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Особливості статистики рівноважного електромагнітного випромінювання

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# Specifics of Equilibrium electromagnetic radiation statistics

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**Особливості статистики рівноважного електромагнітного випромінювання**

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**Анотація.** Розглянута можливість розрахунку густини спектру випромінювання чорного тіла виходячи з класичних фізичних уявлень. Показано, що розгляд енергії електромагнітного випромінювання як сукупності енергій стоячих хвиль в об'ємі, обмеженому дзеркальними стінками, дозволяє розрахувати спектр випромінювання чорного тіла, при використанні статистичної гіпотези максимальної ймовірності рівноважного стану. З точки зору квантової фізики це дуже цікавий результат, оскільки існування сталої Планка доведено в рамках класичної фізики. Але класичний розв'язок все ж виглядає більш простим, так як не вносить нульових коливань вакууму, тобто безмежної енергії в скінченому об'ємі.

**Specifics of equilibrium electromagnetic radiation statistics**

V. Tataryn

**Abstract.** The possibility of calculating the black body radiation spectrum density proceeding from the classical physical ideas is considered. The consideration of electromagnetic radiation energy as a set of standing wave energies in the volume restricted by mirror walls is shown to enable calculating the black body radiation spectrum using the equilibrium state maximum probability statistical hypothesis. It is a very interesting result from the point of view of quantum physics since the existence of Planck's constant is proved within the framework of classical physics. But the classical solution looks simpler as it doesn't introduce zero oscillation of the vacuum, that is, infinite energy in the definite volume.

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## 1. Introduction

The problem of calculating the absolutely black body radiation spectrum was first solved within the scope of quantum physics over 100 years ago by Planck [1]. Later on the same problem was solved by Einstein [2] and Bose [3], also on the basis of quantum ideas. Besides, each new approach to this problem stimulated the development of physical theory as it cast a new light on the problem. However all these solutions have the same deficiency consisting in the introduction of new statistical hypotheses, which would not have been bad if these hypotheses had been necessary and provided better understanding of the nature. But when any hypothesis is introduced only to overcome so called ultraviolet catastrophe and finally results in even greater problems the issue of the necessity of its introduction arises. In such cases the reference to the experimental confirmation of Planck formula is not a convincing argument [4] as the legitimacy of Ptolemy epicycle system with its basic hypotheses could also be confirmed the same way.

If we elaborate upon the results of quantum theory the first question arising concerns the very name of the theory - what is a quantum? A quantum itself or certain amount of energy doesn't induce any questions. It can be both imagined and measured. Some questions begin to arise when we examine the popular formula of quantum physics -

$$E = h \cdot \nu \ .$$

Here $E$ stands for energy, $h$ - constant, but what can be said about $\nu$? This value has the dimension of frequency, but it cannot be frequency, as the photon is postulated by the point object. The point object cannot have an allocated frequency. However the set of such photons behaves as a classically described electromagnetic field with certain given frequency. This is an obvious contradiction.

Such a primitive idea of photon can be considered to follow from the so-called "old quantum mechanics" while a new one based on Schrodinger equation provides answers to these uncomfortable questions. However, citing L.I. Mandelstam we have got "... issues of spectral line shift and expansion and, finally, the issue of radiation. Dirac provided their approximate and non-rigorous examination - there are diverging series etc. I think it depends not on the gist of matter, as, for example, in the question of infinite energy where the deficiencies of the theory are revealed, but on poor approximation methods in calculations." [5] Actually addition term $h\nu/2$ in harmonic oscillator mean energy successfully produces ultraviolet catastrophe at the new level. Admittedly, L.I. Mandelstam made his comment in 1935. Since then several generations of theoreticians has changed, a new branch of knowledge - quantum electrodynamics - appeared, but: "It leads to many right answers but also to some horrendously wrong ones that theorists simply ignore; but it is now known that virtually all the right answers could have been found without, while some of the wrong ones were *caused* by, field quantization." [6]

Summarizing the aforesaid, quantum theory was brought to life by the need to explain the radiation spectrum of the absolutely black body. It could be explained only by introducing new statistical hypotheses. However these very hypotheses brought to life some contradictory ideas. Thus the problem of explaining the radiation spectrum of the absolutely black body is still vital.

## 2. Black body radiation spectrum calculation.

Let us examine the model of the black body in the form of a cavity with still opaque walls with steady temperature. The atoms of the walls continuously exchange energy both among themselves and with radiation in the cavity. It is quite clear that such energy exchange will result in the maximum entropy state as the most probable one. Afterwards the spectral density of radiation energy in the cavity will be definite and it will be easy to prove that it won't depend on the material the walls are made of.

Thus, the radiation in such cavity will be equilibrium, and its properties, namely radiant energy density, its distribution over the frequency spectrum, the direction of propagation and radiation polarization depend neither on the form of cavity walls nor on the material they are made of.

