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Critical slowing down in random anisotropy magnets

M.Dudka [1,2], R.Folk [2], Yu.Holovatch [1,2,3], G.Moser [4]

[1] Institute for Condensed Matter Physics
of the National Academy of Sciences of Ukraine,
79011 Lviv, Ukraine

[2] Institut für Theoretische Physik,
Johannes Kepler Universität Linz,
A–4040, Linz, Austria

[3] Ivan Franko National University of Lviv,
79005 Lviv, Ukraine

[4] Institut für Physik und Biophysik,
Universität Salzburg,
A–5020 Salzburg, Austria

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We study the purely relaxational critical dynamics with non-conserved order parameter (model A critical dynamics) for three-dimensional magnets with disorder in a form of the random anisotropy axis. For the random axis anisotropic distribution, the static asymptotic critical behaviour coincides with that of random site Ising systems. Therefore the asymptotic critical dynamics is governed by the dynamical exponent of the random Ising model. However, the disorder effects considerably the dynamical behaviour in the non-asymptotic regime. We perform a field-theoretical renormalization group analysis within the minimal subtraction scheme in two-loop approximation to investigate asymptotic and effective critical dynamics of random anisotropy systems. The results demonstrate the non-monotonic behaviour of the dynamical effective critical exponent $z_{\mathbf{eff}}$.

Key words: *critical dynamics, disordered systems, random anisotropy, renormalization group*

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1. Introduction

The concept of scaling plays a central role in modern theory of critical phenomena [1]. Introducing a set of appropriate scaling variables, a large amount of experimental and numerical data can be described by a few scaling functions [2]. The most prominent effect in dynamical critical phenomena is *critical slowing down* which consists in an increase of the relaxation time approaching the critical point. This is induced by the divergence of the correlation length ξ at the critical point which also causes the relaxation time τ to diverge with the dynamical critical exponent z:

$$\tau \sim \xi^z. \tag{1}$$

Renormalization group (RG) theory, with its concepts of invariance of the system at the critical point against changes in length scale gives a basis to universality in connection with fixed points and static [3] and dynamic scaling [4].

However, this holds only in the asymptotic region in the vicinity of the critical point. Further away, scaling breaks down and the description of critical phenomena becomes more complicated and involves non-universal characteristics both in statics and in dynamics [1,5].

Another important point in observing the behaviour in certain universality classes is the homogeneity of the system under consideration. Therefore the effect of impurities on critical behaviour is of considerable interest. However it turned out that the disordered systems may also show scaling within a certain universality class. This might be the universality class of a pure system or a new one [6–9]. Moreover the changes introduced by the disorder depend on the type of this disorder; namely, whether they are introduced by dilution (random site [10] or random bond [11] systems), or as a random field [12], random connectivity [9,13] or as an anisotropy [14]. The defects may be correlated [7,8] or noncorrelated. It may even happen that the second order transition of the pure system is destroyed [16,17].

It turned out that considering a specific system with defects, the behaviour near the critical point seems to be non-universal. Knowing that the non-asymptotic behaviour is non-universal it became necessary to study the non-asymptotic behaviour of such systems in more detail. Indeed in many systems (e.g., with site disorder) the effective critical behaviour is capable of explaining the experimental situation [11,18].

RG investigations of dynamic critical behaviour is in many cases technically much more involved in comparison with statics. Thus, the results for dynamics are known in much lower approximations (in most cases only up to two loop order). On the other hand for the dynamics of magnetic systems with mode coupling terms the dynamical critical exponent is known exactly or contains only a static exponent.

In this paper, we will present the analysis of the dynamical critical behaviour of random anisotropy magnets which constitute a large class of disordered systems [14]. In order to give some examples, the majority of the amorphous rare-earth alloys are recognized as random anisotropy magnets [14], certain crystalline compounds with rear-earth component belong to this class as well [19]. Random anisotropy also characterizes the molecular based magnets [20], nanocrystalline materials [21], as well as granular systems [22]. Moreover, the analysis involving random anisotropy also found its application in interpreting the phase transition in liquid crystals in porous media [23].

