Препринти Інституту фізики конденсованих систем НАН України розповсюджуються серед наукових та інформаційних установ. Вони також доступні по електронній комп'ютерній мережі на WWW-сервері інституту за адресою http://www.icmp.lviv.ua/

The preprints of the Institute for Condensed Matter Physics of the National Academy of Sciences of Ukraine are distributed to scientific and informational institutions. They also are available by computer network from Institute's WWW server (http://www.icmp.lviv.ua/)

Михайло Павлович Козловський
Роман Васильович Романік

Рівняння стану 3D ізінгоподібної системи

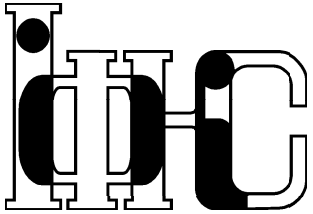
Роботу отримано 9 липня 2009 р.

Затверджено до друку Вченою радою ІФКС НАН України

Рекомендовано до друку відділом статистичної теорії конденсованих середовищ

Виготовлено при ІФКС НАН України
© Усі права застережені

Національна академія наук України



ІНСТИТУТ ФІЗИКИ КОНДЕНСОВАНИХ СИСТЕМ

ICMP–09–07E

M.P.Kozlovskii, R.V.Romanik*

THE EQUATION OF STATE 3D ISING-LIKE SYSTEM

*Lviv National University, 1 Universytetska Str., 79000 Lviv

ЛЬВІВ

УДК: 537.611.2; 538.955

PACS: 05.50+q, 64.60.Fr, 75.10.Hk

Рівняння стану 3D ізінгоподібної системи

М.П.Козловський, Р.В.Романік

Анотація. Виконано розрахунок вільної енергії F та параметра порядку M поблизу точки фазового переходу для $3D$ ізінгоподібної системи в області температур $T < T_c$. Отриманий явний вираз для F як функція температури та поля при довільних значеннях останнього. Знайдено залежність коефіцієнтів виразу для вільної енергії від параметрів гамільтоніану. Шляхом прямого диференціювання F за зовнішнім полем отримано вираз для намагніченості моделі, як функції температури та поля. Встановлено, що у випадку малих полів поведінка намагніченості визначається критичним показником β ($M = M_{0\tau}(-\tau)^{\beta}$), у випадку $T = T_c$ маємо польову залежність ($M = M_{0h}h^{1/\delta}$).

The equation of state 3D Ising-like system

M.P.Kozlovskii, R.V.Romanik

Abstract. The free energy F and the order parameter M are calculated near the phase transition point for a $3D$ Ising-like system in the temperature region $T < T_c$. Explicit expression for the free energy as a function of temperature and field is obtained at arbitrary values of the latter. Hamiltonian parameters dependence of coefficients of the free energy expression is found. Expression for the model's magnetization as a function of temperature and field is derived by direct differentiation of F with respect to external field. It is shown that in the case of small fields the behavior of magnetization is determined by the critical exponent β ($M = M_{0\tau}(-\tau)^{\beta}$), in the case $T = T_c$ we have the field dependence ($M = M_{0h}h^{1/\delta}$).

Подається в Condensed Matter Physics
Submitted to Condensed Matter Physics

© Інститут фізики конденсованих систем 2009
Institute for Condensed Matter Physics 2009

1. Introduction

In the present work we obtain an explicit expression for the free energy of the 3D Ising-like model with the exponentially decreasing potential of interparticle interaction at $T \leq T_c$ near the phase transition point $T = T_c$. It is a continuation of the work [1] where such calculations are presented at $T \geq T_c$. It should be noted that in the case $T = T_c$ both free energy expressions coincide, hence the derivates of this quantity with respect to temperature and field are equal. For calculation purposes we use the collective variables method (CV) [2] generalized in [3] for the presence of a constant external field. The technique of the calculation does not impose any constraints on the field value $h = \beta\mathcal{H}$ (β is the inverse temperature, $\mathcal{H}$ is the magnetic field magnitude). It is known [4] that as the temperature approaches T_c one can distinguish two distinct regions on the field-temperature plane. Far from T_c the field h can take both small and large values relative to the value $h_c = |\tau|^{\beta\delta}$ ($\tau = (T - T_c)/T_c$, β and δ are the critical exponents of the order parameter: the temperature and the field exponents respectively). As the temperature approaches T_c (as τ decreases) the role of the field h becomes more important in the formation of physical characteristics. Technically, in this value range of τ and h the inequality $h \gg h_c$ always holds. Therefore, approaching T_c at a constant value of field $\mathcal{H}$ we get different values of the parameter $\alpha = h/h_c$. Far from T_c $\alpha \ll 1$; near T_c we have $\alpha \gg 1$. We used these inequalities in previous works [3, 4] to calculate explicit expressions for the free energy in the boundary cases of small and large field values. In the region $\alpha \approx 1$ these expressions had to be "joined", which generally speaking remains a drawback of these works. To overcome this difficulty in [5] we applied the procedure for determining the point of exit from the critical regime by means of a certain equation. The point of exit from the critical fluctuation regime is determined by the block structure number $n = n_p$, whose linear sizes are proportional to the correlation length at given τ and h. Solutions of that equation were searched for computationally. An advantage of this approach was the obtained scaling function of the order parameter (graphically), which was in good agreement with data from the parameter equation of state for an Ising model [6]. A drawback of the method was using the results from the numerical solution for the point of exiting the critical fluctuation regime and the impossibility of finding an explicit analytical expression for the quantities sought with only graphical dependences possible to obtain.

To obtain results analytically and to avoid using the results of computational calculations, we use the results of [7]. In this work the author

has proposed an explicit expression for the point of exit from the critical fluctuation regime as a function of temperature and field.

We note that the temperature region $T < T_c$ differs significantly from the region $T > T_c$ for an Ising model. The reason is that in an Ising spin system at $T < T_c$ the order parameter appears spontaneously thus causing the existence of its conjugate field. The presence of such a field should be accounted for when determining field dependence of the system's point of exit from the critical fluctuation regime in the presence of a constant external field. Below we will use the results of [8] where the point of exit from the critical regime was found at $T < T_c$.

2. The model

We use notation and basic formulas of [1] to calculate free energy of an Ising-like system at $T < T_c$ with the Hamiltonian

$$H = -\frac{1}{2}\sum_{i,j}\Phi(r_{ij})\sigma_i\sigma_j + \mathcal{H}\sum_i \sigma_i \tag{2.1}$$

Here $\Phi(r_{ij})$ is a short-range potential of interaction between particles i and j that are located at site of a simple cubic lattice with a period c. Variable σ_i takes two values ± 1, $\mathcal{H}$ is an external field.

The partition function of such a system near T_c is written in the form

$$Z = Z_0[Q(d)]^{N_0}\left(\prod_{n=1}^{n_p'} Q_n\right) Z_{IGR}. \tag{2.2}$$

Here

$$Z_0 = 2^N(\cosh h)^N \exp(\frac{1}{2}N\beta\Phi(0)\bar{\Phi}) \tag{2.3}$$

where N is the number of particles, $h = \beta\mathcal{H}$ is a unitless field, $\Phi(0)$ is the value of the Fourier transform $\Phi(r_{ij})$ at $k = 0$, and $\bar{\Phi}$ is its parameter defined in [6,7]. Quantity $Q(d)$ represents contribution of large values of the wave vector to the statistical sum and is given in [1]. This work also gives an explicit expression for the quantity Q_n which corresponds to the partial partition function of a block structure n [9]. The product is conducted up to the point n_p' that determines the point of the system's exit from the critical regime of the order parameter fluctuations. At $T < T_c$ for n_p' we have

$$n_p' = -\frac{\ln(\tilde{h}^2 + h_{cm}^2)}{2\ln E_1} - 1 \tag{2.4}$$

Here $\tilde{h} = s_0^{d/2}(h/h_0)$ is the renormalized external field and

$$h_{cm}(\tau) = |\tau_1|^{p_0} \tag{2.5}$$

is a certain temperature field where

$$\tau_1 = -\tau\frac{c_{k1}}{f_0}E_2^{n_0}. \tag{2.6}$$

Here $p_0 = \ln E_1/\ln E_2$, E_l are the eigen values of the matrix of the RG transformation linearized near a fixed point [1], quantities f_0, c_{1k} denote coordinates of the fixed point [1,8].

