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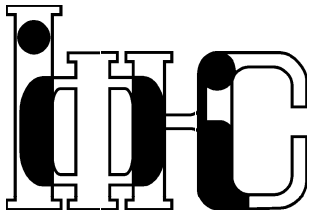

ІНСТИТУТ ФІЗИКИ КОНДЕНСОВАНИХ СИСТЕМ

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Effective critical behaviour of diluted Heisenberg-like magnets

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Ефективна критична поведінка розведених магнетиків гайзенберґівського типу

М. Дудка, Р. Фольк, Ю. Головач, Д. Іванейко

Анотація. За критерієм Гаріса слабке заморожене розведення немагнітною компонентою не впливає на *асимптотичні* критичні показники тривимірних (3d) магнетиків гайзенберґівського типу. У експериментах, однак, знаходять концентраційно і температурно залежні показники. Щоб пояснити ці спостереження, ми використовуємо теоретико-польовий ренормалізаційний груповий підхід, обчислюючи *ефективні* критичні показники слабо розведеного замороженого гайзенберґівського магнетика типу. Будучи неуніверсальними, ці показники змінюються з відстаню до критичної точки T_c і в асимптотичній границі співпадають із показникам чистого 3d гайзенберґівського магнетика, як спостерігається експериментально.

Effective critical behaviour of diluted Heisenberg-like magnets

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Abstract. In agreement with the Harris criterion, *asymptotic* critical exponents of three-dimensional (3d) Heisenberg-like magnets are not influenced by weak quenched dilution of non-magnetic component. However, often in the experimental studies of corresponding systems concentration- and temperature-dependent exponents are found with values differing from those of the 3d Heisenberg model. In our study, we use the field–theoretical renormalization group approach to explain this observation and to calculate the *effective* critical exponents of weakly diluted quenched Heisenberg-like magnet. Being non-universal, these exponents change with distance to the critical point T_c as observed experimentally. In the asymptotic limit (at T_c) they equal to the critical exponents of the pure 3d Heisenberg magnet as predicted by the Harris criterion.

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1. Introduction

Relevance of structural disorder for the critical behaviour remains to be an important problem of modern condensed matter physics. Even a weak disorder may change drastically the behaviour near the critical point and in this respect may be related to the global characteristics of a physical system, such as the space dimension, order parameter symmetry and the origin of interparticle interaction. In this paper, we are going to discuss some peculiarities of a paramegnetic-ferromagnetic phase transition in magnets, where the randomness of structure has the form of substitutional random-site or random-bond quenched disorder. Solid solutions of magnets with small concentration of non-magnetic component as well as amorphous magnets with large relaxation times may serve as an example of such systems.

Intuitively, it is clear that for a weak enough disorder the ferromagnetic phase persists in such systems. Obviously, intuition fails to predict whether the critical exponents characterizing phase transition into ferromagnetic state will differ in a disordered system and in a "pure" one. The answer here is given by the Harris criterion [1] which states that the critical exponents of the disordered system are changed only if the heat capacity critical exponent of a pure system is positive, otherwise the critical exponents of a disordered system coincide with those of a pure one. Returning to $d = 3$ dimensional magnets with $O(m)$ symmetric spontaneous magnetization one is lead to the conclusion, that here only the critical exponents of uniaxial magnets described by the $d = 3$ Ising model ($m = 1$) are the subject of influence by weak quenched disorder. Indeed, the heat capacity diverges $\alpha = 0.109 \pm 0.004 > 0$ [2] for $m = 1$, whereas it does not diverge for the easy-plane and Heisenberg-like magnets: $\alpha = -0.011 \pm 0.004$ and $\alpha = -0.122 \pm 0.010$ for $m = 2$ and $m = 3$, respectively [2].

Note however that the Harris criterion tells about the scaling behaviour *at* the critical point T_c. In other words it predicts (possible) changes in the *asymptotic* values of the critical exponents defined at T_c. In real situations one often deals with the *effective* critical exponents governing scaling when T_c still is not reached [3]. These are non-universal. As far as in our study of particular interest will be the isothermal magnetic susceptibility χ_T let us define the corresponding effective exponent by [3]:

$$\gamma_{\text{eff}}(\tau) = -\frac{\mathrm{d}\ln\chi(\tau)}{\mathrm{d}\ln\tau}, \qquad \text{with} \qquad \tau = |T - T_c|/T_c. \tag{1}$$

In the limit $T \to T_c$ the effective exponent coincides with the asymptotic one $\gamma_{\text{eff}} = \gamma$.

