is found that the key parameter is $8dmJ\vartheta^2$. Herein, m is the particle mass and $\vartheta > 0$ is an anharmonicity parameter. It is proven that the crystal is stable (no phase transitions at all temperatures) whenever $8dmJ\vartheta^2 < 1$. At the same time, a sufficient condition for a phase transition to occur is $8dmJ\vartheta^2 > \phi(d)$, $d \geq 3$, where $\phi(d)$ is an explicitly given function, such that $\phi(d) \to 1$ as $d \to +\infty$.