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FIELD-TYPE TIME-ASYMMETRIC TWO-PARTICLE MODELS IN THE SECOND-ORDER APPROXIMATION IN A COUPLING CONSTANT

ЛЬВІВ

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Часо-асиметричні двочастинкові моделі польового типу в другому наближенні за константою взаємодії

А. Дувіряк, В. Шпитко

Анотація. В рамках явно коваріянтного гамільтонівського формалізму з в'язями розглядається релятивістична двочастинкова система з часо-асиметричними взаємодіями польового типу. В другому наближенні за константою взаємодії в'язь масової оболонки подається як співвідношення між одним з ґенераторів SO(2,1) і квадратом повної маси системи. Пропонується алгебричне квантування класичної задачі, та отримано релятивістичні спектри мас для широкого класу взаємодій польового типу.

Field-Type Time-Asymmetric Two-Particle Models in the Second-Order Approximation in a Coupling Constant

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Abstract. The relativistic two-particle system with field-type time-asymmetric interactions is considered within the framework of manifestly covariant Hamiltonian formalism with constraints. In the second-order approximation in a coupling constant the mass-shell constraint is presented as a relation between one of the generators of SO(2,1) and the total mass squared of the system. An algebraic quantization of the classical problem is proposed and the relativistic mass spectra for a wide range of the field-type interactions are obtained.

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1. Introduction

The relativistic bound state problem stimulates the construction of mechanical models which are related as closely as possible to the field theory. On the classical level this connection is provided by the Fokker action integrals [1,2] which, however, endow few-body systems with an infinite number of degrees of freedom. One of attempts to restrict (within the Fokker formalism) degrees of freedom to a finite number leads to time-asymmetric models. They describe two structureless particles interacting via the retarded field of the first particle and the advanced field of the second one. The two-particle time-asymmetric model with vector-type (electromagnetic) interaction was proposed by Fokker [1]. It has been studied [3] and generalized [4–8] for interactions mediating by massless fields of an arbitrary tensor rank and their superposition.

The present paper is devoted to the quantization problem of these models. We proceed from the appropriate manifestly covariant canonical description with constraints developed in Ref. [4]. Within this framework the models are determined by the pair of Poincaré-invariant first class constraints. One of them, the *light cone* constraint, is purely kinematic. Another, the *mass shell* constraint, determines the classical dynamics of the system. Here we show that in the second-order approximation in a coupling constant (see Ref. [6]) this constraint can be presented as a relation between the total mass of the system and one of canonical generators of SO(2,1) group.

The structure of the mass shell constraint suggests one to use the Lie algebra $\mathsf{so}(2,1)$ as a basis for quantization instead of the Heisenberg algebra. Then the quantum analog of this constraint determines the mass spectrum problem which can be easily solved by taking into account properties of the unitary representations of SO(2,1). This approach is inspired by Barut's dynamical group method [9–11]. It allows one to omit many of details involved usually in a quantization problem of relativistic models (ordering rule, definition of inner product etc.).

The similar quantization has been applied to the exact vector and scalar time-asymmetric models in the case of two-dimensional space-time [12]. Here, using the manifestly covariant description and taking into consideration the Poincaré group (besides SO(2,1)), we generalize this procedure to a wide range of the field-type interactions in the four-dimensional Minkowski space. As a result we obtain a family of relativistic mass spectra. We also consider physically tractable examples and discuss some ambiguities arising from the quantization procedure.

2. The Hamiltonian description of time–asymmetric models with field–type interactions and the canonical realization of $\mathsf{so}(2,1)$ algebra

The manifestly covariant description of time-asymmetric models within the framework of canonical formalism with constraints [4] is based on the phase space $\mathrm{T}^*\mathbb{M}_4^2$ (where $\mathbb{M}_4$ is the Minkowski space) with space-time coordinates and momenta, x_a^μ, $p_{a\mu}$ ($a = 1,2;\ \mu = \overline{0,3}$), satisfying standard Poisson-bracket (PB) relations: $\{x_a^\mu, p_{b\nu}\} = \delta_{ab}\delta_\nu^\mu$.

