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FOKKER–TYPE CONFINEMENT MODELS FROM EFFECTIVE
LAGRANGIAN IN CLASSICAL YANG–MILLS THEORY

Моделі конфайнменту типу Фоккера на основі ефективного лаґранжіяну в класичній теорії Янґа–Міллса

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Анотація. На основі нестандартної теорії поля Янґа–Міллса отримано Абелеві потенціяли рухомих точкових джерел. Вони використовуються для побудови часо-симетричного та часо-асиметричного інтеґралів дії типу Фоккера, що описують класичну динаміку двочастинкової системи з утримуючою взаємодією. Часо-асиметричну модель переформульовано в рамки гамільтонового формалізму. Відповідну задачу двох тіл зведено до квадратур. В рамках квазікласичного розгляду оцінюється поведінка траєкторій Редже.

Fokker–type Confinement Models from Effective Lagrangian in Classical Yang–Mills Theory

A. Duviryak

Abstract. Abelian potentials of pointlike moving sources are obtained from the nonstandard theory of Yang–Mills field. They are used for the construction of the time-symmetric and time-asymmetric Fokker-type action integrals describing the dynamics of two-particle system with confinement interaction. The time-asymmetric model is reformulated in the framework of the Hamiltonian formalism. The corresponding two-body problem is reduced to quadratures. The behaviour of Regge trajectories is estimated within the semiclassical consideration.

ЛЬВІВ

1. Introduction

Potential models of hadrons originate from the quantum chromodynamics (QCD), but they are not rigorously deducible from the theory. Rather, these models are substantiated by various approximate approaches and estimates in QCD [1]. Various models have their own areas of application. In particular, the linear potential which follows from area law in the lattice approximation of QCD, describes, by construction, the static interaction of quarks [2]. Thus it can lawfully be exploited only in non-relativistic potential models [3].

The description of light meson spectroscopy needs the development of appropriate relativistic models. They frequently are built as single-particle wave equations [4–8] which is not satisfactory. Actually, mesons should be treated as composite two–quark relativistic systems. The reliable basis for this purpose is the relativistic direct interaction theory (RDIT) presented by various approaches and formalisms [9], such as Lagrangian formalism with higher derivatives [10], relativistic Hamiltonian mechanics [11,12], canonical formalism with constraints [13], Fokker-type action formalism [14] etc.

Given a nonrelativistic potential, RDIT determines the general structure of its relativistic counterpart. In so going the great arbitrariness arises in the choice of concrete relativistic interaction. Consequently, the variety of relativistic potential models has appeared in the literature [15–25]. Each of them has its own advantages and areas of application, but these models are not substantiated by QCD better than nonrelativistic models.

A possible way to substantiate relativistic direct interactions leads through classical field theory. Especially, we mean the Fokker-type action formalism which, among other approaches to RDIT, is most closely related to this theory. There exists the class of Fokker actions which correspond to particle interactions via linear fields, such as scalar, vector, and other tensor fields [26–28]. These actions are built on the solutions to relevant wave equations. In the nonrelativistic limit they lead to the same Coulomb (or Yukawa) potential.

Working within this scope for the confinement case, one could try to proceed from some solution to classical Yang–Mills equations. But no such solutions leading to confining potentials are known in the literature. Moreover, they are believed to not exist due to the essentially quantum nature of confinement. This is concerned with standard Yang–Mills theory while there exist various nonstandard theories which involve effective Yang–Mills fields arising from QCD. These theories may be used as sources of confining potentials.

In the present paper we find the relation between certain nonstandard classical theory of Yang–Mills field and the Fokker-type confinement model. The former is developed in [29]. This theory describes non-Abelian gauge field averaged over quantum fluctuations. It is based on the effective Lagrangian obtained from the study of infrared behaviour of gluon Green's functions in QCD [30]. Field equations following from this Lagrangian are of 4th order, and some static non-Abelian solutions to them have been used in a sort of bag confinement model [31,32].

