Національна академія наук України

ІНСТИТУТ
ФІЗИКИ
КОНДЕНСОВАНИХ
СИСТЕМ

ICMP–00–08E

V. Blavats'ka,[*] C. von Ferber,[†] Yu. Holovatch

SELF-AVOIDING WALKS IN MEDIA WITH LONG-RANGE-CORRELATED QUENCHED DISORDER

[*]Ivan Franko National University of Lviv, Chair for Theoretical Physics, UA-79005 Lviv, Ukraine

[†]Institut für Theoretische Physik, Heinrich Heine Universität Düsseldorf, D-40225 Düsseldorf, Germany

ЛЬВІВ

УДК: 530.145

PACS: 64.60.Ak, 61.41.+e, 64.60.Fr, 11.10.Gh

Випадкові блукання із самоуниканнями у середовищі із далекосяжно скорельованим безладом

В. Блавацька, К. фон Фербер, Ю. Головач

Анотація. У роботі досліджуються масштабні властивості випадкових блукань із самоуниканнями (SAWs) на невпорядкованих d-вимірних гратках із замороженими домішками, скорельованими за степеневим законом $\sim r^{-a}$ на великих відстанях r. Відомо, що такий тип безладу є суттєвим у магнітних фазових переходах. Застосовано метод теоретико-польової ренормалізаційної групи і виконано обчислення у техніці подвійного розкладу за параметрами $\varepsilon = 4-d$, $\delta = 4-a$. Показано, що асимптотична поведінка SAWs на гратках із далекосяжно скорельованим безладом описується новим показником $\nu^{long} = 1/2 + \delta/8$, $(\varepsilon/2 < \delta < \varepsilon)$, у той час як результат для SAWs на чистій гратці: $\nu^{pure} = 1/2 + \varepsilon/16$ $(\epsilon > 0)$.

Self-avoiding walks in media with long-range-correlated quenched disorder

V. Blavats'ka, C. von Ferber, Yu. Holovatch

Abstract. We study the scaling properties of self-avoiding walks (SAWs) on a d-dimensional disordered lattice with quenched defects obeying a power law correlation $\sim r^{-a}$ for large distances r. Such type of disorder is known to be relevant for magnetic phase transitions. We apply the field-theoretical renormalization group approach and perform calculations in a double expansion in $\varepsilon = 4-d$, $\delta = 4-a$. The asymptotic behaviour of SAWs on a lattice with long-range-correlated disorder is found to be governed by a new exponent $\nu^{long} = 1/2 + \delta/8$, $(\varepsilon/2 < \delta < \varepsilon)$. This is to be compared with a first order result for SAWs on a "pure" lattice: $\nu^{pure} = 1/2 + \varepsilon/16$, $(\varepsilon > 0)$.

Подається в Фізика конденсованих систем
Submitted to Condensed Matter Physics

1. Introduction

Essential progress in understanding scaling properties of polymers in good solvents is due to the application of a renormalization group (RG) [1], [2]. In such description polymer chains are considered as self-avoiding walks (SAWs) on *regular* lattice. Their topological properties in a limit of long chains govern the same scaling laws as those arising at the critical point of an *pure* (undiluted) m-vector model in de Gennes limit $m \to 0$ [2]. In particular, an average square end-to-end distance of a SAW on a regular lattice scales with the number of steps N in asymptotic limit $N \to \infty$ as:

$$\langle \vec{R}^2 \rangle \sim N^{2\nu},$$

where ν is an universal exponent which depends on the space dimensionality d only. Calculated by means of the RG approach in terms of $\varepsilon = 4 - d$–expansion this exponent reads (see, e.g., [1], [2]):

$$\nu = \frac{1}{2} + \frac{\varepsilon}{16} + \cdots, \quad \varepsilon > 0. \tag{1}$$

The problem of SAWs on *randomly diluted* lattices, simulating linear polymers in porous medium, has been the subject of intensive discussion [4], [5], [6], [7], [8]. The resent review on SAWs statistics on random lattices is given in [9]. Whereas the non-universal properties of SAWs on diluted lattices are intensively studied by means of Monte-Carlo simulations, exact enumeration and analytic calculations much less is known about their universal scaling properties.