It should be noted that in the case $T > T_c$ (see [1]) the temperature field $h_c = |\tilde{\tau}|^{p_0}$ was introduced where $\tilde{\tau} = \tau c_{1k}/f_0$. Variable $\tilde{\tau}$ differs from τ_1 in (2.6) since the factor $E_2^{n_0}$ is absent. Quantity n_0 (at $h = 0$) is the difference between the points of exit from the critical fluctuation regime at $T > T_c$ (n_p) and $T < T_c$ (n_p')

$$n_0 = n_p - n_p' \tag{2.7}$$

and implies the presence at $T < T_c$ of a certain internal field conjugate to the spontaneous order parameter.

In expression (2.2) the quantity Z_{IGR} has the form

$$Z_{IGR} = 2^{(N_{n_p'}-1)/2}Q(P_{n_p'})^{N_{n_p'+1}}Z_{n_p'+1}, \tag{2.8}$$

where $N_{n_p'}$ is the number of sites in an effective block lattice with the period $c_{n_p'} = c_0 s^{n_p'}$ ($c_0 = c\cdot s_0$), quantity $Q(P_n)$ is defined in [1] and for $Z_{n_p'+1}$ we have the expression

$$\begin{aligned} Z_{n_p'+1} \;=\; & \int (d\eta)^{N_{n_p'+1}} \exp\Bigg(a_1^{(n_p'+1)} N_{n_p'+1}^{1/2}\eta_0 - \frac{1}{2}\sum_{k\in\mathcal{B}_{n_p'+1}} d_{n_p'+1}(k)\times \\ & \times \eta_{\vec{k}}\eta_{-\vec{k}} - \frac{a_4^{(n_p'+1)}}{4!}N_{n_p'+1}^{-1}\sum_{\vec{k_1},\cdot\vec{k_4}} \eta_{\vec{k_1}}\cdots\eta_{\vec{k_4}}\delta_{\vec{k}_1+\cdots+\vec{k}_4}\Bigg) \end{aligned} \tag{2.9}$$

Here $d_{n_p'+1}(k) = d_{n_p'+1}(0) + 2\beta\Phi(0)b^2k^2$, where b is the range of an exponentially decreasing potential of interaction ($\Phi(r_{ij}) = A\exp(-r_{ij}/b)$) and for $a_1^{(n)}$, $d_n(0)$ and $a_4^{(n)}$ recurrence relations (RR) are valid. Their explicit form is derived in [10]. Introducing notation

$$a_1^{(n)} = s^{-n}w_n, \quad d_n(0) = s^{-2n}r_n, \quad a_4^{(n)} = s^{-4n}u_n$$

RR can be written as follows

$$w_{n+1} = s^{\frac{d+2}{2}} w_n,$$
$$r_{n+1} = s^2[-q + (r_n + q)N(x_n)],$$
$$u_{n+1} = s u_n E(x_n), \tag{2.10}$$

Where the functions $N(x_n)$ and $E(x_n)$ are defined in [1]. The initial values of quantities w_n, r_n and u_n (at $n = 0$) are

$$w_0 = s_0^{d/2} h, \quad r_0 = a_2 - \beta\Phi(0)(1 - \bar{\Phi}), \quad u_0 = a_4, \tag{2.11}$$

where the values of quantities a_2, a_4 are given in [7].

A renormgroup symmetry in the system is present only for $n < n'_p$ i.e. when the period c_n of an effective block lattice is less than $c_{n'_p}$. In this case general RR (2.10) can be substituted by their linearized form near a fixed point and used to calculate the product of partial partition functions Q_n in the expression (2.2). At $n > n'_p$ the RG symmetry is absent and the factor Z_{IGR} reflects the contribution to the statistical sum after the system's exit from the critical regime of the order parameter fluctuations. Unlike CR it was named [11] the inverse Gauss regime (IGR).

It should be noted that the calculations below relate to the temperature region $\tau^* \le \tau \le 0$ where $\tau^* \sim 10^{-2}$. We do not impose any restrictions on the field value.

3. The scheme for calculating free energy near T_c $(T < T_c)$

The free energy of one-component spins near the PTP is calculated based on expression (2.2) that determines partition function at $T \le T_c$. Unlike the case of field absence [11] the generalized point of exit from the critical fluctuation regime is used here. In the case $T < T_c$ expression (2.4) is valid. We present free energy as the sum of several terms

$$F = F_0 + F_{CR}^{(-)} + F_{IGR}. \tag{3.1}$$

Each of them represents contribution of a certain factor in expression (2.2). Thus the first term F_0 which is also present in the expression for the free energy at temperatures higher than T_c [1] has the form

$$F_0 = -kTN[\ln 2 + \ln\cosh h] - \frac{1}{2}N\Phi(0)\bar{\Phi} \tag{3.2}$$

corresponds to the expression for Z_0 and describes free energy of non-interacting spins.

The term $F_{CR}^{(-)}$ represents contribution of the critical fluctuation regime region to the free energy. To calculate it we use the method from [1]. In accordance with (2.2) we have an expression

$$F_{CR}^{(-)} = -kT \sum_{n=1}^{n'_p} N_n f_n, \tag{3.3}$$

where $N_n = N_0 \cdot s^{-3n}$ and for the function $f_n(x_n, y_{n-1})$ we get

$$f_n = \frac{1}{2}\ln y_{n-1} + \frac{9}{4}y_{n-1}^{-2} + \frac{x_n^2}{4} + \ln U(0, x_n), \tag{3.4}$$

where $U(a, x)$ is Weber's parabolic cylinder functions. Expressions for arguments x_n and y_n can be found in [1]. First we note that due to (2.4) the following equalities hold

$$s^{-(n'_p+1)} = (\tilde{h}^2 + h_{cm}^2)^{\frac{1}{d+2}}, \qquad E_1^{n'_p+1} = (\tilde{h}^2 + h_{cm}^2)^{-1/2},$$
$$\tilde{\tau} E_2^{n'_p+1} = -H_{cm}, \qquad H_{cm} = -\tilde{\tau}(\tilde{h}^2 + h_{cm}^2)^{-1/2p_0},$$
$$E_3^{n'_p+1} = H_{3m}, \qquad H_{3m} = (\tilde{h}^2 + h_{cm}^2)^{\Delta/2p_0}. \tag{3.5}$$

Here we have used notation

$$\nu = \frac{\ln s^*}{\ln E_2}, \quad \Delta = -\frac{\ln E_3}{\ln E_2}, \quad p_0 = \frac{\ln E_1}{\ln E_2} = \frac{d+2}{2}\nu, \tag{3.6}$$

where Δ is the critical factor of the scaling correction, p_0 is the crossover critical factor. The value of the renormalization group parameter $s = s^*$ where $s^* = 3.5977$ corresponds to the case when the quantity x_n in a fixed point yields zero ($x^* = 0$). For the model ρ^4 the critical factors from (3.6) take the following values

$$\nu = 0.605, \quad \Delta = 0.465, \quad p_0 = 1.512.$$

As a result of calculating (3.3) we get

$$F_{CR}^{(-)} = -kTN_0(\gamma_{01} + \gamma_{02}\tau + \gamma_{03}\tau^2) + F_{CR}^{(c)} \tag{3.7}$$

where the coefficients γ_l are presented in [1]. We point out that the analytical part of expression (3.7) coincides with the case $T > T_c$. The singular part of this expression has the form

$$F_{CR}^{(c)} = kTN_0\bar{\gamma}^- s^{-3(n'_p+1)}. \tag{3.8}$$

Coefficient $\bar{\gamma}^-$ is the function of H_{cm} and can be expressed as

$$\bar{\gamma}^- = \bar{\gamma}_1 - \bar{\gamma}_2 H_{cm} + \bar{\gamma}_3 H_{cm}^2, \tag{3.9}$$

where $\bar{\gamma}_l$ are constant quantities calculated in [1].

The term F_{IGR} from (3.1) has the form

$$F_{IGR} = -kT \ln Z_{IGR}, \tag{3.10}$$

where the expression for Z_{IGR} is given in (2.8). We point out that $Z_{n'_p+1}$ in (2.9) can be written as the product of partial partition function similar to the way it is done for the contribution from the region of the critical fluctuation regime. Then for contribution of the inverse Gauss regime to the free energy (3.10) (similar to the contribution of the boundary Gauss regime to the free energy at $T > T_c$) we can write

$$F_{IGR} = F_{TR}^{(-)} + F''. \tag{3.11}$$

Here

$$F_{TR}^{(-)} = -kTN_0 s^{-3(n'_p+1)} \sum_{m=1}^{m_0} s^{-3(m-1)} f_{n'_p+m}, \tag{3.12}$$

where for $f_{n'_p+m}$ we have (3.4) at $n = n'_p + m$,, and for F'' we obtain

$$F'' = -kTN \ln Z''. \tag{3.13}$$

Here

$$Z'' = 2^{(N_{n''+1}-1)/2} [Q(P_{n''})]^{N_{n''+1}} Z_{n''+1}, \tag{3.14}$$

where the quantity $n'' = n'_p + m_0$ is the number of a spin block structure and $n'' > n'_p$. For $Z_{n''+1}$ expression (2.9) is valid; in this expression n'_p should be substituted by n''.