Here we obtain from this theory the Abelian retarded and time-symmetric potentials of moving pointlike source. Both of them are of confining type and reduce in the nonrelativistic limit to the linear potential. Then, using these potentials, we construct the time-asymmetric and time-symmetric Fokker-type actions. The latter is already known in the literature. Two equivalent versions of this action have been proposed by Rivacoba [33] and Weiss [34]. It is noteworthy that both the authors proceeded from general preliminaries of RDIT, without referring to field–theoretical interpretation of particle interaction.

The time–symmetric action leads to difference–differential equations of motion which are difficult to deal with. The only circular–orbit solutions to these equations are found in [33,34]. Contrastingly, the dynamics following from the time-asymmetric action is well defined in terms of second-order differential equations of motion. Thus this action can be considered as the classical background model of relativistic two-quark quantum dynamics. Following [35,36], we reformulate this model into the Lagrangian formalism. Then we transit to the Hamiltonian formalism, and integrate the two-body problem in quadratures.

The time-asymmetric analogue of Rivacoba-Weiss model is the simplest version of relativistic confinement model. It can be appropriate for the classical description of light mesons for which the confinement interaction dominates. To include into consideration also heavy mesons one can modify the present model by adding to the action the vector-type interaction term from the time–asymmetric version of the Wheeler-Feynman electrodynamics. This corresponds to the taking account of Abelian solution to the standard Yang–Mills equations (i.e., the classical analogue of one-loop correction in QCD). In the nonrelativistic limit this mixture leads to the well known Coulomb plus linear potential. The modified model becomes appreciably cumbersome but still remains solvable.

Here we do not propose a quantum version of the present model. Instead, we make some estimates of the Regge trajectory from classical and

semiclassical considerations and obtain a physically reasonable result.

The paper is organized as follows. In Section 2 we obtain the Abelian potentials of moving pointlike sources from the standard and nonstandard theories of Yang-Mills field. The formers are the Lienard-Wiechert potentials and their causal modifications while the latters turn out to be the modifications of potentials proposed by Weiss. They are obtained with the Green's functions found in Appendix A. In Section 3 we present equations of particle motion following from the standard and nonstandard theories, and construct corresponding time-symmetric and time-asymmetric Fokker-type integrals. The latter is used as the base of time-asymmetric confinement model. In Section 4 this model is reformulated in the framework of the Hamiltonian formalism. Various special cases of two-body problem are considered in Subsections 4.1–4.3 and Appendix B. Estimates of Regge trajectory are quoted in Section 5. Section 6 is devoted to general discussion of the model.

2. Abelian potentials from the standard and nonstandard theories of Yang-Mills field

We shall consider both the standard and nonstandard classical theories of the Yang-Mills field. The standard theory (ST) is based on the well known Yang-Mills Lagrangian [2]

$$\mathcal{L}_{\mathrm{ST}} = -\frac{1}{16\pi}\langle \mathsf{F}_{\mu\nu}, \mathsf{F}^{\mu\nu}\rangle - \langle \mathsf{J}^{\mu}, \mathsf{A}_{\mu}\rangle. \tag{1}$$

The nonstandard theory (NT) proceeds from the effective Lagrangian [30,29]

$$\mathcal{L}_{\mathrm{NT}} = \frac{1}{16\pi\kappa^2}\langle \nabla_{\lambda}\mathsf{F}_{\mu\nu}, \nabla^{\lambda}\mathsf{F}^{\mu\nu}\rangle + \frac{\xi}{24\pi\kappa^2}\langle \mathsf{F}_{\mu}{}^{\nu}, [\mathsf{F}_{\nu\lambda}, \mathsf{F}^{\lambda\mu}]\rangle \\ - \langle \mathsf{J}^{\mu}, \mathsf{A}_{\mu}\rangle. \tag{2}$$