In the magnetic systems, the presence of point-like uncorrelated (or short-range correlated) quenched disorder has nontrivial effect on their critical behaviour only if the specific heat critical exponent α is positive [10]. This statement is often called Harris criterion. Long-range correlated disorder changes critical behaviour in a more complicated way [11],[12],[13],[14]. Although critical exponent α of a SAW on $d = 3$ pure lattice is positive ($\alpha(d = 3) = 0.236$), the arguments of Harris [15] lead to conclusion that short-range correlated disorder is irrelevant taken the system is far from percolation threshold. This statement was confirmed by the RG results [16]. On the other hand, SAWs on diluted lattice at the percolation threshold obey a new scaling law [17]. For example, $\epsilon = 6 - d$-expansion for the critical exponent ν reads:

$$\nu = \frac{1}{2} + \frac{\epsilon}{42} + \ldots, \quad \epsilon > 0. \tag{2}$$

Influence of the long-range-correlated disorder on scaling properties of SAWs remains unclear and up to our knowledge has not been considered yet. Here, we address the question of the relevance of the long-range-correlated disorder with correlations falling by a power law at large distances [11] on the asymptotic behaviour of SAWs.

The paper is organized as follows: in the next section we present the model, discuss the renormalization procedure and obtain RG functions. In section 3 we analyze fixed points corresponding to various types of critical behaviour and find, that the long-range-correlated disorder causes new scaling properties of SAWs. Section 4 concludes our study.

2. The model

Let us consider a model of an m-vector magnet with quenched long-range-correlated “random-temperature” disorder, introduced by Weinrib and Halperin [11]. Here, the inhomogeneities in the system cause fluctuations in the local transition temperature $T_c(\vec{x})$, characterised by a correlation function $g(|\vec{x} - \vec{y}|) = \langle T_c(\vec{x}) T_c(\vec{y}) \rangle - \langle T_c \rangle^2$, that falls off with distance according to the power law:

$$g(x) \sim x^{-a} \tag{3}$$

for large x, where a is a constant. Performing the Fourier-transformation, one gets for small k:

$$g(k) \sim v_0 + w_0 k^{a-d}. \tag{4}$$

Note, that in the case of random uncorrelated point-like defects the site-occupation correlation function reads: $g(|\vec{x} - \vec{y}|) \sim \delta(\vec{x} - \vec{y})$, so its Fourier transform obeys:

$$g(k) \sim v_0. \tag{5}$$

Comparing (4) and (5), one can conclude, that the case $g(x) \sim x^{-d}$ corresponds to random uncorrelated point-like disorder. Moreover, different integer values of a correspond to uncorrelated extended impurities of random orientations. So, correlation function of a form (3) with $a = d - 1$ describes straight lines of impurities of random orientation whereas random planes of impurities correspond to $a = d - 2$ [20].

By making use of the replica method and by taking average over different configurations of quenched disorder, one gets an effective Hamiltonian of an m-vector model with long-range-correlated disorder [11]:

$$\mathcal{H}_{eff} = \sum_{\alpha=1}^{n} \int \mathrm{d}^d x \Big[\frac{1}{2} (\mu^2 \vec{\phi}_\alpha^2 + (\vec{\nabla} \vec{\phi}_\alpha)^2 + \frac{u_0}{4!} (\vec{\phi}_\alpha^2)^2 \Big] -$$

$$-\sum_{\alpha,\beta=1}^{n}\int \mathrm{d}^d x \mathrm{d}^d y g(|\vec{x}-\vec{y}|)\vec{\phi}_\alpha^2(x)\vec{\phi}_\beta^2(y). \quad (6)$$

Here, $\vec{\phi}_\alpha$ is an m-component field: $\vec{\phi}_\alpha = \{\phi_\alpha^1 \cdots \phi_\alpha^m\}$, μ and u_0 are bare mass and coupling, the interaction vertex $g(x)$ is the correlation function with Fourier image (4), Greek indices denote replicas and replica limit $n \to 0$ is implied.