The model currently used for the description of random anisotropy systems was introduced in the early 70-ies by Harris, Plischke, and Zuckermann [24]. It describes m-component spins located on the sites of a d-dimensional lattice, each spin being subjected to a local anisotropy of random orientation. The Hamiltonian of the random anisotropy model (RAM) reads:

$$\mathcal{H} = -\sum_{\mathbf{R},\mathbf{R}'} J_{\mathbf{R},\mathbf{R}'} \vec{S}_{\mathbf{R}} \vec{S}_{\mathbf{R}'} - D_0 \sum_{\mathbf{R}} (\hat{x}_{\mathbf{R}} \vec{S}_{\mathbf{R}})^2. \tag{2}$$

Here, $\vec{S}_{\mathbf{R}} = (S^1_{\mathbf{R}}, \ldots, S^m_{\mathbf{R}})$, vectors $\mathbf{R}$ span sites of a d–dimensional cubic lattice, $D_0 > 0$ is an anisotropy constant, $\hat{x}$ is a random unit vector specifying the direction of the local anisotropy axis. The interaction $J_{\mathbf{R},\mathbf{R}'}$ is assumed to be ferromagnetic. Note, that for the Ising-like magnets, $m = 1$, the last term in (2) is just a constant, therefore the random anisotropy is present for $m > 1$ only.

Below, we will consider quenched disorder, when the vectors $\hat{x}_{\mathbf{R}}$ in (2) are randomly distributed with a distribution function $p(\hat{x})$ and fixed in a certain configuration. It is well established by now, that the anisotropy axis distribution plays a crucial role in originating the low-temperature phase in the RAM. In particular, when the random vectors $\hat{x}_{\mathbf{R}}$ point with equal probability towards any direction, such a distribution may be called an *isotropic* one, the ferromagnetic ordering is impossible [25] for spatial dimension $d \leqslant 4$. However, it may occur for an anisotropic distribution. In statics, this situation was corroborated by the RG studies of RAM [26–30] restricting $\hat{x}$ to be pointed along one of the $2m$ directions of the axes $\hat{k}_i$ of a hypercubic lattice (*cubic distribution*):

$$p(\hat{x}) = \frac{1}{2m} \sum_{i=1}^{m} \left[\delta^{(m)}(\hat{x} - \hat{k}_i) + \delta^{(m)}(\hat{x} + \hat{k}_i) \right] \tag{3}$$

with Kronecker deltas $\delta(y)$. In this case, there occurs the second order phase transition into the magnetically ordered low-temperature phase. Asymptotically it is characterized by the critical exponents of the random-site Ising model as suggested already in [27] and confirmed later in [28–30].

The studies of static criticality of random anisotropy magnets are far from being as intensive as those of the diluted magnets [10], and even less is known about their dynamic critical behaviour. The dynamical models for systems with *isotropic* distribution of local anisotropy axis were considered in [31–33], the first order RG calculations were performed in [34]. However, the problem of dynamic critical behaviour of a RAM with an anisotropic random axis distribution has remained untouched so far.

In this paper, we consider a purely relaxational dynamics of a three-dimensional ($d = 3$) RAM with non-conserved order parameter (model A in classification of [4]) and a *cubic* random axis distribution (3). The static critical behaviour of such magnets (note, with $m > 1$) belongs to the universality class of a random-site Ising model [27–30], for which the heat capacity does not diverge [10]. Therefore, the critical dynamics of such a model for any m is governed in asymptotics by the model A random-site Ising magnet dynamical critical exponent. However in the non-asymptotic region, the model possesses a rich effective critical behaviour, as will be shown by our subsequent analysis. Assuming that it is this effective behaviour which is observed both in experiments and in the MC simulations, it is important to have a RG prediction for typical scenarios of the approach to criticality in random anisotropy magnets.