Here the components of the gauge field $\mathsf{A}_{\mu}(x)$ ($\mu = \overline{0,3}$) and the current of sources $\mathsf{J}^{\mu}(x)$ take values in the Lie algebra $\mathcal{G}$ of gauge group; $[\mathsf{X}, \mathsf{Y}]$, $\langle \mathsf{X}, \mathsf{Y}\rangle$, and $\nabla_{\mu}\mathsf{X} \equiv \partial_{\mu}\mathsf{X} - [\mathsf{A}_{\mu}, \mathsf{X}]$ are the Lie brackets, the Killing–Cartan metrics, and the covariant derivative, respectively, defined for any $\mathsf{X}, \mathsf{Y} \in \mathcal{G}$; $\mathsf{F}_{\mu\nu} \equiv \partial_{\mu}\mathsf{A}_{\nu} - \partial_{\nu}\mathsf{A}_{\mu} - [\mathsf{A}_{\mu}, \mathsf{A}_{\nu}]$ is the tension tensor; κ is some parameter of the dimension of inverse length, and ξ is meant here as an arbitrary dimensionless parameter. We suppose that the gauge group is the semi-simple compact group, so that the Killing–Cartan metrics is nondegenerate and positively defined. In the real matrix representation

it can be presented in the form

$$\langle \mathsf{X}, \mathsf{Y}\rangle = -\frac{1}{N_R}\mathrm{tr}(\mathsf{X}\mathsf{Y}),$$

where the number N_R depends on the representation chosen. Greece indices move due to the metrics $\eta_{\mu\nu}$ of the Minkowski space–time $\mathbb{M}_4$ which is chosen timelike, i.e., $\|\eta_{\mu\nu}\| = \mathrm{diag}(+, -, -, -)$.

Field equations following from the Lagrangians (1) and (2) are

$$\nabla_{\nu}\mathsf{F}^{\nu\mu} = 4\pi\mathsf{J}^{\mu}, \tag{3}$$

and

$$\left\{2\nabla_{\nu}\nabla^2 - (1+\xi)\nabla_{\lambda}\nabla_{\nu}\nabla^{\lambda} + \xi\nabla^2\nabla_{\nu}\right\}\mathsf{F}^{\nu\mu} = 4\pi\kappa^2\mathsf{J}^{\mu}, \tag{4}$$

respectively. Both of them are compatible provided the current J^{μ} is covariantly conserved,

$$\nabla_{\mu}\mathsf{J}^{\mu} = 0. \tag{5}$$

In the present paper we are interested in the relativistic system of N pointlike charged particles interacting via the Yang–Mills field. The current J^{μ} corresponding to this system is [37]

$$\mathsf{J}^{\mu}(x) \equiv \sum_a \mathsf{J}^{\mu}_a(x) = \sum_a \int d\tau_a \mathsf{Q}_a \dot{z}^{\mu}_a \delta(x - z_a). \tag{6}$$

Here $z^{\mu}_a(\tau_a)$ ($\mu = \overline{0,3}$, $a = \overline{1,N}$) are the space-time coordinates of ath particle world line in $\mathbb{M}_4$ parametrized by an arbitrary evolution parameter τ_a, $\dot{z}^{\mu}_a(\tau_a) \equiv dz^{\mu}_a/d\tau_a$, and $\mathsf{Q}_a(\tau_a)$ is the charge of ath particle. Substituting (6) into (5) one obtains the Wong equations [37] determining the evolution of charges,

$$\dot{\mathsf{Q}}_a = \dot{z}^{\mu}_a[\mathsf{A}_{\mu}(z_a), \mathsf{Q}_a], \qquad a = \overline{1,N}. \tag{7}$$

The total action corresponding to field + particle system can be written down as follows:

$$I = \int d^4x\, \mathcal{L} - \sum_a m_a \int d\tau_a \sqrt{\dot{z}^2_a}, \tag{8}$$

where $\mathcal{L}$ is $\mathcal{L}_{\mathrm{ST}}$ or $\mathcal{L}_{\mathrm{NT}}$, and m_a is the rest mass of ath particle. The variation of the action (8) over A_{μ} yields the field equations (3) or (4). Varying this action with respect to particle positions z^{μ}_a and taking account of (7) one can obtain the following equations of particle motion:

$$\frac{d}{d\tau_a}\frac{m\dot{z}_{a\mu}}{\sqrt{\dot{z}^2_a}} = \langle \mathsf{Q}_a, \mathsf{F}_{\mu\nu}(z_a)\rangle \dot{z}^{\mu}_a. \tag{9}$$

In order to determine motion of particles it is necessary to solve the total set of linked equations, namely, the field equations [(3) or (4)], the Wong equations (7), and the equations of motion (9).