Canonical generators of Poincaré group $\mathcal{P}$,

$$P_\mu = \sum_{a=1}^{2} p_{a\mu}, \qquad J_{\mu\nu} = \sum_{a=1}^{2}(x_{a\mu}p_{a\nu} - x_{a\nu}p_{a\mu}), \tag{1}$$

satisfy PB relations of corresponding Lie algebra p.

By virtue of parametric invariance of the description a Hamiltonian vanishes, and the evolution of the system is determined by the pair of Poincaré-invariant first class constraints. One of them is the *light cone* constraint $x^2 := x_\mu x^\mu = 0$, $\eta x^0 > 0$, where $x^\mu = x_1^\mu - x_2^\mu$, and the sign factor η can be chosen as +1 or –1. This constraint is holonomic. It fixes the relative time variable (say, x^0) and reduces the original configuration space $\mathbb{M}_4^2$ to the 7-dimensional Poincaré-invariant submanifold $\mathcal{C}$. The reduced phase space $\mathrm{T}^*\mathcal{C}$ can be parameterized with 14 canonical variables z which satisfy the relation $\{z, x^2\} = 0$. For convenience we choose them implicitly among the following manifestly covariant variables: x_a^μ, P_μ, and $v_\mu = \frac{1}{2}(p_{1\nu} - p_{2\nu})(\delta_\mu^\nu - P^\nu x_\mu / P\cdot x)$, by taking into account the light-cone constraint and the equality $P\cdot v \equiv 0$. Another, the *mass shell* constraint:

$$\phi(P^2,\ v^2,\ P\cdot x,\ v\cdot x) = 0, \tag{2}$$

determines the dynamics of the system. It is supposed that equation (2) can be solved with respect to the total momentum squared $P^2 = P_\mu P^\mu$ such that $P^2 > 0$.

It is possible to present the mass-shell constraint in the equivalent form:

$$\Phi(P^2,\ K_0,\ K_1,\ K_2) = 0, \tag{3}$$

via the following Poincaré-invariant functions K_0, K_1 and K_2:

$$K_0 = (K_+ + K_-)/2, \quad K_1 = (K_+ - K_-)/2, \quad K_2 = x\cdot v,$$
$$K_+ = -v^2 K_-/P^2 + B(P^2)/K_-, \quad K_- = \eta P\cdot x. \tag{4}$$

Here $B(P^2)$ is an arbitrary function. The functions K_0, K_1, K_2 satisfy PB relations of Lie algebra $\mathsf{so}(2,1)$:

$$\{K_0, K_1\} = K_2, \quad \{K_1, K_2\} = -K_2, \quad \{K_2, K_0\} = K_1. \tag{5}$$

Together with the generators (1) of $\mathcal{P}$ they form the basis of Lie algebra $\mathsf{p} \oplus \mathsf{so}(2,1)$.

Now we restrict the original set of observables to the canonical generators of $\mathcal{P}$ and SO(2,1), i.e., we consider observables defined on the dual $\mathcal{R} = (\mathsf{p} \oplus \mathsf{so}(2,1))^*$ to the Lie algebra $\mathsf{p} \oplus \mathsf{so}(2,1)$. Then the mass shell constraint (3) becomes one of constraints determining the dynamics of the system in $\mathcal{R}$. The generators of the symmetry group $\mathcal{P}$ are the integrals of motion. They characterize a state of the system as a whole. The generators of SO(2,1) describe an internal dynamics. They are not, in general, conserved. Another constraint arises from relations (4). It couples the Casimir functions of p and $\mathsf{so}(2,1)$ algebras P^2, $L^2 = -W^2/P^2 > 0$ (W^μ is Pauli-Lubanski vector) and

$$Q^2 := K_0^2 - K_1^2 - K_2^2 = L^2 + B(P^2). \tag{6}$$

The mass shell constraint for the time-asymmetric models with field-type interaction was obtained in Ref [6]. It follows from the Fokker-type action which, in turn, is related to the classical field theory [2,13,8]. In the second-order approximation in coupling constant α this mass shell constraint takes in terms of generators (4) the form:

$$\begin{aligned} &\tfrac{(m_1^2+m_2^2)^2-4m_1^2m_2^2\mu^2}{8P^4}K_- - \tfrac{\eta(m_1^2-m_2^2)}{2P^2}K_2 + \tfrac{1}{2}K_+ - \tfrac{\alpha m_1 m_2 f(\mu)}{P^2} \\ &\quad - \tfrac{B(P^2)}{2K_-} + \sum\nolimits_a \tfrac{\alpha^2 h(\mu) m_a^2/P^2}{K_- - 2\eta(-)^a K_2} + O(\alpha^3) = 0. \end{aligned} \tag{7}$$

Here m_a is the rest mass of ath particle, $f(\mu)$, $h(\mu)$ are arbitrary functions of

$$\mu = \frac{P^2 - m_1^2 - m_2^2}{2m_1 m_2} \tag{8}$$

defined in a physically reasonable domain $\mu > -1$, i.e., $P^2 > (m_1 - m_2)^2$.

In the case where particles interact via superposition of massless linear field with various tensor rank the functions $f(\mu)$ and $h(\mu)$ have the form [13,6]

$$f(\mu) = \sum\nolimits_n c_n T_n(\mu), \qquad h(\mu) = [(f(\mu) - \mu f'(\mu)]^2 - [f'(\mu)]^2, \tag{9}$$

where c_n are constants, $T_n(\mu)$ are the Chebyshev polynomials and $f'(\mu) = df(\mu)/d\mu$. Each (say, nth) term of the sum in (9) is the contribution of nth rank tensor field into an interaction. We suppose that

$\alpha > 0$, $f(\mu) > 0$ and $f(1) = \sum_n c_n = 1$ which corresponds in the nonrelativistic limit to the Coulomb attractive potential $U = -\alpha/r$.

To simplify the constraint (7) we perform two canonical transformations:

$$\text{(I)} \quad \begin{aligned} &K_2 \to K_2' = \exp\{...,\vartheta K_-\}K_2 = K_2 - \vartheta K_-, \quad K_- = K_-', \\ &K_+ \to K_+' = \exp\{...,\vartheta K_-\}K_+ = K_+ - 2\vartheta K_2 + \vartheta^2 K_-; \end{aligned} \tag{10}$$

$$\text{(II)} \quad K_\pm' \to \tilde{K}_\pm = \exp\{...,\varphi K_2'\}K_\pm' = e^{\mp\varphi}K_\pm', \quad K_2' = \tilde{K}_2, \tag{11}$$

generated by the coadjoint action of SO(2,1). They preserve PB relations (5) even if the parameters ϑ and φ depend on Casimir functions. Let us put

$$\vartheta(P^2) = \eta(m_1^2 - m_2^2)/2P^2, \qquad \varphi(P^2) = \ln\sqrt{|\epsilon(P^2)|}, \quad \epsilon \neq 0; \tag{12}$$

$$\epsilon(P^2) = \frac{m_1^2 m_2^2}{P^4}(\mu^2 - 1) \quad \begin{matrix} < 0 & \text{if} & (m_1-m_2)^2 < P^2 < (m_1+m_2)^2 \\ > 0 & \text{if} & P^2 > (m_1+m_2)^2 \end{matrix} \; . \tag{13}$$

The case $\epsilon < 0$ corresponds to a bounded motion while $\epsilon > 0$ is the scattering case [6].

In the first-order approximation the equation (7) yields the equality:

$$\sqrt{|\epsilon|} = \alpha \frac{m_1 m_2 f(\mu)}{P^2 \tilde{K}_{\theta(\epsilon)}} + O(\alpha^2), \qquad \text{where} \quad \theta(\epsilon) = \begin{matrix} 0 \text{ if } \epsilon < 0 \\ 1 \text{ if } \epsilon > 0 \end{matrix} \; . \tag{14}$$

Taking this into account and choosing the arbitrary function $B(P^2)$ as follows

$$B(P^2) = \alpha^2 h(\mu) \sum\nolimits_a (1 + \mu m_{\bar{a}}/m_a)^{-1}, \qquad \bar{a} = 3 - a, \tag{15}$$

simplifies the mass shell constraint to the following final form:

$$\tilde{K}_{\theta(\epsilon)} - F(P^2) + O(\alpha^3) = 0, \tag{16}$$

where $F(P^2) = \alpha f(\mu)|\mu^2 - 1|^{-1/2}$, $\mu \neq 1$ and, in turn, $\mu = \mu(P^2)$ (see (8)).