The rest of the paper is organized as follows: in the next section 2 we present the Langevin equations governing the model A dynamics and describe the renormalization procedure. In section 3 we give results of our calculations obtained in two-loop approximation and display possible scenarios of effective critical behaviour. Conclusions and outlook are given in section 4.

2. Model equations and renormalization

We consider the dynamics for model (2) with random axis distribution (3) to be relaxational with non-conserved m-component order parameter $\vec{\varphi}_0 \equiv \vec{\varphi}_0(R)$. In this case the relaxation of the order parameter is described by the Langevin equation:

$$\frac{\partial \varphi_{i,0}}{\partial t} = -\mathring{\Gamma}\frac{\partial \mathcal{H}}{\partial \varphi_{i,0}} + \theta_{\varphi_i} \qquad i = 1, \ldots, m, \tag{4}$$

with the Onsager coefficient $\mathring{\Gamma}$, stochastic forces θ_{φ_i} obeying the Einstein relations:

$$\left\langle \theta_{\varphi_i}(R,t)\theta_{\varphi_j}(R',t')\right\rangle = 2\mathring{\Gamma}\delta(R-R')\delta(t-t')\delta_{ij}\,, \tag{5}$$

and the disorder-dependent equilibrium effective Hamiltonian $\mathcal{H}$:

$$\mathcal{H} = \int \mathrm{d}^d R \left\{ \frac{1}{2}\left[|\nabla\vec{\varphi}_0|^2 + \mathring{r}|\vec{\varphi}_0|^2\right] + \frac{v_0}{4!}|\vec{\varphi}_0|^4 - D(\hat{x}\vec{\varphi}_0)^2 \right\}. \tag{6}$$

In (6), the field $\vec{\varphi}_0$ is an m-component vector, D is proportional to the anisotropy constant of the spin Hamiltonian (2) with D_0; $\mathring{r}$ and v_0 are defined by D_0 and the fourth order coupling of the m-vector magnet (see [29] for details).

We treat the critical dynamics of the disordered model within the field theoretical RG method based on the Bausch-Janssen-Wagner formulation [35], where the appropriate Lagrangians are studied. For the model equations (4)–(5) the Lagrangian reads:

$$\mathcal{L} = \int \mathrm{d}^d R \mathrm{d}t \sum_i \tilde{\varphi}_{i,0}\left[\frac{\partial \varphi_{i,0}}{\partial t} + \mathring{\Gamma}\frac{\delta \mathcal{H}}{\delta \varphi_{i,0}} - \mathring{\Gamma}\tilde{\varphi}_{i,0}\right] \tag{7}$$

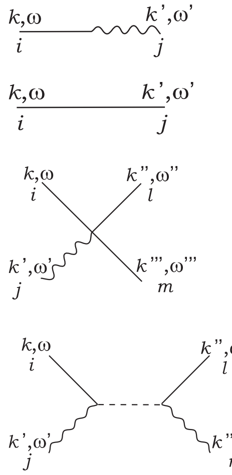
with a new auxiliary response m-component field $\tilde{\vec{\varphi}}_0$. Here and below the sums over field components span values from 1 to m.

Studying critical properties of disordered systems one should average over the random degrees of freedom. In order to treat the quenched disorder, the replica trick is often used in statics [10]. However, it was established in [32] that in dynamics it is not necessary to make use of the replica trick: it is sufficient to average over the random variables $\hat{x}$ with their distribution (3). Then the Lagrangian for the model reads:

$$\mathcal{L} = \Bigg\{ \int \mathrm{d}^d R \mathrm{d}t \sum_i \tilde{\varphi}_{i,0}\Bigg[\left(\frac{\partial}{\partial t} + \mathring{\Gamma}(r_0 - \nabla^2)\right)\varphi_{i,0} - \mathring{\Gamma}\tilde{\varphi}_{i,0} + \frac{\mathring{\Gamma} v_0}{3!}\varphi_{i,0}\sum_j \varphi_{j,0}\varphi_{j,0} + \frac{\mathring{\Gamma} y_0}{3!}\varphi_i^3\Bigg]$$

$$+\int \mathrm{d}t' \sum_i \tilde{\varphi}_{i,0}(t)\varphi_{i,0}(t)\left[\frac{\mathring{\Gamma}^2 u_0}{3!}\sum_j \tilde{\varphi}_{j,0}(t')\varphi_{j,0}(t')+\frac{\mathring{\Gamma}^2 w_0}{3!}\tilde{\varphi}_{i,0}(t')\varphi_{i,0}(t')\right]\Bigg\}. \quad (8)$$