Then the same radiation can exist in the cavity with mirror walls that fully reflect any radiation. Hence these walls can be considered infinitely solid, that is, still. Then an infinitely small (compared to the size of the cavity) black particle of dust is placed in the cavity ensuring the energy exchange of oscillations with different frequencies - radiation equilibrium. Let us use the method suggested by Planck to calculate the radiation energy density at each frequency [7]. It is known that in such cavity with volume $V$ at each frequency $\nu$ within the $d\nu$ range the number of standing waves $A^\nu$ is:

$$A^\nu = 8 \cdot \pi \cdot V \cdot \frac{\nu^2 \cdot d\nu}{c^3} . \qquad (1)$$

Let us assume that each standing wave (mode) has energy $\varepsilon_i^\nu$. Then



the total energy of modes within the interval will make

$$dE^\nu = \sum_i \varepsilon_i^\nu \cdot n_i^\nu ,$$

and total radiation energy in the cavity

$$E = \sum_\nu dE^\nu , \qquad (2)$$

where $n_i^\nu$ – number of modes with energy $\varepsilon_i^\nu$ . In this case the number of possible distributions of radiation energy $W$ by modes will make:

$$W = \prod_\nu \frac{A^\nu!}{\prod_i \cdot n_i^\nu!} .$$

Moreover,

$$A^\nu = \sum_i n_i^\nu . \qquad (3)$$

So far these correlations concerned any radiation that can exist in a mirror cavity. In order to make it equilibrium thermal radiation there should be maximum logarithm of the number of states available at the given energy, namely

$$\delta \ln W = -\sum_\nu \sum_i (1 + \ln n_i^\nu) \delta n_i^\nu = 0 ,$$

with additional conditions $\delta E = 0$ and $\delta A^\nu = 0$. We are going to solve this problem as it was solved by Planck, using the Lagrange method of undetermined factors, i.e.

$$\sum_\nu \sum_i (1 + \ln n_i^\nu + \lambda^\nu + \frac{1}{\beta} \delta n_i^\nu = 0 .$$

Since $\delta n_i^\nu$ variation is an independent value the expression in the brackets should be equal to zero. Thus we obtain

$$n_i^\nu = B^\nu \exp(-\frac{1}{\beta} \varepsilon_i^\nu) ,$$

where $B^\nu = \exp(-1 - \lambda^\nu)$ , i.e. normalizing factor the value of which depends both on the frequency and on some other parameter $\lambda$. Substituting the previous expression in ( 3 ) we will find that

$$A^\nu = \sum_i n_i^\nu = B^\nu \sum_i \exp(-\frac{\varepsilon_i^\nu}{\beta}) .$$

Hence

$$B^\nu = \frac{A^\nu}{\sum_i \exp(-\frac{\varepsilon_i^\nu}{\beta})}$$

and

$$n_i^\nu = 8\pi V \frac{\nu^2 d\nu}{c^3} \cdot \frac{\exp(-\frac{\varepsilon_i^\nu}{\beta})}{\sum_i \exp(-\frac{\varepsilon_i^\nu}{\beta})} \ .$$

In this case the total energy of the modes within this spectrum interval will make

$$dE_i^\nu = 8\pi V \frac{\nu^2 d\nu}{c^3} \cdot \frac{\sum_i \varepsilon_i^\nu \exp(-\frac{\varepsilon_i^\nu}{\beta})}{\sum_i \exp(-\frac{\varepsilon_i^\nu}{\beta})} \ . \tag{4}$$

The last fraction in the expression ( 4 ) is a formula for calculating the mean energy of the mode $\overline{\varepsilon^\nu}$ within this spectrum interval since the value of the expression

$$\frac{\exp(-\frac{\varepsilon_i^\nu}{\beta})}{\sum_i \exp(-\frac{\varepsilon_i^\nu}{\beta})} = p(\varepsilon) \tag{5}$$

is a probability that $i$ mode possesses given energy. Besides, the very value of this probability depends not on the mode number, but on the value of mode energy. That is why we can find the mean energy of the mode $\overline{\varepsilon^\nu}$ as follows:

$$\overline{\varepsilon^\nu} = \frac{\int_{E^\nu_{min}}^{E^\nu_{max}} \varepsilon \exp(-\frac{\varepsilon}{\beta}) d\varepsilon}{\int_{E^\nu_{min}}^{E^\nu_{max}} \exp(-\frac{\varepsilon}{\beta}) d\varepsilon} \ , \tag{6}$$

where $E^\nu_{max}$ - maximum possible mode energy at the given frequency, $E^\nu_{min}$ - minimum possible mode energy at the given frequency. The very values of these energies comprise the specifics of equilibrium electromagnetic radiation statistics. The value of minimum possible energy is beyond any doubts since the energy of each mode is of quadratic dependence on the amplitude of given oscillation. Thus the minimum value of mode energy cannot be negative. Hence

$$E^\nu_{min} = 0 \ .$$

The value of maximum possible mode energy is not so obvious. If we come back to the appearance of probability of existing a mode with such energy $p(\varepsilon)$ (expression ( 5 )), we will see that zero probability corresponds to the infinite energy. It means that the interval of existence of such a mode will be equal to zero with any observation time. The oscillations of zero length don't possess any definite frequency. Thus the existence of modes with infinite energy is impossible at the found probability of mode energy because the oscillation processes are considered.