We intend to formulate a particle dynamics in the scope of RDIT. For this purpose one should eliminate field variables $\mathsf{A}_\mu(x)$ in favour of their expressions in terms of particle positions z_a^μ and, possibly, charges Q_a. In other words, it is necessary to find a solution to field equations. But this task is very complicated because of nonlinearity of the problem.

Here we limit ourselves by search of Abelian solutions to field equations. Let us suppose that

$$\mathsf{A}_\mu(x) = \mathsf{n} A_\mu(x), \qquad \mathsf{J}^\mu(x) = \mathsf{n} J^\mu(x), \qquad \mathsf{Q}_a(\tau_a) = \mathsf{n} Q_a(\tau_a) \qquad (10)$$

etc., where n is a unit constant vector in $\mathcal{G}$. In this case all Lie-bracketed expressions vanish, in particular,

$$\mathsf{F}_{\mu\nu}(x) = \mathsf{n} F_{\mu\nu}(x), \qquad F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x), \qquad (11)$$

and ∇_μ reduces to ∂_μ. The Wong equations (7) yield

$$\dot{Q}_a = 0 \qquad \Longrightarrow \qquad Q_a = q_a = const. \qquad (12)$$

Then the field equations (3) and (4) reduce to

$$\partial_\nu F^{\nu\mu} = 4\pi J^\mu, \qquad (13)$$

and

$$\partial_\nu \square F^{\nu\mu} = 4\pi\kappa^2 J^\mu, \qquad (14)$$

respectively, where the current

$$J^\mu(x) \equiv \sum_a J_a^\mu(x) = \sum_a q_a \int d\tau_a \dot{z}_a^\mu \delta(x - z_a) \qquad (15)$$

is conserved identically, i.e., $\partial_\mu J^\mu \equiv 0$. Due to this fact both the equations (13) and (14) are gauge invariant with respect to the one-parametric (compact) group of residual symmetry.

At this point we have come to the linear field equations which can be solved by means of the Green's function method. In the standard case we deal exactly with the electromagnetic problem. Using the Lorentz gauge fixing condition,

$$\partial_\mu A^\mu = 0, \qquad (16)$$

we reduce the equation (13) to d'Alembert equation,

$$\square A^\mu = 4\pi J^\mu, \qquad (17)$$

and immediately obtain its solution,

$$A^\mu = D_\eta * J^\mu, \qquad (18)$$

where $*$ denotes the convolution, and

$$D_\eta(x) = (1 + \eta \operatorname{sgn} x^0)\delta(x^2), \qquad (19)$$

is one of the retarded ($\eta = +1$), advanced ($\eta = -1$), or time-symmetric ($\eta = 0$) Green's functions of d'Alembert equation.

Let us consider the equation (14) of the nonstandard theory. Using the Lorentz condition (16) one reduces it to the following equation:

$$\square^2 A^\mu = 4\pi\kappa^2 J^\mu, \qquad (20)$$

which is of 4th order. In Appendix A the corresponding retarded, advanced, and time-symmetric Green's functions are calculated. They are:

$$E_\eta(x) = \tfrac{1}{4}\kappa^2(1 + \eta \operatorname{sgn} x^0)\Theta(x^2). \qquad (21)$$

Thus the solution to (20) reads as (18), but with E_η instead of D_η.

Actually, the linearity of equations (17) and (20) allows solutions of more general structure,

$$A^\mu = \sum_a A_a^\mu = \sum_a G_{\eta_a} * J_a^\mu, \qquad (22)$$

where $G_{\eta_a} = D_{\eta_a}$ for ST, and $G_{\eta_a} = E_{\eta_a}$ for NT. Here η_a take values +1, -1, or 0, each own for different particles.