Already in the first experimental studies of weakly diluted uniaxial (Ising-like) $d = 3$ random magnets [4] the asymptotic values of critical exponents were found. For the solid solutions, the exponents do not depend on the concentration of non-magnetic component and belong to the new universality class [5] as predicted by the Harris criterion. We do not know analogous experiments where an influence of disorder on criticality of easy-plane magnets was examined. However its irrelevance was experimentally proven [7] for the superfluid phase transition in He^4 which belongs to the same $O(2)$ universality class as the ferromagnetic phase transition in easy-plane magnets.

As far as the disorder should be irrelevant for the asymptotic critical behaviour of the Heisenberg magnets, the diluted $d = 3$ Heisenberg magnets should belong to the same $O(3)$ universality class as the pure ones. Theoretically predicted values of the isothermal magnetic susceptibility, correlation length, heat capacity, pair correlation function, and the order parameter *asymptotic* critical exponents in this universality class read [2]:

$$\gamma{=}1.3895 \pm 0.0050,\ \nu{=}0.7073 \pm 0.0035,\ \alpha{=}-0.122 \pm 0.009, \tag{2}$$
$$\eta = 0.0355 \pm 0.0025,\ \beta = 0.3662 \pm 0.0025.$$

The experimental picture is more contravensional. The bulk of experiments on critical behaviour of disordered Heisenberg-like magnets performed up to middle 80-ies is discussed in the comprehensive reviews [8,9]. More recent experiments may be found in [10–15] and references therein. We show typical results of measurements of the isothermal magnetic susceptibility effective critical exponent γ_{eff} (1) in Figs. 1. As it is seen from the pictures, the behaviour of γ_{eff} is non-monotonic. The exponent differs from its value predicted in the asymtotic limit (2) and is a subject of a wide crossover behaviour. Before reaching asymtotics γ_{eff} possess maximum (except of the fig. 1.d), the value of the maximum is system dependent: it differs for different magnets.

It is standard now to rely on the renormalization group (RG) method [16] to get a reliable quantitative description of the behaviour in the vicinity of critical point. Namely in this way the cited above values (2) of the critical exponents of $d = 3$ Heisenberg model were obtained. The RG approach appeared to be a powerful tool to describe asymptotic [5] and effective [6] critical behaviour of disordered Ising-like magnets as well. The purpose of the present paper is to describe the crossover behaviour of disordered Heisenberg-like magnets in frames of the field-theoretical RG technique. In particular we want to calculate theoretically