It should be noted that for $n < n'_p$ the values of quantities x_n and y_n are close to their values in a fixed point ($x_n \approx x^*$, $y_n \approx y^*$) and for the region $n'_p < n \le n'_p + m_0$ their deviation from the fixed point should be taken into account. Consequently, contributions of (3.3) to the free energy $F_{CR}^{(-)}$ and those of (3.12) to $F_{TR}^{(-)}$ are essentially different although they have the same functional form.

In accordance with (3.11) the contribution to the free energy F_{IGR} contains two terms. To calculate the first term $F_{TR}^{(-)}$ from (3.12) one should find explicit dependence of $f_{n'_p+m}$ on m. We use solutions of RR

that considering (3.5) take the form

$$\begin{aligned} w_{n'_p+m} &= h_0 E_1^{m-1} \tilde{h} (\tilde{h}^2 + h_{cm}^2)^{-1/2}, \\ r_{n'_p+m} &= \beta\Phi(0) f_0 (-1 - H_{cm} E_2^{m-1} + \\ &\quad R^{(0)} f_0^{-1} \varphi_0^{-1/2} c_{2T} E_3^{m-1} H_{3m}), \\ u_{n'_p+m} &= (\beta\Phi(0))^2 \varphi_0 [1 + \Phi_f H_{cm} E_2^{m-1} + c_{2T} E_3^{m-1} H_{3m}]. \end{aligned} \tag{3.15}$$

Here $\Phi_f = f_0 \varphi_0^{-1/2} R_1^{(0)}$. Near Near the PTP the quantity H_{3m} from (3.5) is small and at $m \gg 1$ the quantity E_3^{m-1} tends to zero ($E_3 < 1$). Thus we neglect the terms proportional to $H_{3m} E_3^{m-1}$.

Taking into account (3.15) and the explicit form for x_n from [1] we find

$$x_{n'_p+m} = -\bar{x} E_2^{m-1} H_{cm} (1 - \Phi_f E_2^{m-1} H_{cm})^{-1/2} \tag{3.16}$$

where we denote $\bar{x} = f_0 \varphi_0^{-1/2} \sqrt{3}$. The absolute value of argument $x_{n'_p+m}$ increases with the increase of m and depends on the value of H_{cm}. It is easy to verify that quantity H_{cm} for each value of $\tilde{h}$ is determined uniquely by the parameter

$$\alpha_m = \tilde{h} / h_{cm}. \tag{3.17}$$

For small fields quantity H_{cm} converges to value $E_2^{-n_0}$ and with the increase of α_m tends to zero. As a result, $x_{n'_p+m}$ from (3.16) has different m dependence for large and small fields. The value $m_0 = 1$ alone leads to rather high absolute values of argument $x_{n'_p+m_0+1}$ which allows one to use the Gauss distribution of fluctuations for all $\rho_{\vec{k}}$ with $|\vec{k}| \in \mathcal{B}_{n'_p+2}$. So the contribution of the transition region of fluctuations to the free energy (3.12) contains only one term, specifically

$$F_{TR}^{(-)} = -kTN_0 f_{n'_p+1} \left(\tilde{h}^2 + h_{cm}^2 \right)^{\frac{d}{d+2}}, \tag{3.18}$$

where for $f_{n'_p+m}$ we have (3.4) at $n = n'_p + 1$.

We point out that for calculation of the contribution of the transition region to the free energy at $T < T_c$ we use the value $m_0 = 1$. There is no need to separate the transition region at $T < T_c$. Parameter m_0 here can take on values $m_0 < 1$ but the purpose of calculating free energy is to obtain an expression that would be valid for temperatures higher and lower than T_c. Thus following the scheme for calculating free energy at $T > T_c$ [1] for the region of temperatures $T < T_c$ we introduce a transition region as well. It does not diminish the generalization of our

study in any way but it does allow us to unify the calculation scheme. Now we can derive the last contribution to the free energy of the system F'' from (3.11). For that we first need to calculate expression for Z'' from (3.13). We have

$$Z'' = 2^{(N_{n'_p+2}-1)/2}[Q(P_{n'_p+1})]^{N_{n'_p+2}} Z_{n'_p+2} \tag{3.19}$$

where

$$Z_{n'_p+2} = \int (d\rho)^{N_{n'_p+2}} \exp\Big(h\sqrt{N}\rho_0 - \tfrac{1}{2}\textstyle\sum_{k\in\mathcal{B}_{n'_p+2}} d_{n'_p+2}(k)\rho_{\vec{k}}\rho_{-\vec{k}} - \\ - \tfrac{a_4^{(n'_p+2)}}{24} N^{-1}_{n'_p+2} \textstyle\sum_{\vec{k}_1,\ldots,\vec{k}_4} \rho_{\vec{k}_1}\cdots\rho_{\vec{k}_4}\delta_{\vec{k}_1+\ldots+\vec{k}_4}\Big). \tag{3.20}$$

For coefficients $d_{n'_p+2}$ and $a_4^{(n'_p+2)}$ we have

$$d_{n'_p+2}(k) = d_{n'_p+2}(0) + 2\beta\Phi(0)b^2k^2, \\ d_{n'_p+2}(0) = s^{-2(n'_p+2)} r_{n'_p+2}, \\ a_4^{(n'_p+2)} = s^{-4(n'_p+2)} u_{n'_p+2}. \tag{3.21}$$

4. Extracting of the macroscopic part of the order parameter

Let us consider explicit forms of coefficients $r_{n'_p+2}$ and $u_{n'_p+2}$ that are part of the expression for quantity $Z_{n'_p+2}$ from (3.20)

$$r_{n'_p+2} = \beta\Phi(0) f_0(-1 - H_{cm}E_2), \\ u_{n'_p+2} = (\beta\Phi(0))^2\varphi_0(1 - \Phi_f H_{cm}E_2). \tag{4.1}$$

The second of them, specifically $u_{n'_p+2}$, remains positive for any values of τ and h as a result the integral in (3.20) converges. In turn, coefficient $r_{n'_p+2}$ remains negative for all $T < T_c$. Therefore, it does not make sense to use the Gauss approximation here. The situation can be improved by introducing in (3.20) the substitution of variables

$$\rho_{\vec{k}} = \eta_{\vec{k}} + \sqrt{N}\delta_{\vec{k}}\sigma_-, \tag{4.2}$$

where σ_- is a constant quantity. The basis for shifting variable ρ_0 (whose average value is tied with the order parameter [11]) is the presence in the system of a spontaneous order parameter. In the presence of an external

field there exists an additional field-induced order parameter. Thus a similar substitution of variables also takes place at $T > T_c$ if the system is under an external field. As a result of (4.2) expression (3.20) would take the form

$$Z_{n'_p+2} = e^{NE_0(\sigma_-)} \int (d\eta)^{N_{n'_p+2}} \exp\Big[A'_0\sqrt{N}\eta_0 - -\frac{1}{2}\sum_{\vec{k}\in\mathcal{B}_{n'_p+2}} \bar{d}'(k)\times \\ \times \eta_{\vec{k}}\eta_{-\vec{k}} - -\frac{1}{6}\bar{b}' N^{-1/2}_{n'_p+2} \sum_{\vec{k}_1,\ldots,\vec{k}_3} \eta_{\vec{k}_1}\cdots\eta_{\vec{k}_3}\delta_{\vec{k}_1+\ldots+\vec{k}_3} - \\ - \frac{1}{24}\bar{a}'_4 N^{-1}_{n'_p+2} \sum_{\vec{k}_1,\ldots,\vec{k}_4} \eta_{\vec{k}_1}\ldots\eta_{\vec{k}_4}\delta_{\vec{k}_1+\ldots+\vec{k}_4}\Big]. \tag{4.3}$$