In this respect classical and quantum approaches differ considerably. Bose [3] presented the same computation being of the opinion that radiation is represented by the set of quanta. Quantum hypothesis is characterized by presenting radiation as a set of quanta with certain energies and certain ascribed frequencies. Besides, there are no reasons preventing all quanta from existing at some single frequency as a result of some fluctuation - the quanta don't interact directly. That is why in quantum theory one can assume that $E^\nu_{max}$ is equal to the total energy of radiation in the cavity and direct this value into infinity as the black body can be of any volume. In the classical theory these values depend on possible speed of amplitude change, or energy of any given mode. Specifically, no mode can appear or disappear because it exists within the whole volume of the black body. The very process of the change of energy of the given mode can be presented as amplitude modulation of some oscillation. The frequency of such modulation cannot be infinite at least owing to some definite speed of propagation of electromagnetic waves. Moreover, totally uniform spectrum of resulting oscillation corresponds to infinite modulation frequencies. The presence of a black particle of dust causes it to radiate rather than absorb in the absence of oscillations at certain frequency. That is why the energy of the mode at any given frequency cannot be infinite. It must depend on the frequency and possible speed of energy change of the given mode. Besides, the change of volume of the black body will not necessarily lead to the change of $E^\nu_{max}$, since such a change results in the change of speed of possible change of energy of the given mode.

Hence we come to the conclusion that the value of maximum possible energy of the mode at the given frequency is finite and possibly dependent on the frequency of existence of the given mode. Taking into account the fact that the upper integral in the expression ( 6 )is a derivative of the lower one we can use derivation by $1/\beta$ to find that

$$\overline{\varepsilon^\nu} = \frac{E^\nu_{max} \exp(-\frac{E^\nu_{max}}{\beta})}{1 - \exp(-\frac{E^\nu_{max}}{\beta})} \ .$$

Substituting the latter expression in (4), we find that

$$dE_i^\nu = 8\pi V \frac{\nu^2 d\nu}{c^3} \cdot \frac{E_{max}^\nu \exp(-\frac{E_{max}^\nu}{\beta})}{1-\exp(-\frac{E_{max}^\nu}{\beta})} \ . \qquad (7)$$

On the other hand, according to Wien's law in its most general form it can be proved that the density of spectral radiation $\rho(\nu)$ of the black body must be the function of two arguments looking as follows [7]:

$$\rho(\nu) = \nu^3 \cdot f(\frac{\nu}{T}) \ ,$$

where $f(\frac{\nu}{T})$ - certain function of one argument, and $T$ - temperature. If we compare the latter expression with our expression ( 7 ), we will see that $E_{max}^\nu \equiv L\nu$, and $\beta \equiv MT$ , where $L$ and $M$ are certain constants. Besides, our expression will coincide with Planck formula [1]

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3} \cdot \frac{h\nu}{\exp(\frac{h\nu}{kT}) - 1}$$

but only when $L \equiv h$ , where $h$ - Planck's constant and $M \equiv k$, where $k$ - Boltzmann's constant.

## 3. Analysis of the results.

We could prove the existence of Planck's constant within the framework of classical physics when examining the non-quantum electromagnetic field. Besides, we have got a new rather interesting result - the value of the oscillation energy of the given mode cannot exceed $h\nu$ if this mode interacts with the black particle of dust, namely the substance absorbing the radiation of all frequencies. That is why one can assume that it is the interaction with the substance that restricts the maximum possible value of the mode energy. This means that the substance consists of non-linear oscillators behaving as harmonic oscillators at low amplitudes and changing suddenly their properties at the oscillation amplitudes with the area of oscillation trajectory exceeding $h$. Such an approach allows us to interpret both traditionally classical effects, such as interference and diffraction of electromagnetic waves, and purely quantum effects, such as photoeffect easily. If one assumes that oscillation absorption takes place only when the energy transmitted by it to the oscillator in the substance reaches $h\nu$, Einstein formula for photo effect results from it without any extra hypotheses.