As it follows from (13) and (14) $\mu = 1 + O(\alpha^2)$. Thus, with the required accuracy we have $B(P^2) \approx \alpha^2 h(1)$, and the constraint (6) becomes as follows:

$$Q^2 - L^2 - \alpha^2 h(1) + O(\alpha^3) = 0. \tag{17}$$

3. Quantization

We have recast the two-body model with field-type interaction into the dynamical system on the presymplectic submanifold $\mathcal{H} \subset \mathcal{R}$ defined by the pair of constraints (16) and (17). This classical description inspires the following quantization.

Let us replace the canonical generators $P_\mu, J_{\nu\lambda}, \tilde{K}_0, \tilde{K}_1, \tilde{K}_2$ by Hermitian operators $\hat{P}_\mu$, $\hat{J}_{\nu\lambda}, \hat{K}_0, \hat{K}_1, \hat{K}_0$, and Poisson brackets $\{...,...\}$ by commutators $[...,...]/\mathrm{i}$. Then we can consider these operators as generators of a unitary representation of $\mathcal{P}\otimes$SO(2,1). This procedure is formal until we specify a Hilbert space of the system. Its construction may be suggested by the group structure of the classical description.

The dual $\mathcal{R}$ can be considered as a unity of orbits of coadjoint representation of $\mathcal{P}\otimes$SO(2,1). Since orbits are homogeneous spaces, their quantum counterparts may be chosen as unitary irreducible representations (UIRs) of this group [14]. Then a quantum analog of $\mathcal{R}$ will be a reducible representation $\mathfrak{R} = \bigoplus_{\ell,M,q} \mathfrak{H}_M^{(\ell)} \otimes \mathfrak{D}^{(q)}$ of $\mathcal{P}\otimes$SO(2,1). Here $\mathfrak{H}_M^{(\ell)}$ and $\mathfrak{D}^{(q)}$ are UIRs of $\mathcal{P}$ and SO(2,1), respectively; quantum numbers M, ℓ, q label eigenvalues of Casimir operators (they are specified below), and $\bigoplus$ denotes the direct sum over discrete variable ℓ and the direct integral over continuous variables M, q. Finally, the subspace $\mathfrak{H} \subset \mathfrak{R}$ determined by quantum counterparts of the constrains is considered as the physical Hilbert space of the system.

Due to physical reasons we construct $\mathfrak{R}$ with the Wigner UIRs $\mathfrak{H}_M^{(\ell)}$ of the special Poincaré group for positive masses $M > |m_1 - m_2|$ and integer spins $\ell = 0, 1, 2, ...$ (half-integer spins are forbidden due to discrete symmetry properties of two-particle system):

$$\begin{aligned} \hat{P}^2|\Psi(M\ell)\rangle &= M^2|\Psi(M\ell)\rangle, \\ \hat{L}^2|\Psi(M\ell)\rangle &= \ell(\ell+1)|\Psi(M\ell)\rangle, \qquad |\Psi(M\ell)\rangle \in \mathfrak{H}_M^{(\ell)}. \end{aligned} \tag{18}$$

There are a few UIRs of SO(2,1) for a given value of the Casimir operator $\hat{Q}^2$ [9]:

$$\hat{Q}^2|\Xi(q)\rangle = Q^2|\Xi(q)\rangle, \qquad Q^2 = q(q+1), \qquad |\Xi(q)\rangle \in \mathfrak{D}^{(q)}. \tag{19}$$

If $Q^2 \geq 0$ ($q \geq 0$), there exist only two series of UIRs $\mathfrak{D}_+^{(q)}$ and $\mathfrak{D}_-^{(q)}$. In the domain $-1/4 < Q^2 < 0$ ($-1/2 < q < 0$) they coexist with the complementary series $\mathfrak{D}^{(q,\varepsilon)}$. The principal series ($Q^2 \leq -1/4$, $q = -1/2 + \mathrm{i}\omega \in \mathbb{C}$) is not relevant to the present problem.