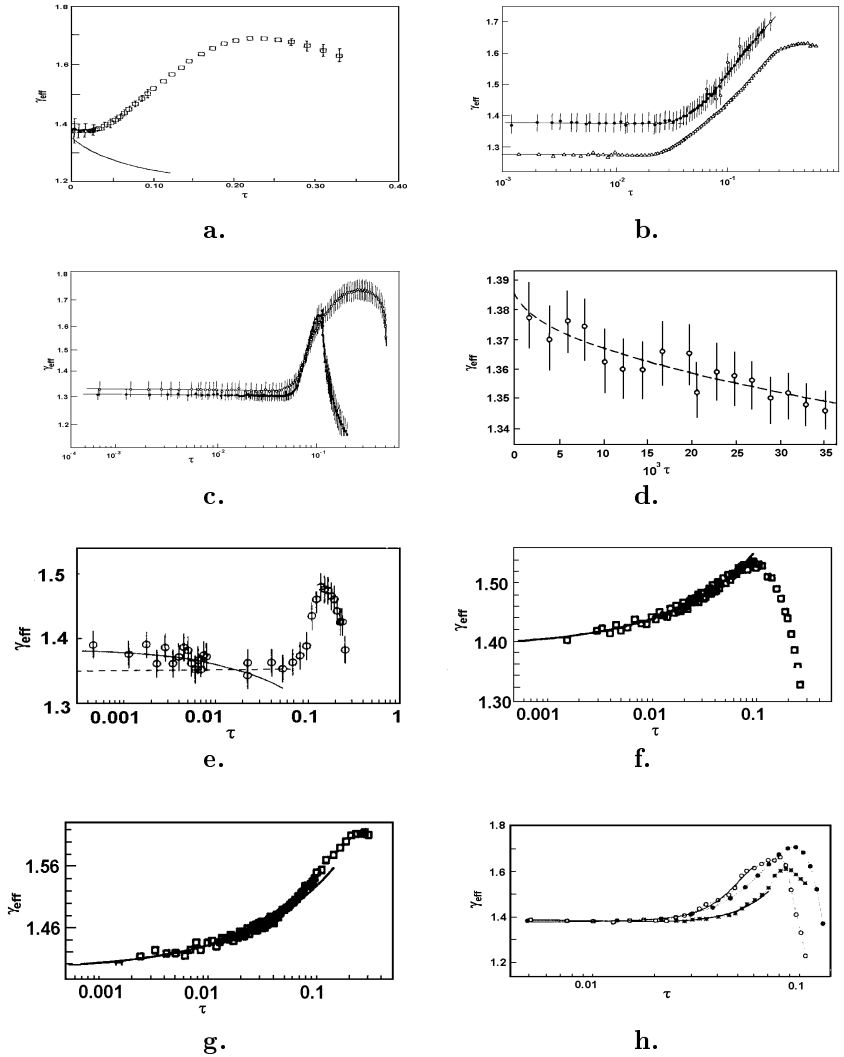

Figure 1. Experimentally measured isothermal magnetic susceptibility effective critical exponent $\gamma_{\rm eff}$ for disordered Heisenberg-like magnets ($\tau = (T - T_c)/T_c$). **a.**: $Fe_{20}Ni_{56}B_{24}$ (Fähnle *et al.*, 1983 [11]); **b.**: $Fe_{32}Ni_{36}Cr_{14}P_{12}B_6$ (Kaul, 1985 [9]); **c.**: $Fe_{20}Ni_{60}P_{14}B_6$, $Fe_{40}Ni_{40}P_{14}B_6$ (Kaul, 1985 [9]); **d.**: $Fe_{10}Ni_{70}B_{19}Si_1$ (Kaul, 1988 [12]); **e.**: $Fe_{16}Ni_{64}B_{19}Si_1$ (Kaul *et al.*, 1994 [13]); **f.**: $Fe_{86}Co_4Zr_{10}$ (Babu *et al.*, 1997 [14]); **g.**: $Fe_{90}Zr_{10}$ (Babu *et al.*, 1997 [14]); **h.**: $Fe_{90-x}Mn_xZr_{10}$ (Perumal *et al.*, 2001 [15]).

the isothermal magnetic susceptibility effective critical exponent and to explain in this way the appearance of the peak in its typical experimental dependencies. The rest of the paper is organized as follows. In the Section 2 we formulate the model and review main theoretical results obtained for it so far by means of the RG technique, effective critical behaviour is analyzed in the Section 3, we end by conclusions and outlook in the Section 4.

2. The model and its RG analysis

The model of a random quenched magnet we are going to consider is described by the following Hamiltonian:

$$H = -\frac{1}{2}\sum_{\mathbf{R},\mathbf{R}'} J(|\mathbf{R}-\mathbf{R}'|)\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'}c_{\mathbf{R}}c_{\mathbf{R}'}. \qquad (3)$$

Here, the sum spans over all sites $\mathbf{R}$ of d-dimensional hypercubic lattice, $J(|\mathbf{R}-\mathbf{R}'|)$ is a short-range (ferro)magnetic interaction between classical "spins" $\vec{S}_{\mathbf{R}}$ and $\vec{S}_{\mathbf{R}'}$. We consider the spins $\vec{S}_{\mathbf{R}}$ to be m-component vectors and the Hamiltonian (3) contains their scalar product. Obviously, for the particular case of Heisenberg spins we will put later $m = 3$. The randomness is introduced into the Hamiltonian (3) by the occupation numbers $c_{\mathbf{R}}$ which are equal 1 if the site $\mathbf{R}$ is occupied by a spin and 0 if the site is empty. Considering the case when occupied sites are distributed without any correlation and fixed in certain configuration one obtains so-called uncorrelated quenched m-vector model.