In an explicit form the solutions (22) can be written down as follows:

$$A^\mu(x) = \sum_a A_a^\mu(x) = \sum_a q_a \int d\tau_a \dot{z}_a^\mu G_{\eta_a}(x - z_a), \qquad (23)$$

where the quantity $A_a^\mu(x)$ represents the relativistic potential created by ath particle. In both the ST– and NT–cases each particle potential (as well as the total sum (23)) satisfies the Lorentz condition (16).

Up to the numerical factor, the only difference between (19) and (21) is that the function $\delta(x^2)$ is replaced by $\Theta(x^2)$. This substitution was guessed by Weiss in [34] where the time-symmetric potential (in our case, Eqs (23) with $G_{\eta_a} = E_0$, $a = \overline{1, N}$) has been proposed for the model of the action-at-a-distance linear confinement.

3. Equations of motion and Fokker-type action integrals

Now the equations of particle motion can be obtained in a closed form by substitution of the relativistic potentials (23) and the constant charges (12) into the right-hand side (r.h.s.) of (9). In the standard case this procedure leads to an appearance of divergent self-action terms which can be regularized in usual way [38]. The resulting equations of motion can be presented in the form:

$$\frac{d}{d\tau_a}\frac{m\dot{z}_{a\mu}}{\sqrt{\dot{z}_a^2}} = q_a \sum_{b \neq a} F_{ab\mu\nu}\dot{z}_a^\nu + R_{a\mu}, \tag{24}$$

where

$$F_{ab\mu\nu} = 2q_b\int d\tau_b(1 + \eta_b\,\mathrm{sgn}\,z_{ab}^0)\delta'(z_{ab}^2)\left\{z_{ab\mu}\dot{z}_{b\nu} - z_{ab\nu}\dot{z}_{b\mu}\right\}, \tag{25}$$

$z_{ab} \equiv z_a - z_b$, and

$$R_{a\mu} = \frac{2}{3}\eta_a q_a^2\left\{\delta_\mu^\nu - \frac{\dot{z}_{a\mu}\dot{z}_a^\nu}{\dot{z}_a^2}\right\}\frac{d}{d\tau_a}\frac{1}{\sqrt{\dot{z}_a^2}}\frac{d}{d\tau_a}\frac{\dot{z}_{a\nu}}{\sqrt{\dot{z}_a^2}}, \tag{26}$$

The self-action terms $R_{a\mu}$ correspond to radiation reaction. They disappear if fields generated by particles are time-symmetric (i.e., if $\eta_a = 0$).

In the nonstandard case no divergences and self-action terms arise. Thus the equations of motion are calculated immediately. They are described by (24) with $R_{a\mu} = 0$ and

$$F_{ab\mu\nu} = \tfrac{1}{2}\kappa^2 q_b\int d\tau_b(1 + \eta_b\,\mathrm{sgn}\,z_{ab}^0)\delta(z_{ab}^2)\left\{z_{ab\mu}\dot{z}_{b\nu} - z_{ab\nu}\dot{z}_{b\mu}\right\}. \tag{27}$$

We have obtained the closed set of equations of particle motion which are not obvious to be directly deducible from the variacion principle. Below we construct the relevant Fokker-type version of the theory and examine it consistency with the equations obtained above.

The purpose is to eliminate field variables from the total action (8). Using (10)–(12) in (1), (2), and then in (8), one obtains the action

$$I = I_{\mathrm{free}} + I_{\mathrm{int}} + I_{\mathrm{field}}, \tag{28}$$

where

$$I_{\mathrm{free}} = -\sum_a m_a\int d\tau_a\sqrt{\dot{z}_a^2}, \tag{29}$$

$$I_{\mathrm{int}} = -\int d^4x\, J^\mu A_\mu, \tag{30}$$

are the same for ST and NT while I_{field} is different:

$$I_{\mathrm{field}} = -\frac{1}{16\pi}\int d^4x\, F^{\mu\nu}F_{\mu\nu}, \tag{31}$$

