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DYNAMICS OF A DISSIPATIVE QUANTUM SYSTEM DRIVEN BY DICHOTOMOUS NOISE: AN EXACT TREATMENT

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Proceeding from the generalized master equation for the reduced density operator of a dissipative N-level quantum system driven by an external field, we arrived at *exact* averaging of this equation in the case of random dichotomous driving. The obtained kinetic equations are used to investigate the problem of the dissipative transfer of a quantum particle in a molecular dimer with dichotomically fluctuating intersite coupling. Furthermore, the problem of relaxation in spin-boson system with dichotomically modulated energy bias is considered. In the latter case, our treatment generalizes the well-known result of the non-interacting blip approximation.

1. Introduction

Quantum-dynamical systems (QDS) with a finite number of states are ubiquitous in physics. Such systems are frequently interacting with an environment which causes the damping and dephasing effects in the QDS dynamics. The vast literature has been devoted to discuss these effects in various contexts (see, for example [1,2] and references therein). There exist two different theoretical approaches to account for the environment influence on the QDS dynamics. Within microscopical approach, the environment is usually modelled by a bath of independent harmonic oscillators under in the thermal equilibrium (thermal bath, TB). This approach relies on the removing of the TB variables from the Liouville equation for the whole system (QDS+TB) by means of an appropriate elimination procedure to get the equation of motion for the reduced density matrix of QDS[1,3]. The main idea of alternative stochastic approach is to treat the QDS-environment interaction semiclassically and phenomenologically in the framework of stochastic Liouville equation[4,5].

Recently, a complementary method which combines the advantages of both above mentioned approaches has been introduced [6–10]. The main idea of works [7,8] was to model the environment influence through both the interaction, H_{int}, with a quantum TB and semiclassical stochastic addition, $\tilde{H}(t)$, into the QDS Hamiltonian, H_0. The latter one can be used, for example, to model non–equilibrium and highly anharmonic degrees of the environment. So far, the two different methods have been used to handle this stochastic addition. Making use of the cumulant expansion method

[6,11], it was possible to obtain the stochastically averaged kinetic equations only in the lowest approximations over the Kubo number $K = \Delta\tau_c$ which characterizes the strength of fluctuations of the stochastic process $\tilde{H}(t)$ [6,7]. Here Δ and τ_c are the parameters related to the amplitude of fluctuations (hereafter $\hbar = 1$) and autocorrelation time of a stochastic term $\tilde{H}(t)$, respectively. For this reason, an alternative method based on the theory of kinetic equations for a QDS in a strong external field has been suggested [8,10]. This method permits to obtain the stochastically averaged kinetic equations without use of the perturbation theory in Kubo number K. However, it was restricted before to the case of fast fluctuations when the autocorrelation time of a stochastic perturbation, τ_c, is much shorter than the relaxation time, τ_r, in QDS. Recently, we have put forward a way to overcome this restriction in the case of perturbation $\tilde{H}(t)$ modelled as dichotomous Markov process (DMP) [12]. Note that DMP is widely used in physics as a simple and exactly tractable model for colored noise [13,14]. Hence, the considered model is of fundamental importance.

The goal of this paper is to obtain the general kinetic equation for the averaged reduced density matrix $\langle\rho_{nm}(t)\rangle = \langle n|\langle\rho(t)\rangle|m\rangle$ of QDS in the state representation. Here $\rho(t) = \mathrm{Tr}\sigma(t)$ is the reduced density operator of the quantum system, $\sigma(t)$ is density operator of the whole system, $|n\rangle$ is a state of the QDS , Tr denotes trace over the quantum TB, and $\langle ... \rangle$ denotes the average over the stochastic process. For this purpose we utilize the theory of quantum relaxation processes in QDS driven by a strong external field [2].