In principle, the above information is enough to apply the RG approach for a study of the critical behaviour of the model (3). One should obtain an effective Hamiltonian corresponding to the model under consideration and then one analyzes its long-distance properties by analyzing appropriate RG equations [16]. But already on this step there are at least two different possibilities to proceed and both were exploited for the model (3). On one hand, to get the free energy of the model one can average the logarithm of configuration-dependent partition function over different possible configurations of disorder [17]. Then, making use of the replica trick [18] one arrives to the familiar effective Hamiltonian [19]:

$$H_{\rm eff} = -\int d^dR\Big\{\frac{1}{2}\sum_{\alpha=1}^{n}\Big[{\mu_0}^2|\vec{\phi}^{\alpha}|^2+|\vec{\nabla}\vec{\phi}^{\alpha}|^2\Big]+\frac{u_0}{4!}\sum_{\alpha=1}^{n}|\vec{\phi}^{\alpha}|^4+$$

$$\frac{v_0}{4!}\left(\sum_{\alpha=1}^{n}|\vec{\phi}^{\alpha}|^2\right)^2\Bigg\} \tag{4}$$

describing in the replica limit $n \to 0$ critical properties of the model (3). Here, μ_0 is a bare mass, $u_0 > 0$ and $v_0 \leq 0$ are bare couplings and $\vec{\phi}^{\alpha} \equiv \vec{\phi}^{\alpha}(\mathbf{R})$ is an α-replica of m-component vector field. The prevailing amount of RG studies of the critical behaviour of quenched m-vector model was performed on the base of the effective Hamiltonian (4) [5].

However, one more effective Hamiltonian corresponding to the model (3) is discussed in the literature [20–23]. It is obtained exploiting the idea that a quenched disordered system can be described as an equilibrium system with additional forces of constraints [24]. In such approach both variables $\vec{S}_{\mathbf{R}}$ and $c_{\mathbf{R}}$ are treated equivalently and one ends up with the effective Hamiltonian which differs from (4) and, consequently, leads to different results for the critical behaviour of the model (3) [20–23]. Whereas the effective Hamiltonian (4) was used in the wide context of general m-vector models [5], the approach of Refs. [20–23] was mainly used in explanations of crossover behaviour in Heisenberg-like systems [25]. Below, we will discuss our results, based on the effective Hamiltonian (4) for $m = 3$ and compare them with those derived in [20–23].

As it is well known, the renormalization group (RG) approach makes use of the scaling symmetry of the system in the asymptotic limit to extract the universal content and at the same time removes divergencies which occur for the evaluation of the bare functions in this limit [16]. A change in the renormalized couplings u, v of the effective Hamiltonian (3) under the RG transformation is described by the flow equations:

$$\ell\frac{\mathrm{d}}{\mathrm{d}\ell}u(\ell) = \beta_u\left(u(\ell), v(\ell)\right), \quad \ell\frac{\mathrm{d}}{\mathrm{d}\ell}v(\ell) = \beta_v\left(u(\ell), v(\ell)\right). \tag{5}$$

Here, ℓ is the flow parameter related to the distance τ to the critical point. The fixed points (u^*, v^*) of the system of differential equations (5) are given by:

$$\beta_u\left(u^*, v^*\right) = 0, \quad \beta_v\left(u^*, v^*\right) = 0. \tag{6}$$

A fixed point is said to be stable if the stability matrix

$$B_{ij} \equiv \partial\beta_{u_i}/\partial u_j, \qquad i, j = 1, 2; \qquad u_i = \{u, v\}, \tag{7}$$

possess in this point eigenvalues ω_1, ω_2 with positive real parts. In the limit $\ell \to 0$, $u(\ell)$ and $v(\ell)$ attain the stable fixed point values u^*, v^*. If the stable fixed point is reachable from the initial conditions (let us

recall that for the effective Hamiltonian (3) they read $u > 0, v \leq 0$) it corresponds to the critical point of the system. The asymptotic critical exponents values are defined by the fixed point values of the RG γ-functions. In particular the isothermal magnetic susceptibility exponent γ is expressed in terms of the RG functions γ_ϕ and $\bar{\gamma}_{\phi^2}$ describing renormalization of the field ϕ and of the two-point vertex function with a ϕ^2 insertion correspondingly [16]:

$$\gamma^{-1} = 1 - \frac{\bar{\gamma}_{\phi^2}}{2 - \gamma_\phi}. \tag{8}$$

In Eq. (8), the functions $\gamma_\phi \equiv \gamma_\phi(u, v)$, $\bar{\gamma}_{\phi^2} \equiv \bar{\gamma}_{\phi^2}(u, v)$ are calculated in the stable fixed point u^*, v^*. In the RG scheme, the effective critical exponents are calculated in the region, where couplings $u(\ell), v(\ell)$ have not reached their fixed point values and depend on ℓ. In particular for the exponent γ_{eff} one gets:

$$\gamma_{\text{eff}}^{-1}(\tau) = 1 - \frac{\bar{\gamma}_{\phi^2}[u\{\ell(\tau)\}, v\{\ell(\tau)\}]}{2 - \gamma_\phi[u\{\ell(\tau)\}, v\{\ell(\tau)\}]} + \ldots. \tag{9}$$

In (9) the second part is proportional to the β–functions (5) and comes from the change of the amplitude part of the susceptibility. In the subsequent calculations we will neglect this part, taking the contribution of the amplitude function to the crossover to be small [28].