for ST, and

$$I_{\mathrm{field}} = \frac{1}{16\pi\kappa^2}\int d^4x\,(\partial^\lambda F^{\mu\nu})(\partial_\lambda F_{\mu\nu}), \tag{32}$$

for NT. The term I_{field} can be transformed to the form

$$I_{\mathrm{field}} = \frac{1}{8\pi}\int d^4x\, A_\mu\partial_\nu H^{\nu\mu} + \begin{pmatrix} \text{surface} \\ \text{terms} \end{pmatrix}, \tag{33}$$

where $H^{\nu\mu} = F^{\nu\mu}$ for ST, and $H^{\nu\mu} = \Box F^{\nu\mu}/\kappa^2$ for NT. Taking into account the field equations (13) and (14) in r.h.s. of (33) and omitting surface terms, we obtain

$$I = I_{\mathrm{free}} + \tfrac{1}{2}I_{\mathrm{int}}. \tag{34}$$

Now substituting the current (15) and the potential (23) into (30), one can present the second term in r.h.s. of (34) in the following form:

$$\tfrac{1}{2}I_{\mathrm{int}} = \sum_a\sum_{<\,b} I_{ab} + \tfrac{1}{2}\sum_a I_{aa}, \tag{35}$$

where

$$I_{ab} = -q_a q_b\iint d\tau_a d\tau_b\,\dot{z}_a\cdot\dot{z}_b\, G_{\eta_{ba}}(z_{ab}), \tag{36}$$

and $\eta_{ba} \equiv \tfrac{1}{2}(\eta_b - \eta_a)$. In the ST–case the self-action term I_{aa} diverges. It can be regularized and unified with ath term of I_{free}. In the NT–case this term vanishes. Thus in the both cases the resulting interaction term $\tfrac{1}{2}I_{\mathrm{int}}$ has the form:

$$\tfrac{1}{2}I_{\mathrm{int}} = -\sum_a\sum_{<\,b} q_a q_b\iint d\tau_a d\tau_b\,\dot{z}_a\cdot\dot{z}_b\, G_{\eta_{ba}}(z_{ab}), \tag{37}$$

where $G_{\eta_{ba}} = D_{\eta_{ba}}$ for ST, and $G_{\eta_{ba}} = E_{\eta_{ba}}$ for NT.

In the case of NT each constituent (36) by means of integration via parts (see [39] for such a technique) can be transformed to the following

form (here we omit all unessential constant factors):

$$
\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} d\tau_a d\tau_b \, \dot{z}_a \cdot \dot{z}_b \, (1 + \eta_{ba} \operatorname{sgn} z_{ab}^0) \Theta(z_{ab}^2)
$$

$$
= -2 \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} d\tau_a d\tau_b \, (z_{ab} \cdot \dot{z}_a)(z_{ab} \cdot \dot{z}_b)(1 + \eta_{ba} \operatorname{sgn} z_{ab}^0)\delta(z_{ab}^2)
$$

$$
- \frac{1}{2}\left[\left[(1 + \eta_{ba} \operatorname{sgn} z_{ab}^0)\Theta(z_{ab}^2)z_{ab}^2\right]_{\tau_a=-\infty}^{\tau_a=+\infty}\right]_{\tau_b=-\infty}^{\tau_b=+\infty}. \tag{38}
$$

The second term in r.h.s. of (38) is divergent, but it does not contribute in equations of motion and can be omitted. Then the interaction term (37) for NT can be put in the equivalent form,

$$
\tfrac{1}{2}I_{\text{int}} = \frac{\kappa^2}{2}\sum_a \sum_{<\,b} q_a q_b \iint d\tau_a d\tau_b \, (z_{ab} \cdot \dot{z}_a)(z_{ab} \cdot \dot{z}_b)D_{\eta_{ba}}(z_{ab}). \tag{39}
$$

Fokker–type equations of motion following from this action differ from those (24) directly obtained from the field theory. Firstly, they do not reproduce the self-action terms $R_{a\mu}$ which, in general, are present in r.h.s. of equations (24) for ST. In this paper we suppose that these terms can be neglected since in QCD a radiation is suppressed by confinement. Secondly, the sign factors η_b in the expressions (25) and (27) for $F_{ab\mu\nu}$ are replaced by η_{ba}. This changes the causal structure of pair particle interactions. Namely, while equations (24) correspond to retarded, advanced, or time-symmetric fields generated by bth particles (and sensed by ath particle) for $\eta_b = +1, -1,$ or 0, respectively, in the Fokker–type equations the causality of interactions is its own for different pairs of particles.