Passing in (6) to the Fourier image and taking into account (4), one ends up with an effective Hamiltonian containing three bare couplings u_0, v_0, w_0. For $a > d$ the w_0-term is irrelevant in a RG sense and one obtains an effective Hamiltonian of a quenched diluted (short-range correlated) m-vector model [19] with two couplings u_0, v_0. For $a < d$ we have, in addition to the momentum-independent couplings u_0 the momentum dependent one $w_0 k^{a-d}$. Note that $g(k)$ must be positively definite as a Fourier image of the correlation function. From here one gets that at small k $w_0 \geq 0$. Coupling u must be positive, otherwise a pure system undergoes a 1st order transition.

The critical behaviour of the model with $m \geq 1$ has been investigated by means of RG approach [11], [13], [14]. We will be interested in mapping model (6) to a polymer limit $m \to 0$ in order to interpret this as a model for SAWs in disordered media. Note, that such a limit is not trivial. So, for the case $u_0 \neq 0, v_0 \neq 0, w_0 = 0$ the "naive" RG analysis leads to controversial results about absence of a stable fixed point and thus to absence of the second order phase transition [3]. As it was shown by Kim [16], once the limit $m, n \to 0$ has been taken, both u_0 and v_0 terms are of the same symmetry, and one gets an effective Hamiltonian with one coupling of $O(mn = 0)$ symmetry. This leads to the conclusion that a weak quenched uncorrelated disorder is irrelevant for SAWs.

Our present analysis is based on the crucial observation of Kim, allowing one to pass in (6) in the limit $m, n \to 0$ to the effective Hamiltonian containing two coupling $U_0 = u_0 - v_0$ and w_0 (in what follows below we will keep the notation u_0 for this new coupling U_0).

In order to describe the peculiarities of the critical behaviour of the model, we shall use a field-theoretical RG method. We choose the massive field theory scheme performing renormalization at non-zero mass and zero external momenta [21] leading to the Callan-Symanzik equation for the renormalized one-particle irreducible vertex functions $\Gamma_R^{(i)}$. However, in our case the normalization conditions are written both in fixed d and a [13].

The renormalized mass m and couplings u, w are defined by:

$$m^2 = \Gamma_R^{(2)}(k, -k, m^2, u, w)|_{k=0},$$

$$m^{4-d}u = \Gamma_{R,u}^{(4)}(\{k\}, m^2, u, w)|_{k=0},$$

$$m^{4-a}w = \Gamma_{R,w}^{(4)}(\{k\}, m^2, u, w)|_{k=0}.$$

Here, $\Gamma_{R,u}^{(4)}$ and $\Gamma_{R,w}^{(4)}$ are the contributions to four-point vertex function $\Gamma_R^{(4)}$, corresponding to u- and w-term symmetry, respectively. Change of couplings u, w under renormalization defines a flow in parametric space, governed by corresponding β-functions $\beta_u(u,w), \beta_w(u,w)$. The fixed points u^*, w^* of this flow are given by solutions of the system of equations: $\beta_u(u^*, w^*) = 0$, $\beta_w(u^*, w^*) = 0$. The stable fixed point is defined as the fixed point where the stability matrix:

$$B_{ij} = \frac{\partial \beta_{u_i}}{\partial u_j}, \quad u_i = \{u, w\} \quad (7)$$

possess eigenvalues λ_{u_i} with positive real parts. The stable fixed point corresponds to the critical point of the system. At this point the critical exponents are defined by: $\eta = \gamma_\phi(u^*, w^*)$, $\nu^{-1} = 2 - \gamma_\phi(u^*, w^*) - \gamma_{\phi^2}(u^*, w^*)$ where $\gamma_\phi, \gamma_{\phi^2}$ are the coefficients of Callan-Symanzik equation, expressed by the renormalizing factors for a field and a two-point function with ϕ^2 insertion. Performing the above described procedure, the RG functions are obtained in a form of series in renormalized couplings. Some details of calculations are given in Appendix. In the one-loop approximation we get:

$$\beta_u = -\varepsilon\left[u - \frac{4}{3}u^2 I_1\right] - \delta 2uw\left[I_2 + \frac{1}{3}I_4\right] + (2\delta - \varepsilon)\frac{2}{3}w^2 I_3, \quad (8)$$

$$\beta_w = \delta\left[w + \frac{2}{3}w^2 I_2\right] - \varepsilon\frac{2}{3}\left[wuI_1 - w^2 I_4\right], \quad (9)$$

$$\gamma_{\phi^2} = \varepsilon\frac{u}{3}I_1 - \delta\frac{w}{3}I_2, \quad \gamma_\phi = \delta\frac{w}{3}I_4. \quad (10)$$

Here, I_i are one-loop integrals which depend on space dimension d and parameter a. Their explicit expressions are given in the Appendix by formulas (14)-(17).