Here, r_0 is proportional to the temperature distance to the mean field critical point and the bare couplings are $u_0 > 0$, $v_0 > 0$, $w_0 < 0$. Moreover, u_0 and w_0 are connected to the moments of the distribution (3) in such a way that $w_0/u_0 = -m$. The term with y_0 does not appear after averaging over disorder. However it should be added since it will be generated within the renormalization procedure. The set of static couplings $\{u_0, v_0, w_0, y_0\}$ will be denoted below by $\{u_{0,i}\}$, $i = 1, \dots, 4$.



$$G(k,\omega)\delta(k+k')\delta(\omega+\omega')\delta_{i,j}$$

$$C(k,\omega)\delta(k+k')\delta(\omega+\omega')\delta_{i,j}$$

$$\Gamma A\delta(k+k'+k''+k''')\delta(\omega+\omega'+\omega''+\omega''')$$

$$\Gamma^2 B\delta(k+k')\delta(k''+k''')\delta(\omega+\omega')\delta(\omega''+\omega''')$$

Figure 1. Elements for construction of Feynman diagrams. The response function and the correlation function read: $G(k,\omega) = (-\mathrm{i}\omega + \mathring{\Gamma}(r+k^2))^{-1}$, $C(k,\omega) = 2\mathring{\Gamma}(|-\mathrm{i}\omega + \mathring{\Gamma}(r+k^2)|)^{-2}$. A stands for $v_0/3! \cdot (\delta_{i,j}\delta_{l,m} + \delta_{i,l}\delta_{j,m} + \delta_{i,m}\delta_{j,l})/3$ or for $y_0/3!\,\delta_{i,j}\delta_{j,l}\delta_{l,m}$ while B is equal to $u_0/3!\,\delta_{i,j}\delta_{l,m}$ or to $w_0/3!\,\delta_{i,j}\delta_{j,l}\delta_{l,m}$.

With the Lagrangian (8) depending on the bare quantities (denoted by the sub- and superscripts "o") we analyze within the field theory [3] the dynamical vertex functions. As far as the static RG functions have been obtained before [15,29], we need only to calculate the two-point dynamical vertex function $\mathring{\Gamma}^{i,j}_{\tilde{\varphi}\varphi}(r_0, \{u_{i,0}\}, \mathring{\Gamma}, k, \omega)$ $= \mathring{\Gamma}_{\tilde{\varphi}\varphi}(r_0, \{u_{i,0}\}, \mathring{\Gamma}, k, \omega)\delta_{i,j}$. The calculations are performed using Feynman diagrams, elements for them are given in figure 1 whereas the one- and two-loop contributions to $\mathring{\Gamma}^{i,j}_{\tilde{\varphi}\varphi}$ are depicted in figure 2.

We perform renormalization of $\mathring{\Gamma}_{\tilde{\varphi}\varphi}$ within the minimal subtraction scheme [3]. In this scheme, in order to define renormalized static (φ, r, $\{u_i\}$) and dynamic ($\tilde{\varphi}$, Γ) fields and couplings, the renormalization factors Z_φ, Z_r, Z_{u_i} and $Z_{\tilde{\varphi}}$, Z_Γ are introduced by:

$$\varphi = Z_\varphi^{-1/2}\varphi_0,\ \tilde{\varphi} = Z_{\tilde{\varphi}}^{-1/2}\tilde{\varphi}_0,\ r = Z_r^{-1}r,\ u_i = \mu^{-\varepsilon} Z_{u_i}^{-1} Z_\varphi^2 A_d u_{0,i},\ \Gamma = Z_\Gamma \mathring{\Gamma}. \quad (9)$$

Here, μ is the external momenta scale, $\varepsilon = 4 - d$, and $A_d = \Gamma(1-\varepsilon/2)\Gamma(1+\varepsilon/2)\Omega_d(2\pi)^{-d}$ is a geometrical factor.