2. Model and Theory

Let

$$H(t) = H_0 + \tilde{H}(t) + H_{int} + H_T \tag{2.1}$$

be the Hamiltonian of a whole system, where

$$H_0 = \sum_{nm} H^{(0)}_{nm}\hat{\gamma}_{nm} \tag{2.2}$$

denotes the Hamiltonian of the QDS written in the basis of the transition operators $\hat{\gamma}_{nm} = |n\rangle\langle m|$,

$$\tilde{H}(t) = \alpha(t)H_1 = \alpha(t)\sum_{nm} H^{(1)}_{nm}\hat{\gamma}_{nm} \tag{2.3}$$

is the stochastic perturbation, and H_T is the Hamiltonian of a quantum thermal bath. In equation (2.3) $\alpha(t)$ denotes DMP with zero mean and the autocorrelation function $\langle\alpha(t+\tau)\alpha(t)\rangle = \exp(-\nu\tau)$, where $\nu = 1/\tau_c$ labels the reverse autocorrelation time [16]. The operator of quantum system-thermal bath interaction is written in the quite general form as

$$H_{int} = \sum_{nm} \hat{F}_{nm}\hat{\gamma}_{nm}, \tag{2.4}$$

where $\hat{F}_{nm} = \hat{F}^{\dagger}_{mn}$ are bath-dependent operators with the zero average over a thermal bath, $\langle\hat{F}_{nm}\rangle_T = \mathrm{Tr}(\rho_T\hat{F}_{nm}) = 0$. The density operator of a thermal bath, ρ_T, is supposed to be equilibrium, one

$$\rho_T = \exp(-H_T/k_BT)[\mathrm{Tr}\exp(-H_T/k_BT)]^{-1}, \tag{2.5}$$

where k_B is the Boltzmann constant, T is the absolute temperature.

Following to Argyres and Kelley[15], we write the reduced density operator equation in the Born approximation in the interaction H_{int} and by the initial factorization assumption, $\sigma(0) = \rho(0)\rho_T$, in the form

$$\frac{d}{dt}\rho(t) = -iL(t)\rho(t) - \int_0^t \Gamma(t,t')\rho(t')dt', \tag{2.6}$$

where

$$L(t) = L_0 + \alpha(t)L_1, L_i = [H_i, (\cdot)], i = 0, 1 \tag{2.7}$$

is stochastic Liouville superoperator, and

$$\begin{aligned}\Gamma(t,t') = \sum_{kk',rr'} \{&K_{rr'kk'}(t-t')[\hat{\gamma}_{rr'}, S(t,t')\hat{\gamma}_{kk'}(\cdot)] \\ &K_{kk'rr'}(t'-t)[\hat{\gamma}_{rr'}, S(t,t')(\cdot)\hat{\gamma}_{kk'}]\}\end{aligned} \tag{2.8}$$

is the memory kernel. In equation (2.8)

$$K_{kk'rr'}(\tau) = \langle \exp(iH_T\tau)\hat{F}_{kk'} \exp(-iH_T\tau)\hat{F}_{rr'}\rangle_T \tag{2.9}$$

is the bath correlation function, and $S(t,t')$ is the evolution superoperator that fulfils the stochastic evolution equation (SEE) written in the "forward" and "backward" forms as

$$\begin{aligned}\frac{d}{dt}S(t,t') &= -i[L_0 + \alpha(t)L_1]S(t,t'), S(t',t') = I, \\ \frac{d}{dt'}S(t,t') &= iS(t,t')[L_0 + \alpha(t')L_1], S(t,t) = I.\end{aligned} \tag{2.10}$$

The main peculiarity of the equations (2.6)-(2.10) is that the dichotomous noise $\tilde{H}(t)$ affects the memory kernel and may be arbitrarily strong.

One must average the master equation (2.6) over the dichotomous process $\alpha(t)$. With this goal in mind, we proceed as follows. Consider the formal expression

$$\langle \Gamma(t,t'+\tau)\alpha(t'+\tau)\alpha(t')\rho(t')\rangle, \tau > 0. \tag{2.11}$$

In the equation (2.11) the $\Gamma(t,t'+\tau)$ and $\rho(t')$ are functionals of the DMP $\alpha(t)$ involving only times, respectively, posterior to $t'+\tau$ and prior to t'. Therefore, this expression meets the conditions of the Bourret and Frisch theorem (the theorem B in [16]) and can be transformed as

$$\begin{aligned}&\langle \Gamma(t,t'+\tau)\alpha(t'+\tau)\alpha(t')\rho(t')\rangle = \\ &= \langle \Gamma(t,t'+\tau)\rangle\langle\alpha(t'+\tau)\alpha(t')\rangle\langle\rho(t')\rangle + \\ &+\langle \Gamma(t,t'+\tau)\alpha(t'+\tau)\rangle\langle\alpha(t')\rho(t')\rangle.\end{aligned} \tag{2.12}$$