3. ε, δ-expansion

In order to obtain the qualitative characteristics of the critical behaviour of the model one can proceed in two ways. The first scheme consists in initial fixing the values d and a and treating power series in renormalized couplings u, w. For the magnet with long-range-correlated impurities such approach was exploited in [13]. Note, that (possible) asymptotic nature of the series for RG functions does not allow their immediate analysis. It postulates knowledge at least of the next order of perturbation theory with application of resummation procedure [13] involving some symmetry arguments [12]. Here, for quantitative analysis of the first order results we apply the double ε, δ-expansion, proposed by Weinrib and Halperin [11]. Substituting the loop integrals in (8)-(10) by their expansion:

$$I_1 = \frac{1}{\varepsilon}\left(1-\frac{\varepsilon}{2}\right), \qquad I_2 == \frac{1}{\delta}\left(1-\frac{\delta}{2}\right),$$

$$I_3 = \frac{1}{2\delta-\varepsilon}\left(1-\frac{2\delta-\varepsilon}{2}\right), \qquad I_4 = \frac{1}{\delta}\left(\frac{\delta-\varepsilon}{2}\right),$$

we find system of equations for the fixed points [22]:

$$u(\varepsilon - u + 3w) - 2w^2 = 0, \tag{11}$$

$$w(\delta - u/2 + w) = 0. \tag{12}$$

u^*	w^*	λ_u	λ_w
0	0	$-\varepsilon$	$-\delta$
ε	0	ε	$\varepsilon/2-\delta$
$\frac{2\delta^2}{(\varepsilon-\delta)}$	$-\frac{\delta(\varepsilon-2\delta)}{(\varepsilon-\delta)}$	$\frac{1}{2}\{\varepsilon-4\delta\pm\sqrt{\varepsilon^2-4\varepsilon\delta+8\delta^2}\}$	

Table 1. Fixed points and stability matrix eigenvalues in the first order of ε, δ - expansion.

Fixed points of (11),(12) and the stability matrix (7) eigenvalues λ_u, λ_w are given in the table 1. One gets following conclusions from the first order results:

- the Gaussian fixed point ($u^*=0, w^*=0$) is stable for $\varepsilon<0$, $\delta<0$, i.e. $d>4$, $a>4$;
- the pure SAW fixed point ($u^*\neq 0, w^*=0$) is stable for $\varepsilon>0$, $\varepsilon/2-\delta>0$, or $d<4$, $a>2+d/2$;

- the long-range SAW fixed point ($u^*\neq 0, w^*\neq 0$) is stable for $d<4$, $a<2+d/2$.

For the SAW square end-to-end distance critical exponent one gets $\nu = 1/2+\varepsilon/16$ in the pure fixed point and $\nu = 1/2+\delta/8$ in the long-range fixed point. However, taking into account that accessible values of couplings are $u>0$, $w>0$ one finds that the long-range stable fixed point is accessible only for $\delta<\varepsilon<2\delta$, or $d<a<2+d/2$. The final results for ν at $d<4$ read:

$$\nu = \begin{cases} 1/2+\varepsilon/16 & \text{for} \quad \delta<\varepsilon/2, \\ 1/2+\delta/8 & \text{for} \quad \varepsilon/2<\delta<\varepsilon. \end{cases} \tag{13}$$

