Review for Yulian Ivan Mykhailo Honchar's thesis
«Scaling properties of phase transitions above the upper critical dimension
and in the description of DNA denaturation»

The manuscript by Yulian Honchar considers features of scaling descriptions in two different problems: (i) second order phase transition above the upper critical dimension and (ii) DNA denaturation transition in therms of conformal properties of polymer networks. This study is important for deeper understanding of physics underlying the power laws describing the above mentioned phenomena. Combining properly extended powerful numerical and analytic methods author obtained a series of relevant results.

The thesis comprises an introduction, a literature review chapter, three research chapters (with two chapters devoted to problem (i) and one chapter to problem (ii)), a conclusions section, a list of references, and appendices. In the literature review, the author condensedly describes the physical concepts concerning the critical behavior of finite-size systems above the upper critical dimension. The review also addresses the Poland-Sheraga model of denatured DNA chains, depicting them as polymers with monomers in both bound and unbound states, and discusses the conformal properties of long polymer molecules in terms of self-avoiding walks.

In the thesis's second chapter, the first research chapter following the review, the author uses Monte Carlo simulations to study the scaling properties of different thermodynamic observables at the second-order phase transition occurring in the Ising model on finite hyperlattices at dimension $d$=5, above the critical dimension $d_c$ = 4, where the mean-field picture of critical properties becomes valid. In this context, one of Yulian's co-advisers, Prof. Ralph Kenna, along with Prof. Bertrand Berche, proposed the so-called 'Q-scaling' to address the apparent violation of the hyperscaling relation. For free boundary conditions, the results of the chapter verified the validity of the finite-size Q-scaling hypothesis for magnetisation at the pseudocritical temperature and demonstrated the crucial impact of boundaries for small lattices. This makes it impossible to conclusively answer the question for other observables for the lattice sizes simulated by the author.

In the third chapter, the same problem was tackled using additional powerful methods, namely the analysis of properties of Fourier modes of magnetisation and the study of the behavior of partition function zeros in the complex magnetic field plane. The use of the first approach confirmed the conclusions of the previous chapter, while the second approach helped elucidate the scaling picture with higher accuracy. In particular, the results obtained from the examination of Lee-Yang zeros of the partition function are in favor of Q-scaling for isothermal susceptibility at the pseudocritical point, providing valuable insights into the consistent Q-scaling behavior observed under free boundary conditions above the upper critical dimension.

The fourth chapter focuses on the scaling of the partition function of the Poland-Sheraga model for thermal DNA denaturation. Its loop closure exponent governs the order of the denaturation transition and can be connected with critical exponents of copolymer networks. To this extent, Yulian Honchar derived new scaling relations for this exponent. By analyzing known high-order perturbative series for critical exponents of copolymer networks, he examined the effects of heterogeneity, considering cases where the denatured loop and bound chains can be described differently, as either self-avoiding walks or random walks, mimicking the $\theta$-point for polymers in the second case. Using sophisticated resummation techniques on perturbative series, the obtained values of the loop closure exponent are shown to be larger for heterogeneous cases compared to homogeneous ones. The study of the impact of the environment in the form of long-correlated disorder demonstrates a further increase in the value of the loop closure exponent and, therefore, the strengthening of the first-order transition in DNA denaturation.

I believe the results obtained in this thesis represent a significant advancement in the modern theory of phase transitions. The novelty and relevance of Yulian Honchar's research, in my opinion, are caused primarily by the deeply thought-out formulation of the problem, thanks to the amazing intelligence of his co-advisers, Prof. R. Kenna and Prof. Yu. Holovatch. Having attended many scientific meetings where Yulian Honchar presented his research, and having had the opportunity to engage in discussions with him, I can affirm that he has mastered modern analytic methods of theoretical physics as well as computational techniques used in numerical simulations of physical problems, thus becoming a skilled scientist. The reliability and novelty of his results are further corroborated by their publication in two authoritative international scientific journals. Additionally, his findings have already been accepted for publication as a regular scientific paper in the *Condensed Matter Physics* journal. Moreover, the results were disseminated and discussed at 11 scientific Ukrainian and international conferences and workshops.

Upon reviewing the thesis, I found no drawbacks in the scientific concepts utilized, the methods applied, or the calculations performed, and therefore I have no essential objections to the thesis. In summary, I consider that Yulian Honchar's thesis fully meets the requirements for the awarding of the Doctor of Philosophy degree, and its author, Yulian Honchar, deserves this degree in natural sciences (104 - physics and astronomy).

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Head of the Department for
Statistical Theory of Condensed Systems of
Insitute for Condensed Matter Systems of
National Academy of Sciences of Ukraine

Dr. Maxym Dudka