Performing the limit $\tau \to +0$ in the equation (2.12) we get, using the remarkable property of the DMP, $\alpha^2(t) = 1$,the following corollary of the theorem (2.12)

$$\langle \Gamma(t,t')\rho(t')\rangle = \Gamma^{(0)}(t-t')\rho_0(t') + \Gamma^{(1)}(t-t')\rho_1(t'), \tag{2.13}$$

where $\Gamma^{(0)}(t-t') = \langle\Gamma(t,t')\rangle$, $\Gamma^{(1)}(t-t') = \langle\Gamma(t,t')\alpha(t')\rangle$, $\rho_0(t) = \langle\rho(t)\rangle$, and $\rho_1(t) = \langle\alpha(t)\rho(t)\rangle$. In the same way we obtain

$$\langle\alpha(t)\Gamma(t,t')\rho(t')\rangle = \Gamma^{(2)}(t-t')\rho_0(t') + \Gamma^{(3)}(t-t')\rho_1(t'), \tag{2.14}$$

where $\Gamma^{(2)}(t-t') = \langle\alpha(t)\Gamma(t,t')\rangle$ and $\Gamma^{(3)}(t-t') = \langle\alpha(t)\Gamma(t,t')\alpha(t')\rangle$.

To get the equation for the correlator $\rho_1(t)$, one can use the Shapiro and Loginov theorem [17,11]. According to this theorem, any functional, $f(t)$, of the dichotomous process $\alpha(t)$ must obey the following equation

$$\frac{d}{dt}\langle\alpha(t)f(t)\rangle = -\nu\langle\alpha(t)f(t)\rangle + \langle\alpha(t)\frac{d}{dt}f(t)\rangle. \tag{2.15}$$

Using equations (2.13)-(2.15), we obtain from the equation (2.6) the set of coupled equations for the averaged reduced density operator $\rho_0(t)$ and the correlator $\rho_1(t)$

$$\begin{aligned}\frac{d}{dt}\rho_0(t) &= -iL_0\rho_0(t) - iL_1\rho_1(t) - \\ &\int_0^t \{\Gamma^{(0)}(t-t')\rho_0(t') + \Gamma^{(1)}(t-t')\rho_1(t')\}dt', \\ \frac{d}{dt}\rho_1(t) &= -(\nu + iL_0)\rho_1(t) - iL_1\rho_0(t) - \\ &\int_0^t \{\Gamma^{(2)}(t-t')\rho_0(t') + \Gamma^{(3)}(t-t')\rho_1(t')\}dt'\end{aligned} \tag{2.16}$$

with initial conditions $\rho_0(0) = \rho_0$ and $\rho_1(0) = 0$. The kernels $\Gamma^{(i)}(t-t')$ in the equation (2.16) are specified in a similar way to the kernel $\Gamma(t,t')$, equation (2.8), in which the evolution operator $S(t,t')$ is replaced by the averaged operator $S^{(0)}(t-t') = \langle S(t,t')\rangle$, $S^{(1)}(t-t') = \langle S(t,t')\alpha(t')\rangle$, $S^{(2)}(t-t') = \langle\alpha(t)S(t,t')\rangle$, or $S^{(3)}(t-t') = \langle\alpha(t)S(t,t')\alpha(t')\rangle$, respectively. Using the Shapiro and Loginov theorem (2.15) together with the SEE (2.10), we find after some algebra the Laplace transforms, $\tilde{S}^{(i)}(p) = \int_0^\infty e^{-p\tau}S^{(i)}(\tau)d\tau$,

$$\begin{aligned}\tilde{S}^{(0)}(p) &= [p + iL_0 + L_1(p+\nu+iL_0)^{-1}L_1]^{-1}, \\ \tilde{S}^{(1)}(p) &= -i\tilde{S}^{(0)}(p)L_1(p+\nu+iL_0)^{-1}, \\ \tilde{S}^{(2)}(p) &= -i(p+\nu+iL_0)^{-1}L_1\tilde{S}^{(0)}(p), \\ \tilde{S}^{(3)}(p) &= iL_1^{-1}(p+iL_0)\tilde{S}^{(1)}(p) = i\tilde{S}^{(2)}(p)(p+iL_0)L_1^{-1}\end{aligned} \tag{2.17}$$

of the operators $S^{(i)}(\tau)$.