However the result looks somewhat strange when taking into account that it refers to a cavity of any size. Namely, in a bigger cavity there are more modes, and correspondingly their amplitudes are smaller than those in smaller cavities. The black particle of dust is placed only in one point and it is not clear how it can know the size of the whole cavity or the energy of the modes interacting with it. It seems to lead to the ideology of quantum mechanics where instantaneous action at a distance is acceptable. However, we are considering the classical approach, and in this case we must consider both the interference of neighboring modes and the fact that oscillators of the black particle of dust are of finite quality factor, i.e. they can interact with several modes existing in the cavity simultaneously.



In this context the Einstein's attempt to perform statistical examination of the movement of resonator in the radiation field is worth mentioning [8]. The paper dealt with the plate reflecting electromagnetic radiation at $\nu$ frequency within certain range $d\nu$. On examining Brownian movement of the plate Einstein got Rayleigh-Jeans' radiation law. In other words, the mean square of electromagnetic energy fluctuation makes

$$\overline{\varepsilon^2} = \frac{c^3 E^2}{8\pi\nu^2 d\nu}$$

while according to Planck formula it should make

$$\overline{\varepsilon^2} = h\nu E + \frac{c^3 E^2}{8\pi\nu^2 d\nu}$$

where $E$ - energy of electromagnetic radiation. Thus, the mean square of fluctuations according to Planck formula is the sum of the mean squares of fluctuations according to Wien's law and Rayleigh-Jeans' law. Hence, we can make a conclusion that "... neither wave, nor corpuscular theory of light can represent its nature." [9].

However on examining Einstein's way of reasoning we'll see that the physical model of selectively reflecting plate wasn't verified by him. The existence of the plate with such characteristics is not obvious. Indeed, we could take a group of oscillators with $\nu$ resonance frequency as a model. Then they could have the reflection coefficient close to the unity at their resonance frequency. At the same time the substance containing oscillators must have light refraction index higher than the unity at the frequencies lower than $\nu$ . At the frequencies higher than $\nu$ this index is speedily trying to attain the unity. The substance with refraction index different from the refraction index of the environment must reflect some part of the incident light. As a result, the expression for Einstein's mean square of electromagnetic energy fluctuation must have some additive summand. The value of this summand could be found on examining the concrete model of oscillator in the substance. However, the very existence of a disregarded summand allows us to state that neither Einstein's work [8] nor any other works of this kind prove that the consideration of fluctuations of classically described electromagnetic field necessarily results in Rayleigh-Jeans' radiation law.

Besides, the result obtained can be interpreted differently, namely comparing it with the results obtained by Ehrenfest who considering the same problem and using the same methods assumed that spectral density of the black body radiation can look as follows [10] :

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3} \cdot \frac{\int_0^\infty E \cdot \exp(-\frac{E}{kT}) \cdot G(\frac{E}{\nu}) dE}{\int_0^\infty \exp(-\frac{E}{kT}) \cdot G(\frac{E}{\nu}) dE},$$

by introducing $G(\frac{E}{\nu})$ - weight function of the distribution of mode energies, or oscillation energies in equilibrium radiation. Analyzing the result by substituting $G(\frac{E}{\nu})$ in several arbitrary functions Ehrenfest came to the conclusion that for so-called violet requirement to be fulfilled the weight function *must* allow for balance other than zero at $nh\nu$ points and be equal to zero at all the other points. Some physicists are of the opinion that Ehrenfest thus proved the impossibility of explaining the black body radiation spectrum from the classical point of view. However simple substitution of Heaviside function as a weight function

$$G(\frac{E}{\nu}) = \Phi(L - \frac{E}{\nu}) \equiv \begin{cases} 1 & \text{at } L \geq \frac{E}{\nu} \\ 0 & \text{at } L < \frac{E}{\nu} \end{cases}$$

allows us to obtain both our relation ( 7 ) and Planck formula exactly fulfilling so-called violet requirement. That is, the impossibility of calculating the black body radiation spectrum by applying classical physical ideas is not proved so far and, taking into account our results can hardly be proved within the scope of quantum physics.

## 4. Conclusion.

All the aforesaid allows us to conclude that the black body radiation spectrum can be calculated within the scope of classical physics if the energy of this radiation is presented in the form of the sum of energies of standing waves (modes) in the given volume. It is a very interesting result from the point of view of quantum physics since the existence of Planck's constant is proved within the framework of classical physics. But the classical solution looks simpler as it doesn't introduce zero oscillation of the vacuum, that is, infinite energy in the definite volume. At the same time this approach provides the possibility of clear interpretation of the values used in $E = h\nu$ formula. Here $E$ is maximum possible energy of the oscillator with $\nu$ resonance frequency, while $h$ is maximum possible area of oscillator phase trajectory. As we can see, these values can exist for the oscillations of definite duration which means that they don't contain any inner contradictions.



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