4. Conclusions

In this paper we studied the scaling behaviour of SAWs on a d-dimensional lattice with quenched defects obeying power-law correlations for large separation. To this end we applied the field-theoretical RG approach, performing renormalization for the fixed mass and zero external momenta [21]. In the case of magnetic systems with long-range-correlated quenched disorder similar approach was used in [13]. Out analysis exploits observation of Kim [16] about symmetry properties of quenched m-vector model in de Gennes limit $m\to 0$. In order to analyse critical behaviour of the model we performed the first order ε, δ-expansion and were lead to conclusion that SAWs on a lattice with long-range-correlated disorder are governed by a new exponent $\nu^{long} = 1/2+\delta/8$, $(\varepsilon/2<\delta<\varepsilon)$. This is to be compared with a first order result for SAWs on a “pure” lattice: $\nu^{pure} = 1/2+\varepsilon/16$, $(\varepsilon>0)$. However these first order results are to be considered as qualitative ones as far as they are based on an analysis of divergent series. In particular, the region of stability of a “long-range” behaviour does not match the region of relevance of the coupling w (similar feature is observed for $m>0$ as well). A more refined analysis relying on higher-order calculations with convenient resummation technique will be a subject of forthcoming study.

Appendix

Here, we give some details of the perturbation theory calculations in the one-loop approximation. Working in the so-called faithful representation [18] for the Feynman graphs of the vertex functions $\Gamma^{(i)}$, we represent two vertices u and wk^{a-d} as it is shown in figure (1).

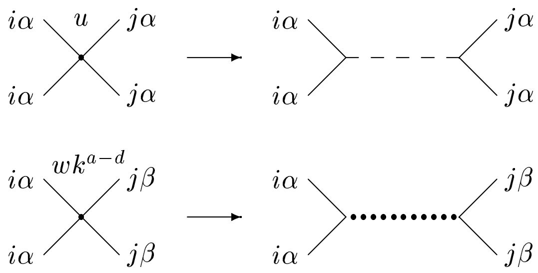

Figure 1. Diagrammatic representation of the interaction vertices. Latin indices correspond to the dimension of field, and Greek indices denote replicas.

Then for the functions $\Gamma^{(2)}$ and $\Gamma^{(4)}$one gets diagrams shown in figures 2-4. Note, that as far as we are working in double limit $n, m \to 0$, the only contributing diagrams are those without closed loops.

Figure 2. One-loop contributions to the two-point vertex function $\Gamma^{(2)}$. Here and in figures 5,6 the indices are omitted.

For the one-loop integrals one gets the following expressions:

$$I_1 = \int \frac{d\vec{q}}{(q^2+1)^2}, \tag{14}$$

this integral corresponds to the 1-st and 2-nd diagrams in figure 3 and to the 2-nd diagram in figure 4,

$$I_2 = \int \frac{d\vec{q}\, q^{a-d}}{(q^2+1)^2} \tag{15}$$

corresponds to 4-th, 5-th and 6-th diagrams in figure 3 and to the 1-st diagram in figure 4,

$$I_3 = \int \frac{d\vec{q}\, q^{2(a-d)}}{(q^2+1)^2} \tag{16}$$

corresponds to 3-rd diagram in figure 3. Note that contrary to familiar $O(m)\,\phi^4$ theory, here because of the k-dependence of one of the vertices

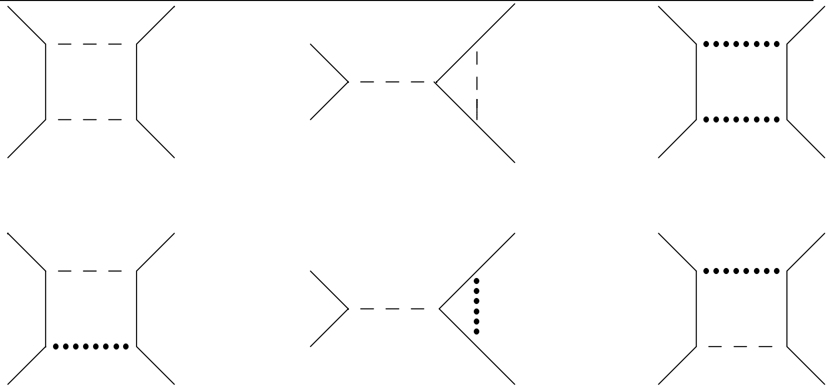

Figure 3. One-loop contributions to the four-point vertex function $\Gamma_u^{(4)}$