For the averaged density matrix $\langle\rho_{nm}(t)\rangle$ and the correlation matrix $\langle\alpha(t)\rho_{nm}(t)\rangle$ one can obtain from equation (2.16) the final set of the coupled kinetic equations

$$
\begin{aligned}
\frac{d}{dt}\langle\rho_{nm}(t)\rangle &= -i\sum_{n'm'}\{L^{(0)}_{nmn'm'}\langle\rho_{n'm'}(t)\rangle + \\
&+ L^{(1)}_{nmn'm'}\langle\alpha(t)\rho_{n'm'}(t)\rangle\} - \\
&- \sum_{n'm'}\int_0^t dt'\{\Gamma^{(0)}_{nmn'm'}(t-t')\langle\rho_{n'm'}(t')\rangle + \\
&+ \Gamma^{(1)}_{nmn'm'}(t-t')\langle\alpha(t)\rho_{n'm'}(t')\rangle\}, \qquad (2.18)\\
\frac{d}{dt}\langle\alpha(t)\rho_{nm}(t)\rangle &= -\sum_{n'm'}\{[\nu\delta_{nn'}\delta_{mm'} + iL^{(0)}_{nmn'm'}]\langle\alpha(t)\rho_{n'm'}(t)\rangle + \\
&+ iL^{(1)}_{nmn'm'}\langle\rho_{n'm'}(t)\rangle\} - \\
&- \sum_{n'm'}\int_0^t dt'\{\Gamma^{(2)}_{nmn'm'}(t-t')\langle\rho_{n'm'}(t')\rangle + \\
&+ \Gamma^{(3)}_{nmn'm'}(t-t')\langle\alpha(t)\rho_{n'm'}(t')\rangle\},
\end{aligned}
$$

where

$$
\begin{aligned}
L^{(i)}_{nmn'm'} &= H^{(i)}_{nn'}\delta_{mm'} - H^{(i)}_{m'm}\delta_{nn'},\\
\Gamma^{(i)}_{nmn'm'}(\tau) &= \sum_{rr'}\{K_{nrr'n'}(\tau)S^{(i)}_{m'r'mr}(-\tau) + \\
&\quad + K_{m'r'rm}(-\tau)S^{(i)}_{r'n'rn}(-\tau) - \\
&\quad - K_{m'r'nr}(-\tau)S^{(i)}_{r'n'mr}(-\tau) - \\
&\quad - K_{rmr'n'}(\tau)S^{(i)}_{m'r'rn}(-\tau)\}, \qquad (2.19)
\end{aligned}
$$

and $S^{(i)}_{nrr'm'}(\tau) = \langle n|S^{(i)}(\tau)\hat{\gamma}_{r'm'}|r\rangle$ are the elements of the Liouville operator L_i , kernel $\Gamma^{(i)}(\tau)$ and averaged operator $S^{(i)}(\tau)$ in a tetradic representation, respectively. It should be particularly emphasized that the derived equations are exact in the perturbation $\tilde{H}(t)$. To illustrate their advantage, we enlarge on examples below.

3. Dimer with random fluctuating intersite coupling

Let us consider the transfer of a quantum particle in a symmetric dimer with quantum fluctuating site energies and dichotomically fluctuating intersite matrix element. Then, the QDS Hamiltonian can be specified as follows

$$H_0 = E_0\hat{\gamma}_{11} + E_0\hat{\gamma}_{22} + \frac{1}{2}V[\hat{\gamma}_{12} + \hat{\gamma}_{21}]\,. \qquad (3.1)$$