For the effective Hamiltonian (4), the fixed point structure is well established [5]. It is schematically shown in Figs. 2.a, 2.b. Two qualitatively different scenarios are observed: for $m > m_c$ the critical behaviour of the disordered magnet is governed by the fixed point of the pure magnet ($u^* > 0$, $v^* = 0$), whereas for $m < m_c$ the new stable fixed point ($u^* > 0$, $v^* < 0$) governs the asymptotic critical behaviour of the disordered magnet. At the marginal dimensionality m_c which separates these two regimes, the α exponent of the pure magnet equals zero in agreement with the Harris criterion.

Best theoretical estimates of m_c definitely support $m_c < 2$: $m_c = 1.942 \pm 0.026$ [26], $m_c = 1.912 \pm 0.004$ [27]. Consequently, the fixed point structure of the model of diluted Heisenberg-like magnet ($m = 3$) is given by Fig. 2.**a**: the stable reachable fixed points of the diluted and pure Heisenberg-like magnets do coincide ($u^* \neq 0, v^* = 0$), hence their *asymptotic* critical exponents do coincide as well. However the last statement does not concern the *effective* exponents. These are defined by the running values of the couplings $u(\ell) \neq 0, v(\ell) \neq 0$ and will be calculated in the next section.

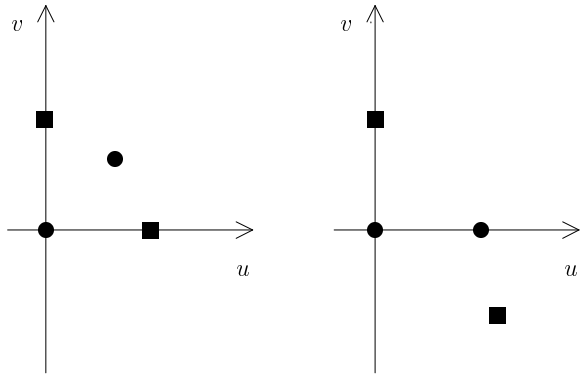

Figure 2. Fixed points structure for the effective Hamiltonian (4) at $d = 3$ and arbitrary m. **a**: $m > m_c$, **b**: $m < m_c$. Stable fixed points are shown by filled boxes, unstable ones are shown by filled circles. Only stable fixed points with coordinates $u^* > 0$, $v^* \leq 0$ are reachable for the model of the quenched magnet (3).

3. The RG flows and the effective critical behaviour

The RG functions of the model (4) are known by now in pretty high orders of the perturbation theory [5,29]. For the purpose of present study we will restrict ourselves by the first approximation where the described crossover phenomena manifests itself for the Heisenberg-like disordered magnets in non-trivial way. Within the two loop approximation in the minimal subtraction RG scheme [30] the RG-functions read [31]:

$$\beta_u(u,v) = -u(\varepsilon - \frac{m+8}{6}u - 2v + \frac{3m+14}{12}u^2 + \frac{5mn+82}{36}v^2 + \frac{11m+58}{18}uv), \quad (10)$$

$$\beta_v(u,v) = -v(\varepsilon - \frac{m+2}{3}u - \frac{mn+8}{6}v + \frac{5(m+2)}{36}u^2 + \frac{3mn+14}{12}v^2 + \frac{11(m+2)}{18}uv), \quad (11)$$

$$\gamma_\phi(u,v) = \frac{m+2}{72}u^2 + \frac{mn+2}{72}v^2 + \frac{m+2}{36}uv, \quad (12)$$

$$\bar{\gamma}_{\phi^2}(u,v) = \frac{m+2}{6}u + \frac{mn+2}{6}v - \frac{m+2}{12}u^2 - \frac{mn+2}{12}v^2 - \frac{m+2}{6}uv. \quad (13)$$

Here, $\varepsilon = 4 - d$ and replica limit $n = 0$ is to be taken.