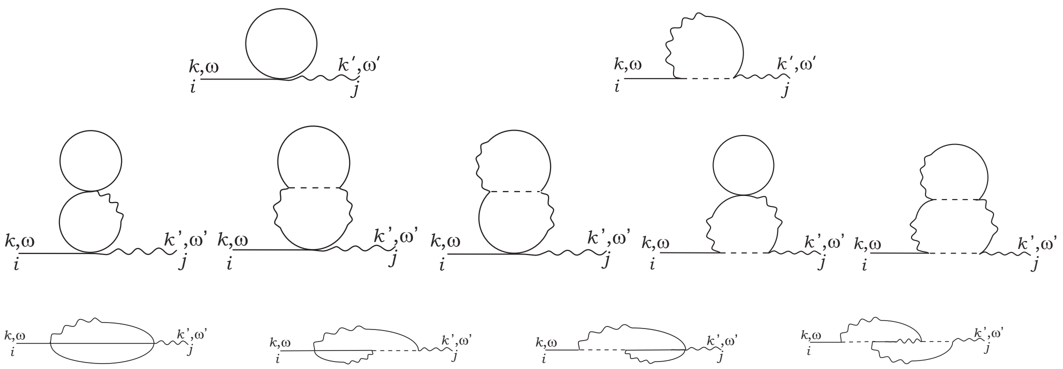


Figure 2. Diagrams of the function $\mathring{\Gamma}^{i,j}_{\tilde{\varphi}\varphi}(r_0, \{u_{i,0}\}, \mathring{\Gamma}, k, \omega)$ up to two-loop order. First two terms represent one-loop contribution, while the rest of the diagrams are of two-loop order.

The critical behaviour of the system is described by the following RG functions:

$$\beta_{u_i}(\{u_i\}) = \mu \frac{\partial u}{\partial \mu}\bigg|_0, \qquad \zeta_r(\{u_i\}) = -\mu \frac{\partial \ln Z_r}{\partial \mu}\bigg|_0, \qquad \zeta_\Gamma(\{u_i\}) = \mu \frac{\partial \ln Z_\Gamma}{\partial \mu}\bigg|_0, \tag{10}$$

where the symbol $\big|_0$ means differentiation at fixed bare parameters. The β-functions determine the RG flow of couplings under renormalization:

$$\ell \frac{\mathrm{d}u_i}{\mathrm{d}\ell} = \beta_{u_i}(\{u_i\}), \qquad i = 1, \ldots, 4, \tag{11}$$

and the flow parameter ℓ is related to the distance from the critical point. Subsequently, an information about the critical behaviour of a system can be obtained from the analysis of the fixed points (FPs) of the flow equations (11). A FP $\{u_i^*\}$ is defined as simultaneous zero of all β-functions:

$$\beta_{u_i}(\{u_i^*\}) = 0, \qquad i = 1, \ldots, 4. \tag{12}$$

The stable and accessible FP (from initial conditions) corresponds to the critical point of the system. A FP is stable if all eigenvalues ω_i of the stability matrix $B_{i,j} = \partial\beta_{u_i}/\partial u_j$ calculated at this FP have positive real parts.

The FP values of the RG ζ-functions (10) determine the asymptotic values of critical exponents. In particular, the dynamical asymptotic critical exponent z (1) is given at the stable and accessible FP by:

$$z = 2 + \zeta_\Gamma(u^*, v^*, w^*, y^*). \tag{13}$$

While the effective dynamical exponent z_{eff} is calculated in the non-asymptotical region, where the renormalized couplings did not reach their FP values and it is defined by the solutions of the flow equations (11):

$$z_{\mathrm{eff}} = 2 + \zeta_\Gamma(u(\ell), v(\ell), w(\ell), y(\ell))\,. \tag{14}$$

We neglect contributions to z_{eff} coming from the amplitude function because they are considered to be small.