Here, E_0 is the site energy of the quantum particle in the basis of localized states $\mid 1\rangle$ and $\mid 2\rangle$, and V is the mean intersite matrix element. The stochastic perturbation $\tilde{H}(t)$ is given by

$$\tilde{H}(t) = \frac{1}{2}\Delta\alpha(t)[\hat{\gamma}_{12} + \hat{\gamma}_{21}], \qquad (3.2)$$

where Δ is the amplitude of fluctuations. The interaction with the thermal bath,

$$H_{int} = \frac{1}{2}\hat{F}(\hat{\gamma}_{11} - \hat{\gamma}_{22}), \tag{3.3}$$

includes fluctuations of the site energies difference caused by the generalized force

$$\hat{F} = \sum_{\lambda} \kappa_{\lambda}(\hat{b}^{+}_{\lambda} + \hat{b}_{\lambda}) \ . \tag{3.4}$$

The TB is modelled by a number of harmonic oscillators

$$H_T = \sum_{\lambda} \omega_{\lambda}(\hat{b}^{+}_{\lambda}\hat{b}_{\lambda} + \frac{1}{2}) \ . \tag{3.5}$$

In equations (3.3-3.5) ω_{λ} is the frequency of the λ -th bath mode, $\hat{b}^{+}_{\lambda}(\hat{b}_{\lambda})$ is the creation (annihilation) operator, and κ_{λ} is the coupling constant. Taking into account equations (3.4), (3.5), the bath correlation function (2.9) can be expressed as

$$K(\tau) = \frac{1}{2\pi}\int\limits_0^{\infty} J(\omega)\frac{\cosh(\hbar\omega/2k_BT - i\omega t)}{\sinh(\hbar\omega/2k_BT)}d\omega \ . \tag{3.6}$$

Here the bath spectral function $J(\omega) = 2\pi\sum_{\lambda}\kappa^2_{\lambda}\delta(\omega - \omega_{\lambda})$ has been introduced [1].

To treat the intersite transfer dynamics in the problem considered it is convenient to change to the Bloch variables $\sigma_z(t) = \rho_{11}(t) - \rho_{22}(t)$, $\sigma_y(t) = i(\rho_{21}(t) - \rho_{12}(t))$, $\sigma_x(t) = \rho_{21}(t) + \rho_{12}(t)$. Then, the state populations, $\rho_{11}(t)$ and $\rho_{22}(t)$, are defined through $\sigma_z(t)$ as $\rho_{11}(t) = (1 + \sigma_z(t))/2$ and $\rho_{22}(t) = (1 - \sigma_z(t))/2$. From the equations (2.17)-(2.19), which are relevant for the current problem, we get after some transformations desired kinetic equations :

$$\begin{aligned}
\frac{d}{dt}\langle\sigma_z(t)\rangle &= V\langle\sigma_y(t)\rangle + \Delta\langle\alpha(t)\sigma_y(t)\rangle \ , \\
\frac{d}{dt}\langle\alpha(t)\sigma_z(t)\rangle &= -\nu\langle\alpha(t)\sigma_z(t)\rangle + V\langle\alpha(t)\sigma_y(t)\rangle + \Delta\langle\sigma_y(t)\rangle \ , \\
\frac{d}{dt}\langle\sigma_y(t)\rangle &= -V\langle\sigma_z(t)\rangle - \Delta\langle\alpha(t)\sigma_z(t)\rangle - \\
&\quad - \int\limits_0^t K_s(t-t')\langle\sigma_y(t')\rangle dt' \ , \\
\frac{d}{dt}\langle\alpha(t)\sigma_y(t)\rangle &= -\nu\langle\alpha(t)\sigma_y(t)\rangle - V\langle\alpha(t)\sigma_z(t)\rangle - \\
&\quad -\Delta\langle\sigma_z(t)\rangle - \int\limits_0^t e^{-\nu(t-t')}K_s(t-t')\langle\alpha(t')\sigma_y(t')\rangle dt'
\end{aligned} \tag{3.7}$$

for expectations $\langle\sigma_{z,y}(t)\rangle$ and correlators $\langle\alpha(t)\sigma_{z,y}(t)\rangle$. Here