Starting form the expressions (10)–(13) one can either develop the ε-expansion, or work directly at $d = 3$ putting in (10), (11) $\varepsilon = 1$ and considering renormalized couplings u, v as the expansion parameters [32]. However, such RG perturbation theory series with several couplings are known to be asymptotic at best [16]. One should apply appropriate resummation technique to improve their convergence to get reliable numerical data on their basis. We used several different resummation schemes for this purpose. Here we will give the results obtained by the method which allowed to analyze the largest region in the parametric $u-v$ space. The method was proposed in Ref. [33] and was successfully applied to study random $d = 3$ Ising model [29]. Moreover, it was shown that the RG functions of the $d = 0$ random Ising model are Borel-summable by this method [33]. The main idea proposed in Ref. [33] is to consider resummation in variables u and v separately. Taken that the RG function $f(u,v)$ is given to the order of p loops, one first rewrites it as a power series in v:

$$f(u,v) = \sum_{k=0}^{p} A_k(u) v^k. \quad (14)$$

Then each coefficient $A_k(u)$ is considered as power series in u and resummed as a function of a single variable u thus obtaining the resummed functions $A_k^{res}(u)$. Next one substitutes these functions into (14) and resumes the RG function f in single variable v. For the resummation in a single variable one may use any of familiar methods. Our results are obtained by making use of the Padé-Borel-Leroy method [34].

First, applying the above described resummation procedure to the β-functions (10), (11) we get the pure Heisenberg fixed point coordinates $u^* = 0.8956$, $v^* = 0$. The stability matrix (7) eigenvalues are positive at this fixed point ($\omega_1 = 0.577, \omega_2 = 0.147$) providing its stability. Then for the resummed values of the asymptotic critical exponents we get [35]:

$$\gamma = 1.382,\ \nu = 0.701,\ \alpha = -0.104,\ \eta = 0.030,\ \beta = 0.361. \quad (15)$$

We do not give the confidence intervals in (15), as far as they can be estimated only by comparison of changes introduced by different orders of perturbation theory. Note however that the results (15) are in a good agreement with the most accurate estimates of the exponents in the $O(3)$

universality class (2). This brings about that both the considered here two-loop approximation as well as the chosen resummation technique give an adequate description of asymptotic critical phenomena.

Before passing to the effective critical exponents let us first analyze the corrections to scaling. For the pure Heisenberg magnet, taking into account the leading correction to scaling results in the following formula for the isothermal susceptibility:

$$\chi(\tau) = \Gamma_0 \tau^{-\gamma}(1 + \Gamma_1 \tau^{\Delta}), \tag{16}$$

where the correction-to-scaling exponent is given by $\Delta = \omega\nu$ with $\omega = \partial\beta_u(u)/\partial u|_{u=u^*}$ and non-universal critical amplitudes Γ_0, Γ_1. For the diluted Heisenberg magnet the corresponding formula includes two leading corrections Δ_1, Δ_2 (see e.g. [12]):

$$\chi_(\tau) = \Gamma_0' \tau^{-\gamma}(1 + \Gamma_1' \tau^{\Delta_1} + \Gamma_2' \tau^{\Delta_2}), \tag{17}$$

with critical amplitudes $\Gamma_0', \Gamma_1', \Gamma_2'$. The exponents Δ_i are expressed in terms of the stability matrix (7) eigenvalues ω_i in the pure Heisenberg fixed point: $\Delta_i = \nu\omega_i$. At this fixed point, the eigenvalues of the stability matrix (7) read:

$$\omega_1 = \frac{\partial\beta_u(u,v)}{\partial u}|_{u^*\neq 0, v^*=0}, \qquad \omega_2 = \frac{\partial\beta_v(u,v)}{\partial v}|_{u^*\neq 0, v^*=0}. \tag{18}$$

It is straightforward to see that the value ω_1 coincides with the exponent ω of the pure model whereas it may be shown (see e.g. [12,31]) that the exponent $\omega_2 = |\alpha|/\nu$ where α and ν are the heat capacity and correlation length critical exponent of the pure Heisenberg model. On the base of the numerical values of the exponents (15) we get:

$$\Delta_1 = 0.405, \qquad \Delta_2 = 0.104. \tag{19}$$

Again, obtained by us in the two-loop approximation numbers (19) can be compared with those in the six-loop approximation making use of the data (2) together with the value of ω of pure 3d Heisenberg model $\omega = 0.782 \pm 0.0013$ [2]. As we have noted above, in order to get the numerical values of the correction-to-scaling exponents of diluted Heisenberg model it is no need to consider the RG functions (10)–(13) in the whole region of couplings u, v: it is enough to know them for the case of the pure model (i.e. for $u \neq 0, v = 0$). However, to get the effective exponents it is necessary to study complete set of the RG functions (10)–(13) working also in the region where both couplings u and v differ from zero.