Here

$$E_0(\sigma_-) = h\sigma_- - \frac{1}{2}d_{n'_p+2}(0)\sigma_-^2 - \frac{1}{24}a_4^{(n'_p+2)} s_0^3 s^{3(n'_p+2)}\sigma_-^4 \tag{4.4}$$

and for coefficients A'_0, $\bar{d}'(k)$, $\bar{b}'$ and $\bar{a}'_4$ we have expressions

$$A'_0 = h - d_{n'_p+2}(0)\sigma_- - \tfrac{1}{6}a_4^{n'_p+2} s_0^3 s^{3(n'_p+2)}\sigma_-^3, \\ \bar{d}'(k) = \bar{d}'(0) + 2\beta\Phi(0)b^2k^2, \\ \bar{d}'(0) = s^{-2(n'_p+2)}(r_{n'_p+2} + \tfrac{1}{2}u_{n'_p+2}s_0^3 s^{(n'_p+2)}\sigma_-^2), \\ \bar{b}' = u_{n'_p+2}s_0^{3/2} s^{\frac{5}{2}(n'_p+2)}\sigma_-, \quad \bar{a}'_4 = u_{n'_p+2}s^{-4(n'_p+2)}. \tag{4.5}$$

Similar to [1], we find the shift quantity σ_- from the condition

$$\frac{\partial E_0(\sigma_-)}{\partial\sigma_-} = 0.$$

Considering (4.4) we obtain an equation

$$A'_0 = 0. \tag{4.6}$$

We will be looking for the solution in the form

$$\sigma_- = \sigma'_0 s^{-(n'_p+2)/2}. \tag{4.7}$$

For quantity σ'_0 we obtain a cubic equation

$$\sigma_0'^3 + p'\sigma'_0 + q' = 0 \tag{4.8}$$

where for coefficients p' and q' we have expressions

$$p' = 6s_0^{-3} r_{n'_p+2}/u_{n'_p+2},$$
$$q' = -6s_0^{-9/2} s^{5/2} \frac{h_0}{u_{n'_p+2}} \frac{\tilde{h}}{(\tilde{h}^2+h_{cm}^2)^{1/2}}. \quad (4.9)$$

In general, quantities p' and q' are functions of temperature and field. The form of solutions (4.8) depends on the sign of the discriminant

$$Q = (p'/3)^3 + (q'/2)^2. \quad (4.10)$$

As we know, for $Q > 0$ we have one real root and for a negative Q we have three real roots for equation (4.8). It is easy to verify that there exists a value $\tau = \tau_0$ ($\tau_0 < 0$) such that at $\tau_0 < \tau \leq 0$ quantity $Q > 0$, and at $\tau < \tau_0$ quantity $Q < 0$. The value of τ_0 can be found from the condition

$$Q = 0. \quad (4.11)$$

For all $\tau_0 < \tau \leq 0$ we look for the solution of equation (4.8) using the Kardano method $\sigma'_0 = A + B$ where

$$A = (-q'/2 + Q^{1/2})^{1/3}, \quad B = -(q'/2 + Q^{1/2})^{1/3}.$$

For the region $\tau < \tau_0$ we have three real solutions

$$\sigma'_1 = 2(-p'/3)^{1/2} \cos(\alpha_r/3),$$
$$\sigma'_{2,3} = -2(-p'/3)^{1/2} \cos\left(\tfrac{\alpha_r}{3} \pm \tfrac{\pi}{3}\right),$$

Where the angle α_r can be determined from the relation

$$\cos\alpha_r = -\frac{q'}{2(-p'/3)^{3/2}}.$$

The curve $\sigma'_0(\tau)$ is presented at fig.1 for the field value $h = 10^{-4}$. As the external field decreases the value of τ_0 tends to zero. This dependence is shown at Fig.2. It should be noted that similar to [1] we use here the following numerical values of parameters

$$s_0 = 2, \quad b/c = 0.3, \quad h_0 = 0.760. \quad (4.12)$$

With the decrease of the field the quantity σ_0 ($T > T_c$) decreases as well [1] and yields zero at $h = 0$, whereas for all $\tau < 0$ (at $h = 0$) equation (4.8) evidently has three real solutions, two of which are symmetric and the third one is zero. Quantity σ'_0 takes one of the three possible real values.

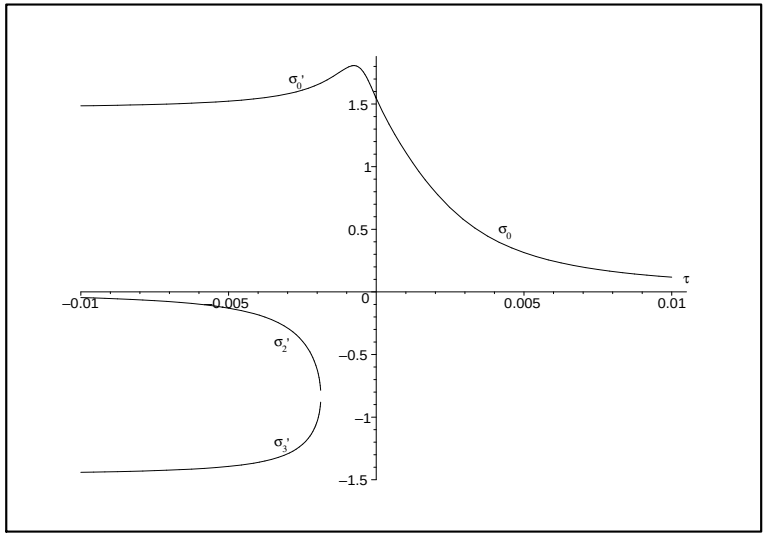


Figure 1. Temperature dependence of solutions σ'_0 of equation (4.8) (the region with $\tau < 0$), and quantity σ_0, found in [1] (the region with $\tau > 0$). The external field value is $h = 10^{-4}$

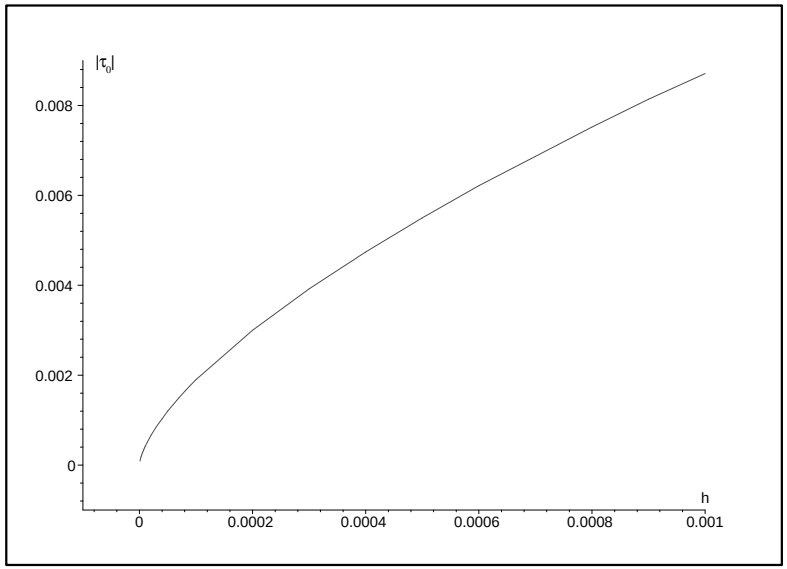


Figure 2. External field dependence of τ_0

We express renormalized coefficients (4.5) as

$$\bar{d}'(0) = s^{-2(n'_p+2)} r'_R,$$
$$\bar{b}' = s^{-3(n'_p+1)} v'_R,$$
$$\bar{a}' = s^{-4(n'_p+2)} u'_R, \qquad (4.13)$$

where

$$r'_R = r_{n'_p+2} + \tfrac{1}{2} u_{n'_p+2} s_0^3 \sigma_0'^2,$$
$$v'_R = u_{n'_p+2} s_0^{3/2} \sigma'_0, \quad u'_R = u_{n'_p+2}. \qquad (4.14)$$

Due to small values of coefficients v'_R and u'_R relative to r'_R in (4.3) we can use the Gauss approximation. Substituting the variables

$$\eta_{\vec{k}} = \rho_{\vec{k}} s^{(n'_p+2)} \sqrt{2/r'_R}$$

for $Z_{n'_p+2}$ from (4.3) we get

$$Z_{n'_p+2} = e^{NE_0(\sigma_-)} s^{(n'_p+2)N_{n'_p+2}} (2/r'_R)^{N_{n'_p+2}/2} Z_{IG} \qquad (4.15)$$

where

$$\begin{aligned} Z_{IG} \;=\; & \int (d\rho)^N_{n'_p+2} \exp\Big[- \sum_{\vec{k}\in\mathcal{B}_{n'_p+2}} \Big(1 + 2\beta\Phi(0) b^2 k^2 s^{2(n'_p+2)}/r'_R\Big) \times \\ & \times \rho_{\vec{k}} \rho_{-\vec{k}} - x_5 N^{-1/2}_{n'_p+2} \sum_{\vec{k}_1,\ldots,\vec{k}_3} \rho_{\vec{k}_1} \ldots \rho_{\vec{k}_3} \delta_{\vec{k}_1+\ldots+\vec{k}_3} - \\ & - x_6 N^{-1}_{n'_p+2} \sum_{\vec{k}_1,\ldots,\vec{k}_4} \rho_{\vec{k}_1,\ldots,\vec{k}_4} \delta_{\vec{k}_1+\ldots+\vec{k}_4} \Big]. \end{aligned} \qquad (4.16)$$