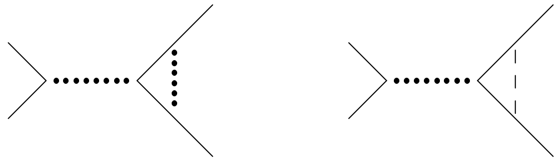

Figure 4. One-loop contributions to the four-point vertex function $\Gamma_w^{(4)}$

one gets non-zero contribution to $\partial\Gamma^{(2)}/\partial k^2|_{k^2=0}$ already on the one-loop level, i.e. derivative of the 2-nd diagram in figure 2 is non-zero and reads:

$$I_4 = \frac{\partial}{\partial k^2}\left[\int \frac{d\vec{q}\, q^{a-d}}{[q+k]^2+1)}\right]_{k^2=0}. \tag{17}$$

References

1. des Cloizeaux J. and Jannink G. Polymers in Solution. Clarendon Press, Oxford, 1990; Schäfer L. Universal Properties of Polymer Solutions as Explained by the Renormalization Group. Berlin, Springer, 1999.
2. de Gennes G.-P. Scaling Concepts in Polymer Physics. Ithaca and London, Cornell University Press, 1979.
3. Chakrabarti B.K., Kertész J. // Z. Phys. B - Condensed Matter, 1981, vol. 44, p. 221-223.

4. Lee S., Nakanishi H. // Phys. Rev. Lett., 1987, vol. 61, No. 18, p. 2022-2025.
5. Lee S., Nakanishi H., Kim Y. //Phys. Rev. B, 1988, vol. 39, No. 13, p. 9561-9572.
6. Machta J., Kirkpatrick T. R. //Phys. Rev. A, 1990, vol. 41, No. 10, p. 5345-5356.
7. Izyumov A.V., Samokhin K.B. // J. Phys. A: Math. Gen., 1999, vol. 32, p. 7843-7849.
8. Altenberger A.R., Siepmann I.I., Dahler J.S. // Phys. A, 1999, vol. 272, p. 22-47.
9. Barat K., Chakrabarti B.K. // Phys. Reports, 1995, vol. 258, p. 377-411.
10. Harris A. B. // J. Phys. C, 1974, vol. 7, p. 1671.
11. Weinrib A., Halperin B. I. // Phys. Rev. B, 1983, vol. 27, No. 1, p. 417.
12. Dorogovtsev S. // J. Phys. A: Math. Gen., 1984, vol. 17, p. 677-679.
13. Prudnikov V. V., Prudnikov P. V., Fedorenko A. A. // J. Phys. A: Math. Gen., 1999, vol. 32, p. L399; Prudnikov V. V., Prudnikov P. V., Fedorenko A. A. // J. Phys. A: Math. Gen., 1999, vol. 32, p. 8587.
14. Korutcheva E., de la Rubia F.G. // Phys. Rev. B, 1998, vol. 58, No. 9, p. 5153-5156.
15. Harris A.B. // Z. Phys. B - Condensed Matter, 1983, vol. 49, p. 347-349.
16. Kim Y. // J. Phys. C: Solid State Phys., 1983, vol. 16, p. 1345-1352.
17. Meir Y., Harris A. // Phys. Rev. Lett., 1989, vol. 63, No. 26, p. 2819-2822.
18. see, e.g. Amit D. Field Theory, Renormalization Group and Critical Phenomena. New-York, Mc-Grew Hill, 1978.
19. Grinstein G., Luther A. // Phys. Rev. B, 1976, vol. 13, p.1329.
20. The case of extended impurities of parallel orientation is considered in: Boyanovsky D., Cardy J. // Phys. Rev. B, 1982, vol. 26, p.154.
21. Parisi G., in Proceedings of the Cargrése Summer School, 1973, (unpublished); J. Stat. Phys., 1980, vol. 23, p. 49-82.
22. To simplify the resulting expressions, we passed to new variables: $u \equiv 4/3u$, $w \equiv 2/3w$.

Вікторія Богданівна Блавацька
Крістіан фон Фербер
Юрій Васильович Головач

Випадкові блукання із самоуниканнями у середовищі із далекосяжно скорельованим безладом

Роботу отримано 25 липня 2000 р.

Затверджено до друку Вченою радою ІФКС НАН України

Рекомендовано до друку семінаром відділу СТеКС

Виготовлено при ІФКС НАН України