To this end we use the above described resummation technique in order to restore the convergence of the RG expansions in couplings u, v. First we solve the system of differential equations (5) and get the running values of couplings $u(\ell)$, $v(\ell)$ (10)–(13). They define the flow in the parametric space u, v and in the limit $\ell \to 0$ attain the stable fixed point value (shown by the filled box in Fig. 3). Character of the flow

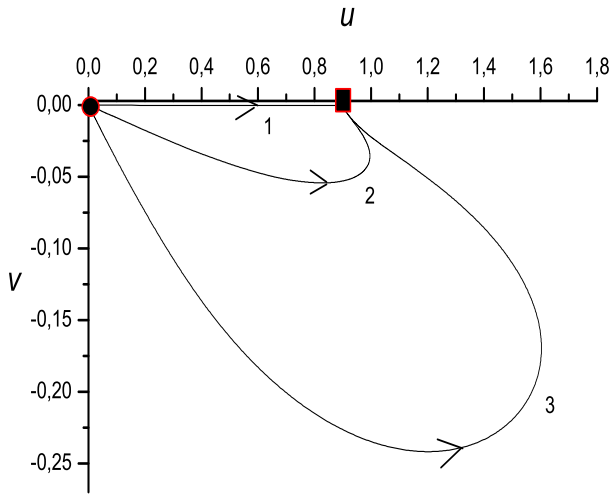

Figure 3. Flows in the parametric space of couplings. The filled box denotes the stable fixed point $u^* = 0.8956$, $v^* = 0$. Curve 1 corresponds to the flow from initial values with $v_0 = 0$, curve 2 starts with a small ratio $|v_0/u_0|$ whereas flow 3 corresponds to larger $|v_0/u_0|$.

depends on the initial conditions u_0, v_0 for solving the system of differential equations (5). For the model (3), the coupling v is proportional to variance of disorder [5] thus one can use the ratio $|v_0/u_0|$ to define the degree of dilution. Typical flows which are obtained for different ratios $|v_0/u_0|$ are shown in Fig. 3 by curves 1-3. We choose the starting values in the region with the appropriate signs of couplings $u > 0$, $v < 0$ near the origin (in the vicinity of the Gaussian fixed point $u^* = v^* = 0$ shown by the filled circle in the figure). The flow No 1 is obtained for $v_0 = 0$, it corresponds to the pure Heisenberg model. The flow No 2 results from the small ratio $|v_0/u_0|$ and corresponds to the weak disorder whereas the flow No 3 is obtained for large $|v_0/u_0|$ and corresponds to the stronger dilution.

Obtained running values of coupling constant presented by flows in Fig. 3 allow one to get the effective critical exponents. Calculating resummed expression for the effective exponent $\gamma_{\rm eff}$ (9) along the flows 1-3 we get the results shown in the Fig. 4. Again, the curve 1 corresponds

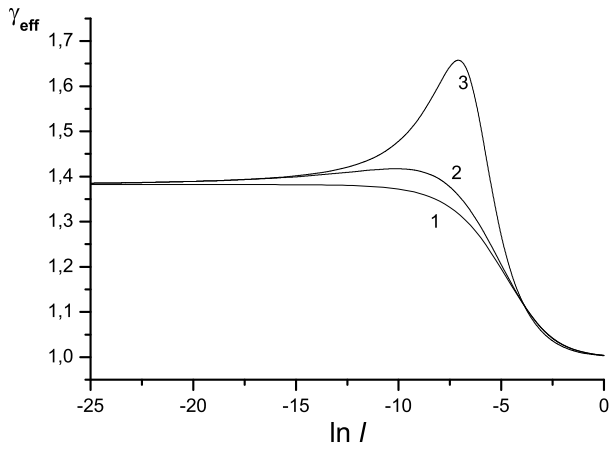

Figure 4. Effective critical exponent $\gamma_{\rm eff}$ versus logarithm of the flow parameter. The curves correspond to flows from the Fig. 3 denoted by corresponding numbers.