Here we choose the representations $\mathfrak{D}_+^{(q)}$ of discrete series existing for $q > -1/2$ ($\mathfrak{D}_-^{(q)}$ leads to the same result). Then the quantum counterpart of constraint (17),

$$(\hat{I} \otimes \hat{Q}^2 - \hat{L}^2 \otimes \hat{I} - \alpha^2 h(1) \hat{I} \otimes \hat{I})|\Psi\rangle = 0, \qquad |\Psi\rangle \in \mathfrak{R} \tag{20}$$

(here $\hat{I}$ is the unit operator), determines in $\mathfrak{R}$ the subspace $\mathfrak{H}' = \bigoplus_{\ell,M} \mathfrak{H}_M^{(\ell)} \otimes \mathfrak{D}_+^{(q_\ell)}$, where

$$q_\ell = -1/2 + \sqrt{(\ell + 1/2)^2 + \alpha^2 h(1)}. \tag{21}$$

The construction of physical Hilbert space $\mathfrak{H} \subset \mathfrak{H}'$ needs for $\mathfrak{D}_+^{(q)}$ a realization of operator commuting with the quantum counterpart of the mass shell constraint (16). Thus, we put $\mathfrak{H}' = \mathfrak{H}'_- \oplus \mathfrak{H}'_+$, where $\mathfrak{H}'_-$ corresponds to $|m_1 - m_2| < M < m_1 + m_2$ and $\mathfrak{H}'_+$ corresponds to $M > m_1 + m_2$. In the subspace $\mathfrak{H}'_-$ the mass shell equation reads:

$$\hat{\Phi}|\Psi\rangle := (\hat{I} \otimes \hat{K}_0 - F(\hat{P}^2) \otimes \hat{I})|\Psi\rangle = 0, \qquad |\Psi\rangle \in \mathfrak{H}'_-. \tag{22}$$

The generator $\hat{K}_0$ of U(1) $\subset$ SO(2,1) commutes with $\hat{\Phi}$ and has a discrete spectrum:

$$\hat{K}_0|q;\kappa\rangle = \nu|q;\kappa\rangle, \qquad \begin{array}{c} \nu = \kappa + 1 + q, \\ \kappa = 0, 1, 2, ..., \end{array} \qquad |q;\kappa\rangle \in \mathfrak{D}_+^{(q)}. \tag{23}$$

Using (23), (22) reduces each subspace $\bigoplus_M \mathfrak{H}_M^{(\ell)} \otimes |q_\ell;\kappa\rangle \subset \mathfrak{H}'_-$ to $\mathfrak{H}_{M_{\kappa\ell}}^{(\ell)} \otimes |q_\ell;\kappa\rangle \simeq \mathfrak{H}_{M_{\kappa\ell}}^{(\ell)}$. The physical subspace $\mathfrak{H}_- = \bigoplus_{\ell=0}^\infty \bigoplus_{\kappa=1}^\infty \mathfrak{H}_{M_{\kappa\ell}}^{(\ell)}$ has the structure of a reducible unitary representation of $\mathcal{P}$. The discrete eigenvalues $M_{\kappa\ell}$ are positive solutions of the equations:

$$F(M_{\kappa\ell}^2) = \nu_{\kappa\ell} := \kappa + 1 + q_\ell. \tag{24}$$

For $\mathfrak{H}'_+$ we choose the realization of non-compact operator $\hat{K}_1$ with a continuous spectrum. Then the relevant mass shell equation reduces $\mathfrak{H}'_+$ to $\mathfrak{H}_+ = \bigoplus_{\ell=0}^\infty \int \oplus \, d\rho(M) \, \mathfrak{H}_M^{(\ell)}$, where the direct integral runs over $m_1 + m_2 < M < \infty$ with some measure $d\rho(M)$.