3. Results

The static RG functions within the minimal subtraction scheme are known in two-loop [15] approximation. Within the massive renormalization they have been calculated already in five-loop [30] approximation. Calculating the dynamical function ζ_Γ within two-loop order we use the static RG functions of the same order [15]. Since the series for these static functions are known to be asymptotic at best we use Padé-Borel resummation scheme [36] described in detail in [15]. The FPs values and solutions of flow equations [15] are obtained based on these functions. In the present study we use the new two-loop expression for the function ζ_Γ. The last is derived from the vertex function $\mathring{\Gamma}^{i,j}_{\tilde{\varphi}\varphi}$ discussed in section 2 and reads:

$$\begin{aligned}\zeta_\Gamma \quad = \quad & -\frac{(u+w)}{3}+\frac{(6\ln(4/3)-1)}{24}(y^2+\frac{2}{3}vy+\frac{m+2}{3}v^2) \\ & +\frac{1}{36}\big((m+2)uv+5u^2+5w^2+10uw+3yw+3vw+3uy\big)\,. \qquad (15)\end{aligned}$$

The FP equations (12) solved for the static β-functions at fixed space dimension $d=3$ [37] result in 16 FPs [15,29]. The region of physical importance $u>0$, $v>0$, $w<0$ includes 10 FPs. Below we list the most interesting FPs together with the asymptotic value for the z exponent (for the numerical values of the FPs coordinates obtained in two-loop approximation using the Padé-Borel resummation see table 2 of [15], the value of the z exponent, however, is calculated by a direct substitution of FP coordinates into equation (13)):

- Gaussian FP I: $u^*=v^*=w^*=y^*=0$; $z(\forall m)=2$;
- pure FP II: $v^*\neq 0$, $u^*=w^*=y^*=0$; $z(m=2)=2.053$, $z(m=3)=2.051$;
- polymer FP III: $u^*\neq 0$, $v^*=w^*=y^*=0$; $z(\forall m)=1.815$;
- Ising FP V: $y^*\neq 0$, $u^*=v^*=w^*=0$; $z(\forall m)=2.052$;
- cubic FP VIII: $v^*\neq 0$, $y^*\neq 0$, $u^*=w^*=0$; $z(m=2)=2.157$, $z(m=3)=2.042$;
- Ising FP X: $y^*\neq 0$, $u^*=-w^*$, $v^*=0$; $z(\forall m)=2.052$;
- random Ising FP XV: $w^*\neq 0$, $y^*\neq 0$, $u^*=v^*=0$; $z(\forall m)=2.139$.

Here, we keep the FP numbering of [15,26,27,29,30]. From the above list, only the random Ising (XV) and polymer (III) FPs are stable. However the polymer FP is not accessible from the physical initial conditions. This leads to the conclusion [27–30] that the random Ising FP XV governs the critical behaviour. Therefore, the m-vector magnets with *cubic* random axis distribution belong to the universality class of the random-site Ising magnets. The non-asymptotic critical behaviour of the RAM essentially differs from that of the random-site model as was demonstrated in statics in [15]. The same concerns the non-asymptotic dynamical critical behaviour: the critical slowing down in RAM is governed by $z_{\rm eff}$ exponent as explained below.

The crossovers between different FPs lead to rich pictures of possible RG flows [15]. Many flows are effected by the Ising FPs V and X. Introducing into (14) several typical RG flows which start from the physical region of initial couplings one obtains different regimes for approaching the effective dynamical exponent z_{eff} to asymptotics. The dependences on the flow parameter ℓ of z_{eff} for easy-plane ($m = 2$) and Heisenberg ($m = 3$) magnets are shown in figures 3 and 4 correspondingly. Flow 3 was chosen to be affected by both Ising FPs V and X. Therefore, both curves 3 of figures 3 and 4 demonstrate that a large region for z_{eff} might exist with dynamical exponent values of the pure one-component (Ising) model A. Curves 6 correspond to the flows which come near the pure FP II and curve 7 in figure 4 corresponds to the flow which comes near the cubic FP VIII.