to the effective critical exponent of the pure Heisenberg model, whereas curves 2 and 3 provide two possible scenarios for the effective exponents of the disordered Heisenberg model. Curve 2 corresponds to the weak dilution region: here, the exponent increases with approach to the critical point, although the crossover region is larger in comparison with the pure magnet (compare curves 1 and 2 in Fig. 4). This may lead to the peculiar situation that the asymptotic value of the exponent is reached earlier than the asymptotic values of the coupling. The effective exponents for the flows originating from non-zero ratio $|v_0/u_0|$ always attain the value which are larger than the asymptotic one. But the absolute value of this "overshooting" for small enough $|v_0/u_0|$ is too small to be observed experimentally. An experimental observation of such type of $\gamma_{\rm eff}$ behaviour of the disordered Heisenberg-like magnet is provided e.g. by Fig. 1**d**. Different behaviour of $\gamma_{\rm eff}$ is demonstrated by the curve 3 in Fig. 4. Here, before reaching the asymptotic region the exponent possess a distinct peak. Such behaviour is in agreement with observed experimental data presented by Figs. 1**a**–1**c**, 1**e**–1**h**. The value of maximum depends on

the initial values for the RG flows. Larger ratio $|v_0/u_0|$ (i.e. stronger disorder) leads to the larger maximum. Thus, within unique approach one may explain both scenarios observed in the diluted Heisenberg-like magnets effective critical exponent $\gamma_{\rm eff}$ behaviour.

As we have noticed in the section 2, the crossover behaviour of random Heisenberg-like magnets was analyzed by means of an alternative approach in [20–23]. There, the quenched disordered magnet was described as an equilibrium one with additional forces of constraints [24]. This resulted in an effective Hamitonian which differs from (4). The fixed point structure of this Hamiltonian differs from those given in Fig. 2 and, for different concentrations, leads to different crossover regimes. In particular, it predicts that there exists a limiting value of concentration where the critical behaviour is governed by Fisher-renormalized tricritical exponents [23] which coincide with those of a $d = 3$ spherical model: $\gamma = 2$, $\nu = 1$, $\alpha = -1$, $\eta = 0$, $\beta = 1/2$. There exist two more fixed points which may be stable in the weak dilution regime. Their stability differs in different orders of the perturbation theory (compare [20] and [22]) but the numerical values of the critical exponents do not differ essentially at these fixed points. The maximal possible value of the effective critical exponent $\gamma_{\rm eff}$ has been estimated as $\gamma_{\rm eff} \simeq 2.6$ [22]. However, the distinct feature of the behaviour of $\gamma_{\rm eff}(\tau)$ obtained in [21] is its monotonic dependence. Hence, the experimentally observed peaks (see Fig. 1) can not be explained within such approach.

4. Conclusions

In the present paper we used the field-theoretical RG technique to study the effective critical behaviour of diluted Heisenberg-like magnets. The question of particular interest was to explain the peak in the exponent $\gamma_{\rm eff}$ as function of distance from T_c observed in some experiments. Our two-loop calculations refined by the resummation of the perturbation theory series resulted in typical behaviour of diluted Heisenberg-like magnets $\gamma_{\rm eff}$ exponent represented by curves 2 and 3 in Fig. 4. The exponent can either reach it asymptotic value without demonstrating distinct maximum or it can first reach the peak and then cross-over to the asymptotic value from above. The strength of disorder is a physical reason which discriminates between these two regimes.

Our calculations are quite general and do not specify any particular object. In order to fit our curves to certain experiment one should include into consideration non-universal parameters to specify the magnetic system. The same concerns the flow parameter ℓ which as we have

already noted is related to the distance to the critical point τ. In principle such calculations may be done. However we want to emphasize that our analysis shows the reason of the peak in $\gamma_{\rm eff}(\tau)$ dependence for different disordered magnets which belong to the $O(3)$ universality class and this reason may be explained within the traditional RG approach. This concerns not only the magnetic susceptibility effective critical exponent. One more example is given by the order parameter effective exponents $\beta_{\rm eff}$ which has minimum when τ goes to zero (see e.g. [15]). Interpretation of this effect will be the goal of a separate study

In conclusion we want to note that similar peculiarities of the effective critical behaviour may be observed in studies of disordered easy-plane magnets which belong to the $O(2)$ universality class. Since the heat capacity does not diverge in such systems, the RG fixed point scenario is given by the Fig. 2**a** as for the Heisenberg-like disordered magnets. Up to our knowledge such experiments have not been performed yet and we hope that our calculations may stimulate them.

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35. We calculate exponents γ and ν from the relations (8) and $\nu^{-1} =$

$2 - \bar{\gamma}_{\phi^2} - \gamma_\phi$. Numerical values of the rest of exponents are obtained from the familiar scaling relations.

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