Quantities x_l have the form

$$x_5 = \frac{\sqrt{2}}{3} v'_R/(r'_R)^{3/2}, \quad x_6 = \frac{1}{6} u'_R/(r'_R)^2 \qquad (4.17)$$

and are small compared to unity, in particular near $T = T_c$. Therefore (4.16) reduces to the product of singular integrals

$$Z_{IG} = \prod_{\vec{k}\in\mathcal{B}_{n'_p+2}} \int d\rho_{\vec{k}} \exp\Big(- \Big(1 + 2\beta\Phi(0) \tfrac{b^2}{r'_R} k^2 s^{2(n'_p+2)}\Big) \rho_{\vec{k}} \rho_{-\vec{k}} \Big).$$

Contribution of (4.15) to the free energy has the form

$$\begin{aligned} F_{n'_p+2} = & -kTNE_0(\sigma_-) - \tfrac{1}{2} kTN_{n'_p+2} \ln\pi - \\ & - kTN_{n'_p+2}(n'_p+2)\ln s + \tfrac{1}{2} kTN_{n'_p+2} \ln r'_R + \\ & + \tfrac{1}{2} kT \textstyle\sum_{\vec{k}\in\mathcal{B}_{n'_p+2}} \ln\Big(1 + 2\beta\Phi(0) b^2 k^2 s^{2(n'_p+2)}/r'_R\Big). \end{aligned} \qquad (4.18)$$

Here

$$E_0(\sigma_-) = h e_0^{(-)} \Big(\tilde{h}^2 + h_{cm}^2\Big)^{\frac{1}{2(d+2)}} - e_2^{(-)} \Big(\tilde{h}^2 + h_{cm}^2\Big)^{\frac{d}{d+2}} \qquad (4.19)$$

where we have introduced the notation (similar to the one in the case of $T > T_c$ [1])

$$e_0^{(-)} = \sigma'_0 s^{-1/2},$$
$$e_2^{(-)} = \tfrac{1}{2} \sigma_0'^2 s^{-3} \left(r_{n'_p+2} + \tfrac{1}{12} u_{n'_p+2} s_0^3 \sigma_0'^2 \right). \qquad (4.20)$$

The sum over $\vec{k} \in \mathcal{B}_{n'_p+2}$ in (4.18) can be calculated by switching to integration using the technique from [1].

Now we write down the contribution of (3.19) to the free energy as the sum of two terms (see (4.15))

$$F'' = F_0^{(-)} + F_I \qquad (4.21)$$

where the term

$$F_0^{(-)} = -kTNE_0(\sigma_-) \qquad (4.22)$$

corresponds to the extracted macroscopic part of the order parameter, and for F_I we have

$$F_I = -kTN_{n'_p+2} f_I. \qquad (4.23)$$

Coefficient f_I has the form

$$\begin{aligned} f_I = & \tfrac{1}{2}\ln 2 - \tfrac{1}{4}\ln 3 + \ln s + \tfrac{1}{4}\ln u_{n'_p+1} - \tfrac{1}{2}\ln r'_R - \\ & - \tfrac{1}{2}\ln U(x_{n'_p+1}) - \tfrac{3}{8}\tfrac{1}{y^2_{n'_p+1}} - \tfrac{1}{2} f''_I. \end{aligned} \qquad (4.24)$$

Here

$$u_{n'_p+1} = (\beta\Phi(0))^2 \varphi_0 (1 - \Phi_f H_{cm}),$$
$$x_{n'_p+1} = -\bar{x} H_{cm} (1 - \Phi_f H_{cm})^{-1/2}, \qquad (4.25)$$

and for f_I'' we have

$$f_I'' = \ln(1+a_I^2) - \frac{2}{3} + \frac{2}{a_I^2} - \frac{2}{a_I^3}\arctan a_I \tag{4.26}$$

where

$$a_I = \frac{\pi}{s_0}\frac{b}{c}\left(\frac{2\beta\Phi(0)}{r_R'}\right)^{1/2}. \tag{4.27}$$

Let us add up the contributions to the system free energy that we have found near the PTP at $T < T_c$. In according with representation (3.1) we have several types of terms. Expression for F_0 from (3.2) contains only analytical dependence on field h. Since the first term in the expression for $F_{CR}^{(-)}$ depends on the field only analytically too, we can combine these contributions by introducing the notation

$$F_a = F_0 + (F_{CR}^{(-)} - F_{CR}^{c}). \tag{4.28}$$

An identical term is present in the expression for the free energy at $T > T_c$ [1]. It is the analytical part of the free energy. It has the form

$$F_a = -kTN\ln\cosh h - \tfrac{1}{2}N\Phi(0)\bar{\Phi} - kTN(\gamma_0 + \gamma_1\tau + \gamma_2\tau^2) \tag{4.29}$$

where coefficients γ_l are found also in [1].

Contributions to the free energy that come from $F_{CR}^{(c)}$(3.8), $F_{TR}^{(-)}$ (3.18) and F'' (4.22, 4.23) contain only non-analytical dependence on temperature τ and field h. Their sum can be expressed with two terms. One of them $F_0^{(-)}$ is related to the shift of variable ρ_0 and has the form (4.22). The other one $F_s^{(-)}$ is the sum of the remaining non-analytical contributions (the region of $T < T_c$)

$$F_s^{(-)} = F_{CR}^{(c)} + F_{TR}^{(-)} + F_I \tag{4.30}$$

and can be written as

$$F_s^{(-)} = -kTN\gamma_s^{(-)}\left(\tilde{h}^2 + h_{cm}^2\right)^{\frac{d}{d+2}} \tag{4.31}$$

where

$$\gamma_s^{(-)} = s_0^{-3}(f_{n_p'+1} - \bar{\gamma}^- + f_I/s^3). \tag{4.32}$$

So the free energy of the system can be written as

$$F = F_a + F_s^{(-)} + F_0^{(-)} \tag{4.33}$$

where F_a has the form (4.29), for $F_s^{(-)}$ we have (4.31) and the term $F_0^{(-)}$ from (4.22) takes on the form

$$F_0^{(-)} = -kTN\left(he_0^-\left(\tilde{h}^2 + h_{cm}^2\right)^{\frac{1}{2(d+2)}} - e_2^-\left(\tilde{h}^2 + h_{cm}^2\right)^{\frac{d}{d+2}}\right) \tag{4.34}$$

Free energy representations (3.1) and (4.33) are equivalent.

Comparing temperature regions above and below the T_c, we point out that expressions for the free energy are functionally similar. The main difference is that for $T < T_c$ the value of the exit point of the critical fluctuation regime is somewhat smaller than for $T > T_c$. This is caused by the presence in the system of a spontaneous order parameter at temperatures that are lower than T_c. For the same reason we have different temperature measurement scales for each temperature interval. They are tied as follows

$$\tau_1 = -\tilde{\tau}E_2^{n_0}. \tag{4.35}$$

Thus using the results of [1] we can write a general expression for the free energy

$$F = F_a + F_s + F_0 \tag{4.36}$$

where the analytical part F_a is common to both temperature intervals and the contributions of F_s and F_0 are given by expression

$$F_s = \begin{cases} F_s^{(-)}, & \texttt{at}\ T < T_c \\ F_s^{(+)}, & \texttt{at}\ T > T_c \end{cases} \tag{4.37}$$

and

$$F_0 = \begin{cases} F_0^{(-)}, & \texttt{at}\ T < T_c \\ F_0^{(+)}, & \texttt{at}\ T > T_c \end{cases} \tag{4.38}$$

respectively.