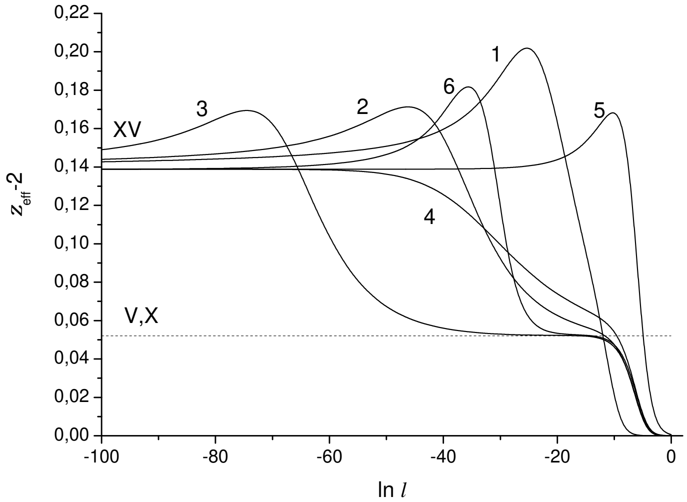


Figure 3. Effective critical exponent z_{eff} as a function of the logarithm of the flow parameter for order parameter dimension $m = 2$. Dashed line indicate the value of z at the FPs V, X. See text for details.

Although the asymptotic exponents of the random anisotropy magnets considered here are the same as those of the random-site (diluted) Ising magnets, the approach to the asymptotical region essentially differs from the diluted magnets. It is defined by the smallest static stability exponent $\omega = -0.0036$ [15] which is equal in absolute value to the ratio of heat capacity critical exponent α_r and correlation length critical exponent ν_r of random-site Ising model [30]. As a consequence the Wegner correction to scaling is $\Delta = \omega\nu_r = -\alpha_r$. The high-loop estimate gives $\Delta \approx 0.049 \pm 0.009$ [30]. Such a small value of Δ means that the approach to the asymptotic values is very slow. Therefore, practically only the non-asymptotic critical behaviour governed by effective critical exponents will be observed experimentally or in the numerical simulations.

As it is seen from figures 3, 4, another particular feature of z_{eff} seems to be that it always reaches its asymptotic value z from the region $z_{\mathrm{eff}} > z$. Therefore, in an experimental situation, a decrease of z_{eff} may serve as an evidence of approaching the asymptotics. Note that such a scenario is an intrinsic feature of the critical slowing down in random-anisotropy magnets. When disorder is implemented by dilution

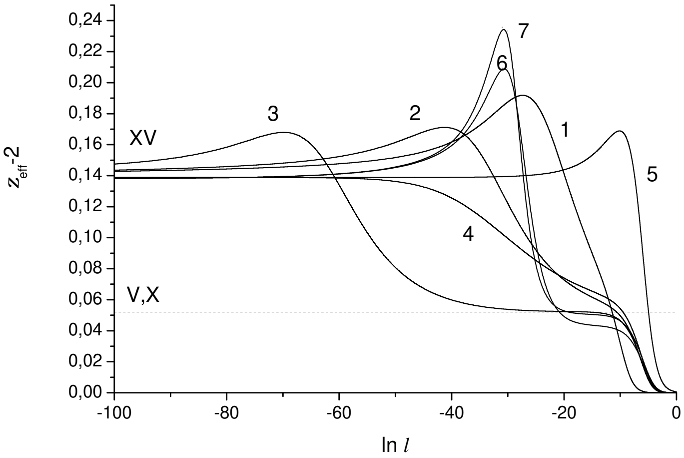


Figure 4. Effective critical exponent z_{eff} as a function of the logarithm of the flow parameter for the order parameter dimension $m = 3$. Dashed line indicates the value of z at the FPs V, X. See the text for details.

of the non-magnetic component, the z_{eff} approaching its asymptotic value is not necessarily only from the above [38].