$$K_s(\tau) = \frac{1}{2}(K(\tau) + K(-\tau)) \ , \tag{3.8}$$

denotes the symmetrized autocorrelation function. For the sake of simplicity, we accept the high temperature limit and classical description of the TB with white noise spectrum $J(\omega) = \eta\omega$ such that $K_s(\tau) = \eta k_B T \delta(\tau)$. This choice of the correlator $K_s(\tau)$ corresponds to the generalization of the Haken-Strobl-Reineker (HSR) model [5,14] to the case of dichotomically fluctuating intersite matrix element. It allows an analytical treatment and yields four rate constants

$$\begin{aligned}
\lambda_{1,2} &= \frac{\nu}{2} + \frac{\xi}{2} \pm \\
&\pm \frac{\sqrt{\nu^2 + \xi^2 + 2\sqrt{\xi^2\nu^2 - 4V^2\nu^2 + 16V^2\Delta^2} - 4\Delta^2 - 4V^2}}{2}, \\
\lambda_{3,4} &= \frac{\nu}{2} + \frac{\xi}{2} \pm \\
&\pm \frac{\sqrt{\nu^2 + \xi^2 - 2\sqrt{\xi^2\nu^2 - 4V^2\nu^2 + 16V^2\Delta^2} - 4\Delta^2 - 4V^2}}{2},
\end{aligned} \tag{3.9}$$

where we denote $\xi = \eta k_B T$. Depending on the relation between parameters Δ, ν, V, and ξ the three different transfer regimes (coherent, incoherent, and a combined one) are possible (see [18] for more details).

4. Spin–boson model with dichotomically fluctuating energy bias

Another example is given by the spin–boson model,

$$H(t) = \frac{1}{2}\Delta\alpha(t)\hat{\sigma}_z + V\hat{\sigma}_x + \frac{1}{2}\hat{\sigma}_z \sum_\lambda \kappa_\lambda(b_\lambda^\dagger + b_\lambda) + \sum_\lambda \omega_\lambda(b_\lambda^\dagger b_\lambda + \frac{1}{2}), \tag{4.1}$$

extended to the case of dichotomically fluctuating energy bias (the mean energy bias is assumed to be zero). Bearing in mind the relations $\hat{\sigma}_z = \hat{\gamma}_{11} - \hat{\gamma}_{11}$ and $\hat{\sigma}_x = \hat{\gamma}_{12} + \hat{\gamma}_{21}$, it is easily seen the model (4.1) is similar to that of Section 3. The difference consists in another choice of perturbation $\tilde{H}(t)$ which is now:

$$\tilde{H}(t) = \frac{1}{2}\Delta\alpha(t)[\hat{\gamma}_{11} - \hat{\gamma}_{22}]. \tag{4.2}$$

Besides, the system–bath interaction H_{int} is considered as a strong one. Then, upon use of the spin-polaron transformation [1,2]

$$U = \exp(\frac{1}{2}\hat{R}\hat{\sigma}_z), \quad \hat{R} = \sum_\lambda \frac{\kappa_\lambda}{\omega_\lambda}(B_\lambda^\dagger - B_\lambda), \tag{4.3}$$

where, e.g., the new annihilation operator reads $B_\lambda = U^\dagger b_\lambda U$, one can represent the total Hamiltonian (4.1) in the basis of dressed states, $|\tilde{n}\rangle = U^\dagger|n\rangle$, as follows

$$\begin{aligned}
\bar{H}(t) = U^\dagger H(t) U &= \frac{1}{2}\Delta\alpha(t)(|\tilde{1}\rangle\langle\tilde{1}| - |\tilde{2}\rangle\langle\tilde{2}|) + \\
&+ \hat{F}_{12}|\tilde{1}\rangle\langle\tilde{2}| + \hat{F}_{21}|\tilde{2}\rangle\langle\tilde{1}| + \sum_\lambda \omega_\lambda(B_\lambda^\dagger B_\lambda + \frac{1}{2}).
\end{aligned} \tag{4.4}$$