5. The order parameter of a spin system near the PTP under an external field

We use (4.36) to calculate temperature and field dependence of the system magnetization. For this we use the definition

$$M = -\frac{1}{N}\left(\frac{dF}{d\mathcal{H}}\right). \tag{5.1}$$

For convenience we express quantity M as the sum of three terms

$$M = M_a + M_s^{(\pm)} + M_0^{(\pm)} \tag{5.2}$$

that correspond to the contributions of different terms (4.36) and have the form

$$M_a = -\tfrac{1}{N}\left(\tfrac{dF_a}{d\mathcal{H}}\right)_T,$$
$$M_s^{(\pm)} = -\tfrac{1}{N}\left(\tfrac{dF_s}{d\mathcal{H}}\right)_T,$$
$$M_0^{(\pm)} = -\tfrac{1}{N}\left(\tfrac{dF_0}{d\mathcal{H}}\right)_T. \qquad (5.3)$$

Here the signs "+" and "−" indicate the regions the system is at (above and below the T_c respectively). The following calculations are done in detail for the temperature region of $T < T_c$.

According to (4.29) the first term in (5.2) which corresponds to the analytical part, has the form

$$M_a = \tanh h \approx h. \qquad (5.4)$$

For the region of $T < T_c$ quantity $M_s^{(-)}$ is calculated in accordance with (4.31) and can be expressed as

$$M_s^{(-)} = \left(\tilde{h}^2 + h_{cm}^2\right)^{\frac{1}{2(d+2)}} \times$$
$$\times \left(\tfrac{d\gamma_s^{(-)}}{dh'}(\tilde{h}^2 + h_{cm}^2)^{1/2} + \tfrac{6}{5}\gamma_s^{(-)}\tfrac{s_0^{3/2}}{h_0}\tfrac{\tilde{h}}{(\tilde{h}^2+h_{cm}^2)^{1/2}}\right) \qquad (5.5)$$

The derivative of $\gamma_s^{(-)}$ is calculated in accordance with (4.32) where an explicit expression for each term is known.

To calculate $M_0^{(-)}$ we use relation (4.34) which is valid for $T < T_c$. Here the derivatives of σ_0' lead to an expression that is identical with condition (4.6). Therefore when calculating $M_0^{(-)}$ we consider quantity σ_0' independent of the field

$$M_0^{(-)} = (\tilde{h}^2 + h_{cm}^2)^{\frac{1}{2(d+2)}}\left[e_0^{(-)}\left(1 + \tfrac{1}{5}\tfrac{\tilde{h}^2}{\tilde{h}^2+h_{cm}^2}\right) -\right.$$
$$\left. -\tfrac{6}{5}e_2^{(-)}s_0^{3/2}\tfrac{1}{h_0}\tfrac{\tilde{h}}{(\tilde{h}^2+h_{cm}^2)^{1/2}} - (\tilde{h}^2 + h_{cm}^2)^{1/2}\tfrac{de_2^{(-)}}{dh'}\right], \qquad (5.6)$$

Based on (5.2) we find the total contribution to the order parameter $M^{(-)}$ in the presence of an external field

$$M^{(-)} = \sigma_{00}^{(-)}\left(\tilde{h}^2 + h_{cm}^2\right)^{\frac{1}{2(d+2)}} \qquad (5.7)$$

where for coefficient $\sigma_{00}^{(-)}$ we have

$$\sigma_{00}^{(-)} = e_0^{(-)}\left(1 + \tfrac{1}{5}\tfrac{\tilde{h}^2}{\tilde{h}^2+h_{cm}^2}\right) + e_{00}^{(-)}\tfrac{\tilde{h}}{(\tilde{h}^2+h_{cm}^2)^{1/2}} + e_{02}^{(-)}. \qquad (5.8)$$

Here quantity $e_0^{(-)}$ is defined in (4.20) and for $e_{00}^{(-)}$ and $e_{02}^{(-)}$ we find expression

$$e_{00}^{(-)} = \tfrac{6}{5}\tfrac{s_0^{3/2}}{h_0}\left(\gamma_s^{(-)} - e_2^{(-)}\right),$$
$$e_{02}^{(-)} = \left(\tfrac{d\gamma_s^{(-)}}{dh'} - \tfrac{de_2^{(-)}}{dh'}\right)\left(\tilde{h}^2 + h_{cm}^2\right)^{1/2}. \qquad (5.9)$$

The first two terms on the right side of equality (5.8) depend only on variable α_m from (3.17). For amplitude $\sigma_{00}^{(-)}$ to be a function of only α_m, it is necessary for coefficient $e_{02}^{(-)}$ to be such a function. One can verify this by direct differentiation of quantities $\gamma_s^{(-)}$ and $e_2^{(-)}$. We get

$$\frac{de_2^{(-)}}{dh'} = -\frac{s_0^{3/2}}{h_0}q_s'\sigma_0'^2\left(1 + \frac{q_l}{12}\sigma_0'^2\right)\left(\tilde{h}^2 + h_{cm}^2\right)^{-1/2} \qquad (5.10)$$

where

$$q_s' = -\tfrac{E_2}{2p_0}\beta\Phi(0)f_0 s^{-3}H_{cm}\tfrac{\alpha_m}{(1+\alpha_m^2)^{1/2}}, \qquad q_l = \Phi_f s_0^3\varphi_0 f_0^{-1}\beta\Phi(0)$$

And for the derivative of $\gamma_s^{(-)}$ from (4.32) we find

$$\frac{d\gamma_s^{(-)}}{dh'} = \frac{s_0^{d/2}}{h_0}f_{\delta_1}\left(\tilde{h}^2 + h_{cm}^2\right)^{-1/2} \qquad (5.11)$$

where

$$f_{\delta_1} = s_0^{-3}\left(f_p^- + \gamma_p^- + s^{-3}f_{Iv}\right).$$

Quantities r_p^-, γ_p^-, f_{Iv} depend only on α_m and are presented in Appendix A. The final explicit expression for the critical magnetization amplitude $\sigma_{00}^{(-)}$ in the low-temperature region can be written as

$$\sigma_{00}^{(-)} = e_0^{(-)}\left(1 + \frac{1}{5}\frac{\alpha_m^2}{1+\alpha_m^2}\right) + e_{00}^{(-)}\frac{\alpha_m}{(1+\alpha_m^2)^{1/2}} + e_{02}^{(-)} \qquad (5.12)$$

where for $e_{02}^{(-)}$ we have

$$e_{02}^{(-)} = \frac{s_0^{3/2}}{h_0}\left(f_{\delta_1} + q_s'\sigma_0'^2\left(1 + \frac{q_l}{12}\sigma_0'^2\right)\right) \qquad (5.13)$$

Based on the results of [1] we can in a similar way calculate magnetization at $T > T_c$. We obtain an expression

$$M^{(+)} = \sigma_{00}^{(+)}\left(\tilde{h}^2 + h_c^2\right)^{\frac{1}{2(d+2)}}, \qquad (5.14)$$

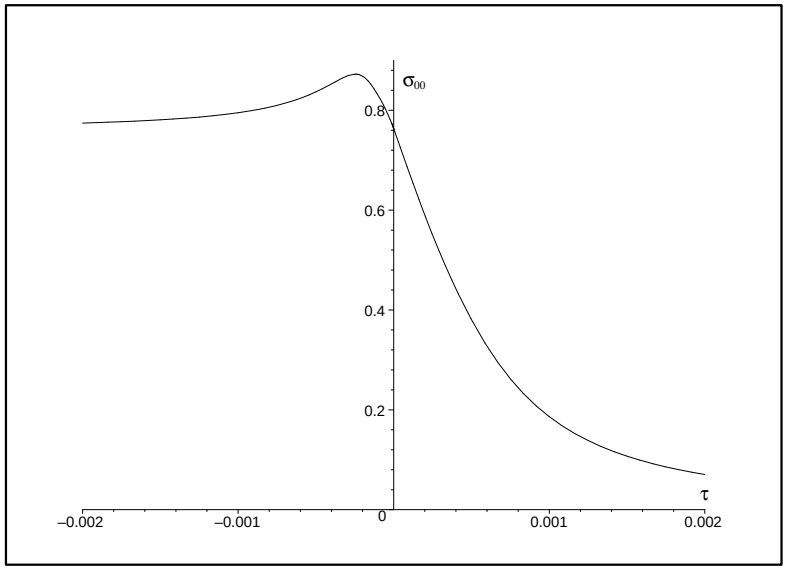


Figure 3. Temperature dependence of the critical magnetization amplitude σ_{00}. External field value $h = 10^{-5}$