4. Conclusions

In this paper, we have analyzed the critical slowing down in magnets effected by random anisotropy. These magnets have a second order phase transition to the ferromagnetic order for an anisotropic (*cubic*) random axis distribution. Therefore, our goal was to study relaxational dynamics of the non-conserved order parameter in the vicinity of the phase transition point. For this purpose we completed previous static RG calculations [15,29,30] by calculating the two-loop dynamical RG function ζ_Γ given in equation (15). Combining this result with the former data for the static critical behaviour we obtained numerical values for the effective critical exponent z_{eff} which governs the critical slowing down of the relaxational time when T_{c} is approached. In figures 3,4 we give the results for two most physically interesting cases $m = 2$ and $m = 3$, which correspond to the easy-plane and Heisenberg random anisotropy magnets.

Although the asymptotic dynamical critical behaviour of random anisotropy systems with *cubic distribution* is the same as for the random-site Ising systems, the crossover between different fixed points considerably effects the non-asymptotic critical properties. Different scenarios of dynamical critical behaviour are observed. Since asymptotics is approached very slowly, it might be observed in real and numerical experiments. The effective exponents measured may take values essentially differing from the asymptotic one (in our calculation $z = 2.139$). For example in a large region z_{eff} can be equal to the exponent of the pure Ising model ($z = 2.052$). Another particular feature of critical slowing down in random anisotropy magnets which is predicted by our analysis is that, contrary to the diluted magnets, z_{eff} seems to always reach its asymptotic value z from the region $z_{\mathrm{eff}} > z$.

Another important contribution to the effective dynamical exponent could come from the coupling of the order parameter to a conserved density (changing from model A dynamics to model C [4]). Since the stable fixed point has a non-diverging specific heat, the asymptotic discussed here would not be changed [39]. A more detailed account of that is in preparation.

It is our pleasure and honour to contribute this paper to the conference on the occasion of Prof. I.R.Yukhnovskii's 80th birthday. His early work on the phase transitions theory and on the non-asymptotic criticality [5] preceded in many respects many later contributions to this field. One of us (Yu.H.) is deeply indebted to Prof. I.R.Yukhnovskii for introducing him into the fascinating field of phase transitions and critical phenomena. R. F. acknowledges the fruitful cooperation with the Institute for Condensed Matter Physics.

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Критичне сповільнення в магнетиках з випадковою анізотропією

М.Дудка [1,2] , Р.Фольк [2] , Ю.Головач [1,2,3] , Г.Мозер [4]

[1] Інститут фізики конденсованих систем НАН України, 79011 Львів, Україна

[2] Інститут теоретичної фізики, Университет Йогана Кеплера міста Лінц, A–4040 Лінц, Австрія

[3] Львівський національний університет ім. І.Франка, 79005 Львів, Україна

[4] Інститут фізики і біофізики, Університет міста Зальцбург, A–5020 Зальцбург, Австрія

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Ми вивчаємо релаксаційну динаміку з незбережним параметром порядку (критична динаміка моделі А) для тривимірного магнетика з безладом у формі випадкової осі анізотропії. Для анізотропного розподілу випадкових осей асимптотична критична поведінка співпадає з поведінкою ізингівських систем з випадковими вузлами. Таким чином асимптотична критична динаміка керується динамічним показником випадкової моделі Ізинга. Однак безлад значно впливає на динамічну поведінку в неасимптотичному режимі. Ми проводимо теоретико-польовий ренормалізаційно-груповий аналіз в рамках схеми мінімального віднімання в двопетлевому наближенні щоб дослідити асимптотичну і ефективну критичну динаміку систем з випадковою анізотропією. Результати демонструють немонотонну поведінку динамічного ефективного критичного показника $z_{\rm eff}$.

Ключові слова: *критична динаміка, невпорядковані системи, випадкова анізотропія, ренормалізаційна група*

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