4. Mass spectra of bounded states

An explicit form of the discrete spectrum requires the equation (24) be solved. It has a positive solution provided q_ℓ (21) is real which, if $h(1) < 0$, leads to the restriction:

$$\alpha < (2\sqrt{|h(1)|})^{-1}. \tag{25}$$

The exact solution of eq. (24) can be found only in few special cases (see below). In the general case we use the power series in α which is considered as a small parameter. Then the condition (25) holds, and $-1/2 < q_\ell \in \mathbb{R}$. In the second-order approximation the expansion series of $M_{\kappa\ell}$ is valid up to α^4 and yields the mass spectrum:

$$M_{\kappa\ell} \approx m - \frac{m_r\alpha^2}{2n^2} + \frac{m_r\alpha^4}{n^3}\left[\frac{h(1)}{2\ell+1} + \frac{1}{2n}\left(f'(1) - \frac{1}{4} - \frac{m_r}{4m}\right)\right]. \quad (26)$$

Here $m = m_1 + m_2$, $m_r = m_1 m_2/m$, the second term in r.h.s. of (26) is the Coulomb (nonrelativistic) energy, and the third term is the second-order correction depending on two, in general, arbitrary constants $h(1)$ and $f'(1) = df(\mu)/d\mu|_{\mu=1}$.

This result correlates well with the mass spectrum obtained within the quasi-relativistic approach to two-body problem [15]. The latter, however, includes the additional second-order Darwin-type term $Cm_r\alpha^4\delta_{0\ell}/n^3$ which contributes in the spectrum of S-states. The constant C depends on both the type of interaction and a quantization rule. In our case this term can be introduced via replacement of $\mathfrak{D}_+^{(q_0)}$ in the construction of the space $\mathfrak{H}'$ by UIR of complementary series $\mathfrak{D}^{(q_0,\varepsilon)}$ for which $-1/2 < q_0 < 0$ and $\hat{K}_0|q_0\,\varepsilon;\kappa\rangle = \tilde{\nu}_{\kappa 0}|q_0\,\varepsilon;\kappa\rangle$ with $\tilde{\nu}_{\kappa 0} = \kappa + 1 + \varepsilon$, $|\varepsilon| < |q_0|$, $\kappa \in \mathbb{Z}$. Indeed, the case $-1/2 < q_\ell < 0$ occurs only if $h(1) < 0$ and $\ell = 0$ (provided the condition (25) holds). Thus, only the spectrum of S-states can be modified (as it should). Changing $\nu_{\kappa 0}$ in r.h.s. of eq. (24) (for $M_{\kappa 0}$) by $\tilde{\nu}_{\kappa 0}$ with $\varepsilon = \lambda q_0$, $|\lambda| < 1$, and taking into account the equality $q_0 \approx -\alpha^2|h(1)|$ leads to the Darwin-type term with $C = (1-\lambda)|h(1)|$.

Below we consider a few particular cases of physical interest. Although our consideration is approximated, we present (where it is possible) exact solutions of eq. (24) which are convenient to compare to results obtained in the literature from other approaches.

Vector (electromagnetic) interaction: $f(\mu) = T_1(\mu) = \mu$, $h(\mu) = -1$,

$$M_{\kappa\ell}^2 = m_1^2 + m_2^2 + \frac{2m_1m_2}{\sqrt{1+\alpha^2/\nu_{\kappa\ell}^2}}. \quad (27)$$

This mass spectrum has been obtained on the base of quasipotential approach [16,17], from an infinite-component wave equation [10], within the semiclassical quantization of relativistic two-body problem [18] etc. It represents the relativistic spectrum of hydrogen-like atom. The Darwin term with $C = m_r/m$ obtained within the quasipotential approach [17] corresponds, in our case, to the choice $\lambda = 1 - m_r/m$, so that $\frac{3}{4} < \lambda < 1$.