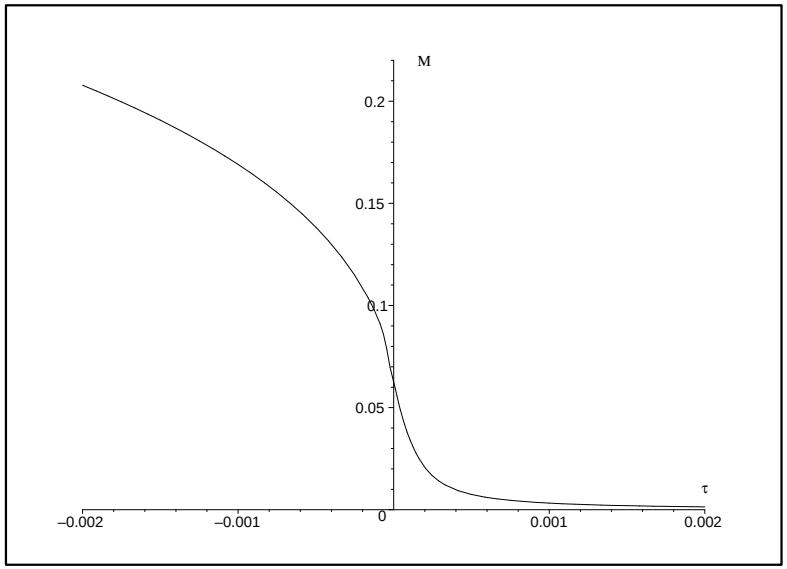


Figure 4. Magnetization of the spin system versus reduced temperature τ. External field value $h = 10^{-6}$

where $\sigma_{00}^{(+)}$ depends on variable α ($\alpha = h/h_c$) only. When passing the point of $T = T_c$ the critical amplitude $\sigma_{00}^{(-)}$ gradually turns into $\sigma_{00}^{(+)}$ which has the same sense but now for the region with $T > T_c$. This allows us to construct a general expression for magnetization of the system near the phase transition point both for the region with $T > T_c$ and that with $T < T_c$

$$M = \sigma_{00}\left(\tilde{h}^2 + h_{CR}^2\right)^{\frac{1}{2(d+2)}}, \tag{5.15}$$

where the critical amplitude σ_{00} is given by expression

$$\sigma_{00} = \begin{cases} \sigma_{00}^{(-)}, & \texttt{at } T < T_c \\ \sigma_{00}^{(+)}, & \texttt{at } T > T_c \end{cases} . \tag{5.16}$$

Quantity h_{CR} equals h_{cm} at temperatures lower than T_c and equals h_c at temperatures higher than T_c. At Fig. 3 one can see a graph of critical amplitude of the order parameter for both temperature intervals. Magnetization of the system depending on reduced temperature τ is shown at Fig. 4.

Therefore, equation of state for a 3D Ising-like system in the presence of a field is described by expression (5.15). For convenience we will be referring to it as the crossover equation of state. We use the term "crossover" here for the reason that expression (5.15) allows for a natural transition to cases when one of the variables (temperature or field) determines the behavior of the order parameter.

For small values of the field we have

$$M = M_{0\tau}|\tau|^{\beta} \tag{5.17}$$

Where quantity $M_{0\tau}$ can be written in the form

$$M_{0\tau} = \begin{cases} \sigma_{00}^{(-)}(1+\alpha_m^2)^{\frac{1}{2(d+2)}}\left(\frac{c_{1k}}{f_0}E_2^{n_0}\right)^{\frac{p_0}{d+2}}, & T < T_c \\ \sigma_{00}^{(+)}(1+\alpha^2)^{\frac{1}{2(d+2)}}\left(\frac{c_{1k}}{f_0}\right)^{\frac{p_0}{d+2}}, & T > T_c \end{cases} , \tag{5.18}$$

And the critical exponent $\beta = p_0/(d+2) = 0.302$. Evidently, in the absence of an external field the order parameter appears at the point $T = T_c$ ($\tau = 0$) which can be seen at Fig. 5. Here the results are presented for different values of the ratio of the potential range to the lattice constant [11]. In particular, $b/c = b_I = 0.2887$ corresponds to the nearest neighbor approximation, $b/c = b_{II} = 0.3379$ applies when we take into account interaction of the second neighbors, $b/c = b_{III} = 0.3584$ corresponds to interaction of the third neighbors. In addition, we have made some

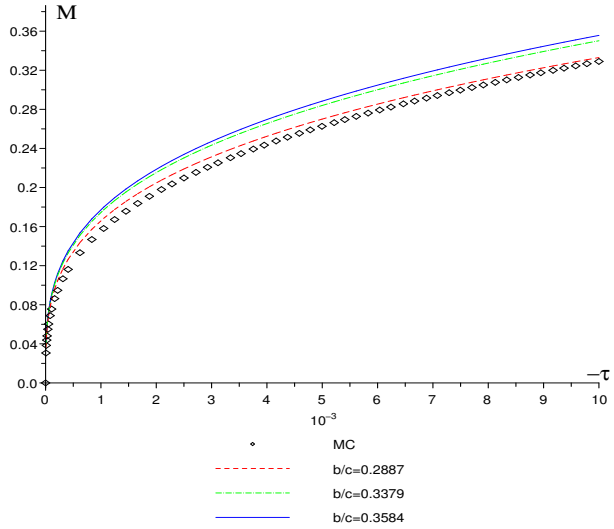


Figure 5. Magnetization of the spin system in the absence of an external field. MC are the results of work [12]

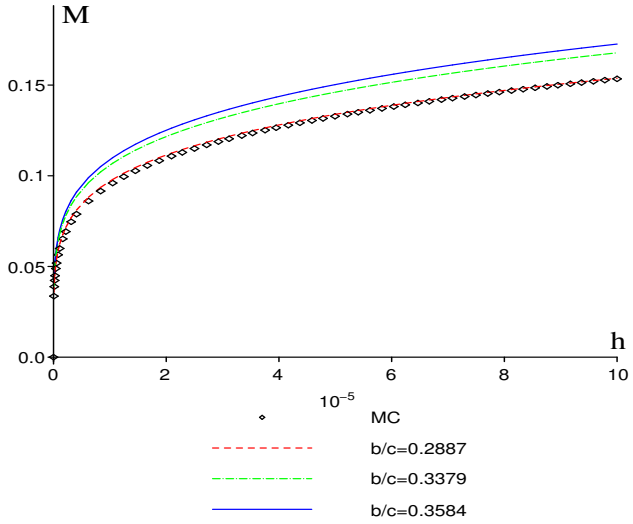


Figure 6. Field dependence of the order parameter of the spin system at the point $T = T_c$. MC are the results of work [12]

comparisons with the corresponding results that were obtained in [12] by numerical methods.

In cases when external field is the determining variable, equation of state is written

$$M = M_{0h} h^{1/\delta} \tag{5.19}$$

where for the scaling function M_{0h} we have

$$M_{0h} = \sigma_{00}^{(-)} \Big(1 + \big(\frac{h_{CR}}{\tilde{h}}\big)^2\Big)^{\frac{1}{2(d+2)}} \Big(\frac{s_0^{3/2}}{h_0}\Big)^{\frac{1}{d+2}} \tag{5.20}$$

And the critical exponent $\delta = d + 2 = 5$. Field dependence of the order parameter at temperature $T = T_c$ ($\tau = 0$) is shown at Fig. 6. These results are obtained for different values of the potential range and compared to the results of work [12].

As we have previously noted, the critical amplitude of the equation of state (5.15) depends only on quantities α_m ($T < T_c$) or α ($T > T_c$). But in turn, they are functions of the microscopic parameters of the hamiltonian. Notably, they depend on quantity s_0 that characterizes the Fourier transform of the interaction potential

$$\alpha_m = (s_0^{3/2}/h_0)\Big(\frac{f_0}{c_{1k}} E_2^{-n_0}\Big)^{p_0} z^{-p_0}, \tag{5.21}$$

$$\alpha = (s_0^{3/2}/h_0)\Big(\frac{f_0}{c_{1k}}\Big)^{p_0} z^{-p_0}. \tag{5.22}$$

Here variable z

$$z = \frac{\tau}{h^{1/p_0}} \tag{5.23}$$

is a certain ratio of the initial temperature-to-field values. For this reason dependence of the critical amplitude $\sigma_{00}^{(-)}$ or the scaling functions $M_{0\tau}$ and M_{0h} specifically on the variable z is more common. A similar variable $x = \tau/M^{1/\beta}$ is used in the equation of state that was proposed by Widom [13]. But it contains an order parameter M which from the standpoint of the microscopic approach has to be derived, not introduced from the "outside". Thus, using variable z is more natural than using variable x, although at small and large values of τ these quantities are equivalent. Dependence of the scaling function of the equation of state (5.19) on variable z is shown at Fig. 7.