Here $\hat{F}_{12} = \hat{F}_{21}^{\dagger} = V \exp(\hat{R})$ is the operator of the dressed intersite coupling. This operator is considered as a weak perturbation. Furthermore, let us assume that $\langle \hat{F}_{12} \rangle_T = 0$. (This assumption is valid, for example, in the case of the Ohmic TB, $J(\omega) \sim \omega$ at $\omega \to 0$.) Then, we get from equations (2.17)-(2.19) the following integro–differential equation for the expectation $\langle \sigma_z(t) \rangle$:

$$\frac{d}{dt}\langle \sigma_z(t) \rangle = -\int\limits_0^t \exp(-\frac{\nu}{2}(t-t'))[\cosh(\frac{1}{2}\sqrt{\nu^2 - 4\Delta^2}(t-t')) +$$
$$+\frac{\nu}{\sqrt{\nu^2-4\Delta^2}}\sinh(\frac{1}{2}\sqrt{\nu^2-4\Delta^2}(t-t'))] f_0(t-t')\langle \sigma_z(t') \rangle dt', \quad (4.5)$$

where

$$f_0(t) = 4V^2 \exp[-G_s(t)] \cos[G_a(t)],$$
$$G_s(t) = \frac{1}{2\pi}\int\limits_0^\infty \frac{J(\omega)}{\omega^2} \coth[\frac{\omega}{2k_B T}](1-\cos\omega t) d\omega,$$
$$G_a(t) = \frac{1}{2\pi}\int\limits_0^\infty \frac{J(\omega)}{\omega^2} \sin\omega t d\omega. \quad (4.6)$$

It should be stressed here that in the considered case the kinetic equations for the expectation $\langle \sigma_z(t) \rangle$ and correlator $\langle \alpha(t)\sigma_z(t) \rangle$ are independent.

For the Laplace-transform $\tilde{\sigma}_z(p) = \int\limits_0^\infty \exp(-pt)\langle \sigma_z(t) \rangle dt$, we obtain from equation (4.5):

$$\begin{aligned}\tilde{\sigma}_z(p) = \{p &+ \frac{1}{2}\{\tilde{f}_0(p+\frac{\nu}{2}+\sqrt{\frac{\nu^2}{4}-\Delta^2}) + \\ &+ \tilde{f}_0(p+\frac{\nu}{2}-\sqrt{\frac{\nu^2}{4}-\Delta^2})\} - \\ &- \frac{\nu}{2\sqrt{\nu^2-4\Delta^2}}\{\tilde{f}_0(p+\frac{\nu}{2}+\sqrt{\frac{\nu^2}{4}-\Delta^2}) - \\ &- \tilde{f}_0(p+\frac{\nu}{2}-\sqrt{\frac{\nu^2}{4}-\Delta^2})\}\}^{-1}, \quad (4.7)\end{aligned}$$

where $\tilde{f}_0(p)$ is the Laplace-transform of $f_0(t)$.

equation (4.7) provides the formal solution to the problem and describes generally non-exponential dynamics. This dynamics can be coherent or incoherent depending on the energetic structure of the TB, the strength of coupling between the QDS and the TB, the temperature, and the external field parameters. Different extremes of equation (4.7) are considered in Ref. [19] concerning the problem of the long–range electron transfer driven by the energy gap fluctuations.

Here, we note finally that in the absence of the external driving force ($\Delta = 0$) the equation (4.7) reduces to the well-known result of the celebrated non-interacting blip approximation (NIBA) [1]:

$$\tilde{\sigma}_z(p) = \frac{1}{p + \tilde{f}_0(p)}. \tag{4.8}$$

Thus, our result (4.7) can be thought as a generalization of the NIBA result to the case of dichotomically fluctuating energy bias.

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ДИНАМІКА ДИСИПАТИВНИХ КВАНТОВИХ СИСТЕМ, КЕРОВАНИХ ДИХОТОМІЧНИМ ШУМОМ: ТОЧНІ РЕЗУЛЬТАТИ

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Виходячи з узагальненого керуючого рівняння для зведеного оператора густини дисипативної N-рівневої квантової системи, керованої зовнішнім полем, отримуємо точне усереднення цього рівняння при випадковому дихотомичному управлінні.

Отримані кінетичні рівняння використовуються для дослідження проблеми дисипативного переносу квантової частинки в молекулярному димері з міжвузловою взаємодією, що флуктує дихотомічно. Розглядається проблема релаксації в спін-бозонній системі з дихотомічно модульованою енергією. У цьому випадку запропонований підхід узагальнює добре відомий результат наближення невзаємодіючого відображення.