Scalar interaction: $f(\mu) = T_0(\mu) = 1$, $h(\mu) = 1$,

$$M_{\kappa\ell}^2 = m_1^2 + m_2^2 + 2m_1m_2\sqrt{1-\alpha^2/\nu_{\kappa\ell}^2}. \quad (28)$$

This mass spectrum has the same form as that obtained within the quasipotential [16,17,19] and variational [20] approaches from the Yukawa model. The only difference is that the Yukawa interaction leads to an integer quantum number $n = 1, 2, ...$ instead of $\nu_{\kappa\ell}$. Besides, it imposes the restriction for the coupling constant $\alpha < 1$. Our case corresponds to the "minimal" scalar interaction (see [18]) and gives no restriction for α. Other known versions of scalar interaction [21] correspond to formal replacement of the constant $h(1) = 1$ in r.h.s. of eqs (24), (21) by some functions of $M_{\kappa\ell}$.

Scalar-vector equal-weighted mixture: $f(\mu) = (1+\mu)/2$, $h(\mu) = 0$,

$$M_{\kappa\ell}^2 = m_1^2 + m_2^2 + 2m_1m_2\frac{4n^2-\alpha^2}{4n^2+\alpha^2}. \quad (29)$$

This model possesses the O(4)-symmetry [5]. Moreover, in this case the mass shell constraint (7) and thus the spectrum (29) is exact [12]. It coincides with the phenomenological result presented in Refs. [22,18].

Second rank tensor interaction. Gravitation: $f(\mu) = T_2(\mu) = 2\mu^2 - 1$,

$$M_{\kappa\ell}^2 = m_1^2 + m_2^2 + \frac{m_1m_2}{\sqrt{2}}\sqrt{4 - \frac{\nu_{\kappa\ell}^2}{\alpha^2} + \frac{\nu_{\kappa\ell}}{\alpha}\sqrt{8 + \nu_{\kappa\ell}^2/\alpha^2}}. \quad (30)$$

This case may correspond to the gravitational interaction with $\alpha = \Upsilon m_1 m_2/c\hbar$, where Υ is the gravitational constant. Due to the nonlinearity of gravitational field the constant $h(1) = -7$ evaluated by means of eq. (9) should be replaced by $h_{\rm gr}(1) = -6$ [6]. Then the condition (25) restricts the maximal mass of elementary particle $m_{\max} \le \sqrt{\frac{c\hbar}{2\sqrt{6}\Upsilon}} = 0.98\times10^{-5}$g which is closed to estimates from more profound theories.

Tensor interaction of arbitrary rank ($s = 0, 1, 2, ...$): $f(\mu) = T_s(\mu)$, $h(1) = 1 - 2s^2$. In the hypothetic case $s > 2$ the equation (24) is too cumbersome to be solved exactly, and we only write down the restriction of coupling constant: $\alpha < \left(2\sqrt{2s^2-1}\right)^{-1}$. It becomes more and more strong as the rank s of field grows.

5. Concluding remarks

The considered time-asymmetric models are based on the special superposition of retarded and advanced relativistic potentials [6,8]. This superposition satisfies formally the classical field equations with point-like sources and provides the classical canonical description with a finite number of degrees of freedom. The reformulation of this description into an algebraic language allows one to quantize the classical problem using a simple group-theoretic scheme. Ambiguity of the quantization procedure manifests itself in the spectrum of S-states via the Darwin-type term.

The obtained mass spectra for vector and scalar interactions generalize the exact spectra of these models in a two-dimensional space-time [12] to the case of $\mathbb{M}_4$. They agree well (at least in the second-order approximation) with results derived via various methods from the quantum field theory. This fact extends the correlation between the time-asymmetric models and the field theory onto the quantum level.

The phenomenological generalization of mass spectra beyond the vector and scalar cases was proposed in Ref. [22]. In our terms this family of spectra is described by eq. (24) with an unspecified function $f(\mu)$ (determining the function $F(M^2)$) and an integer quantum number n (instead of $\nu_{\kappa\ell}$). Here it is shown how the expressions for $f(\mu)$ and $\nu_{\kappa\ell}$ are related to the tensor structure of an interaction.

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