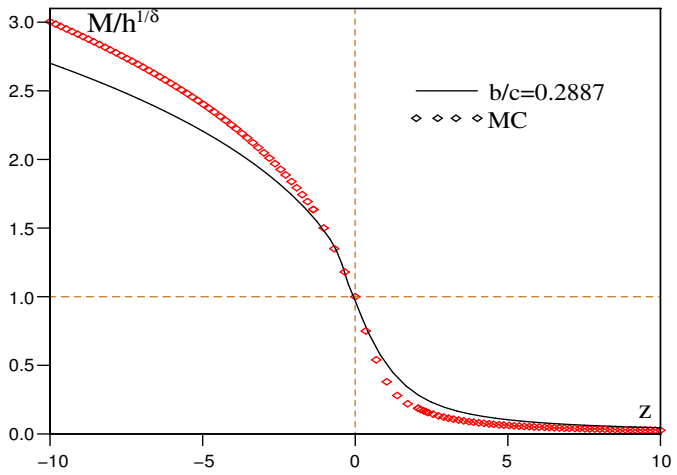


Figure 7. Scaling function of the equation of state (5.7) versus z. MC are the results of work [12]

6. Conclusions

In this work we have found an explicit expression for the free energy of a 3D Ising-like system near the phase transition point (PTP) for temperatures $T < T_c$ as a function of temperature and field. Based on the method proposed in [1], the free energy is expressed as the sum of contributions coming from different regions of fluctuation of the order parameter. In particular, contributions come from the region of the critical fluctuation regime (CR), in which general recurrence relations (RR) for the coefficient of block structures can be substituted by their approximated (linearized near a fixed point) solutions, as well as from the region of the inverse Gauss regime (IGR) that is characterized by the Gauss distribution of fluctuations. Using [1] we have proposed an expression for F of a 3D Ising-like system, which describes critical behavior at temperatures that are higher and lower than T_c.

By direct differentiation of free energy F with respect to external field we have obtained an explicit expression for magnetization of the system as a function of temperature and field. Expression (5.15) is a new form of the equation of state. We have referred to this form of the equation of state as the crossover, since in boundary cases it turns

into the well known representations of this equation. We have found the scaling function of the equation of state in terms of its conventional representation.

References

1. Kozlovskii M.P., Condens. Matter Phys. **12**, 130-140,(2009)
2. Yukhnovskii I.R. Phase Transition of the Second Order. Collective Variables Method (Singapore: World Scientific), 1987.
3. Kozlovskii M.P., Pylyuk I.V., Prytula O.O., Phys.Rev.B. **73** 174406 (2006).
4. Kozlovskii M.P.,Pylyuk I.V., Prytula O.O., Physica A. **369**, 562-576 (2006).
5. Kozlovskii M.P., Pylyuk I.V., Prytula O.O.,Nucl.Phys.B.**753**,242-251 (2006).
6. Guida R., Zinn-Justin J., Nucl. Phys.B **489**(1977) p.626-652.
7. Kozlovskii M.P., Phase Transitions **80**,3-9 (2007).
8. Kozlovskii M.P., Analytical method for description of the critical behaviour of 3D Ising-like model in an external field (Preprint of the Institute for Condensed Matter Physics, ICMP-08-10U, Lviv 2008, 85p.)(in Ukrainian).
9. Kadanoff L.P., Physics. **2**, 263-273 (1966).
10. Kozlovskii M.P., Condens. Matter Phys.**8**,473-506 (2005).
11. Yuchnovskii I.R., Kozlovskii M.P., Pylyuk I.V., Microscopic Theory of Phase Transition in the three dimensional systems (Lviv: Eurosvit, 2001, 592p.)(in Ukrainian).
12. Engels J., Fromme L., Seniuch M. Nucl. Phys. B **655** (2003) p. 277-299.
13. Widom B.J. Chem.Phys. **43**,p.3898 (1965).

Appendix A

The quantities r_p^-, γ_p^-, f_{Iv} depend on α' only

$$f_p^- = \frac{1}{2} r'_p g_p^- \left(1 - 9/y_{n'_p}^2\right) - \frac{1}{2} g_{p+1}^- U(x_{n'_p+1}),$$
$$\gamma_p^- = H_{cmd}\left(-\gamma_2 + 2\gamma_3 H_{cm}\right),$$
$$f_{Iv} = \frac{\varphi_0 \Phi_f}{4u_{n'_p+1}} H_{cmd}\left(\beta\Phi(0)\right)^2 - \frac{1}{2}\left(g_R^-/r'_R + g_{aI}\cdot a_{Ig}\right) +$$
$$+ g_{p+1}^- \left(\frac{3}{4}\frac{r'_{p+1}}{y_{n'_p+1}^2} - \frac{1}{2}\frac{U'(x_{n'_p+1})}{U(x_{n'_p+1})}\right)$$

To reduce the expression we used the following notations

$$r'_{p+m} = \frac{U'(x_{n'_p+m})}{U(x_{n'_p+m})} - \frac{1}{2}\frac{\varphi'(x_{n'_p+m})}{\varphi(x_{n'_p+m})},$$
$$H_{cmd} = \frac{H_{cm}}{p_0}\frac{\alpha_m}{(1+\alpha_m^2)^{1/2}},$$
$$g_p^- = \frac{\bar{x}}{E_2} H_{cmd}\left(1 - \Phi_f \frac{H_{cm}}{E_2}\right)^{-1/2}\left(1 + \frac{\Phi_f H_{cm}}{2E_2}\left(1 - \Phi_f\frac{H_{cm}}{E_2}\right)^{-1}\right),$$
$$g_{p+1}^- = \bar{x} H_{cmd}\left(1 - \Phi_f H_{cm}\right)^{-1/2}\left(1 + \frac{1}{2}\Phi_f H_{cm}\left(1 - \Phi_f H_{cm}\right)^{-1}\right)$$

and also

$$g_{aI} = -\frac{a_I g_R^-}{2r'_R},$$
$$a_{Ig} = \frac{2a_I}{1+a_I^2} - \frac{4}{a_I^3} + \frac{6}{a_I^4} arctg\, a_I - \frac{2}{a_I^3}\frac{1}{1+a_I^2}$$

CONDENSED MATTER PHYSICS

The journal **Condensed Matter Physics** is founded in 1993 and published by Institute for Condensed Matter Physics of the National Academy of Sciences of Ukraine.

AIMS AND SCOPE: The journal **Condensed Matter Physics** contains research and review articles in the field of statistical mechanics and condensed matter theory. The main attention is paid to physics of solid, liquid and amorphous systems, phase equilibria and phase transitions, thermal, structural, electric, magnetic and optical properties of condensed matter. Condensed Matter Physics is published quarterly.

ABSTRACTED/INDEXED IN:

- Chemical Abstract Service, Current Contents/Physical, Chemical&Earth Sciences
- ISI Science Citation Index-Expanded, ISI Alrting Services
- INSPEC
- Elsevier Bibliographic Databases (EMBASE, EMNursing, Compendex, GEOBASE, Scopus)
- "Referativnyi Zhurnal"
- "Dzherelo"

EDITOR IN CHIEF: Ihor Yukhnovskii

EDITORIAL BOARD: T. Arimitsu, *Tsukuba*; J.-P. Badiali, *Paris*; B. Berche, *Nancy*; T. Bryk, *Lviv*; J.-M. Caillol, *Orsay*; C. von Ferber, *Freiburg*; R. Folk, *Linz*; D. Henderson, *Provo*; F. Hirata, *Okazaki*; Yu. Holovatch, *Lviv*; M. Holovko, *Lviv*; O. Ivankiv, *Lviv*; W. Janke, *Leipzig*; M. Korynevskii, *Lviv*; Yu. Kozitsky, *Lublin*; M. Kozlovskii, *Lviv*; H. Krienke, *Regensburg*; R. Levitskii, *Lviv*; V. Morozov, *Moscow*; I. Mryglod, *Lviv*; O. Patsahan (Assistant Editor), *Lviv*; N. Plakida, *Dubna*; G. Röpke, *Rostock*; I. Stasyuk (Associate Editor), *Lviv*; M. Tokarchuk, *Lviv*; I. Vakarchuk, *Lviv*; M. Vavrukh, *Lviv*; A. Zagorodny, *Kyiv*

CONTACT INFORMATION:
Institute for Condensed Matter Physics
of the National Academy of Sciences of Ukraine
1 Svientsitskii Str., 79011 Lviv, Ukraine
Tel: +38(032)2760908; Fax: +38(032)2761978
E-mail: cmp@icmp.lviv.ua http://www.icmp.lviv.ua