There are only two cases in which the direct interaction can be treated as a field-type one. The first case corresponds to the time-symmetric interaction, for which $\eta_a = \eta_{ba} = 0$, $a, b = \overline{1, N}$. For ST the action (34), (29), (37) in this case coincides with the Wheeler–Feynman action of time-symmetric electrodynamics [40]. For NT it corresponds to the action-at-a distance confinement model in the form by Weiss [34]. The Rivacoba's form of this action integral [33] follows from (39).

The second case which is tractable in terms of field interaction realizes only for two-particle systems. It corresponds to the choice $\eta_2 = -\eta_1 = \eta_{21} \equiv \eta = \pm 1$. For ST this is the case of the time-asymmetric

electromagnetic interaction proposed by Staruszkiewicz [41], Rudd and Hill [42], and studied in more detail in [43,44]. For NT the corresponding time-asymmetric Fokker-type action can be taken as the classical base for relativistic confinement model.

4. Time–asymmetric model with confinement interaction

In this section we consider the two-particle model available for the classical description of mesons. It is based on the time-asymmetric Fokker-type action which combines interaction terms (37) from ST and (39) from NT. Since mesons are chargeless systems, we put $q_1 = -q_2 \equiv q$. Then the time-asymmetric Fokker-type action has the form:

$$
I = -\sum_{a=1}^{2} m_a \int d\tau_a \sqrt{\dot{z}_a^2} + \iint d\tau_1 d\tau_2 D_\eta(z_{12}) \times
$$

$$
\times \left\{\alpha \, \dot{z}_1 \cdot \dot{z}_2 - \beta(z_{12} \cdot \dot{z}_1)(z_{12} \cdot \dot{z}_2)\right\}, \tag{40}
$$

where $\alpha \equiv q^2$, $\beta \equiv \frac{1}{2}q^2\kappa^2$, and $\eta = \pm 1$. In the nonrelativistic limit this action leads to the well known interquark potential $U = -\alpha/r + \beta r$.

Integrating the second term of the action (4) once, we reduce the latter to a single-time form [35,36]. Thus we obtain the description of our model in the framework of a manifestly covariant Lagrangian formalism with the Lagrangian function

$$
L = \theta F(\sigma_1, \sigma_2, \delta), \tag{41}
$$

where $\theta \equiv \eta \dot{y} \cdot z > 0$, $z \equiv z_1 - z_2$, $y \equiv (z_1 + z_2)/2$, $\sigma_a \equiv \sqrt{\dot{z}_a^2}/\theta > 0$, $\delta \equiv \dot{z}_1 \cdot \dot{z}_2/\theta^2 > 0$, and with the holonomic constraint $z^2 = 0$, $\eta z^0 > 0$. All variables in (41) depend on an arbitrary common evolution parameter τ. In our case the function F has the form:

$$
F \equiv \sum_{a=1}^{2} m_a \sigma_a - \alpha\delta + \beta. \tag{42}
$$

We note that quantities θ, σ_a, δ in r.h.s. of (41) and (42) are well defined and positive if particle world lines are timelike.

The transition to the manifestly covariant Hamiltonian description with constraints leads to the mass-shell constraint which determines the dynamics of the model and has the following form [35,36]:

$$
\phi(P^2, \, v^2, \, P \cdot z, \, v \cdot z) \equiv \phi_{\text{free}} + \phi_{\text{int}} = 0. \tag{43}
$$

Here $v_\mu \equiv w_\mu - z_\mu\, Pw/Pz$; P_μ and w_μ are canonical momenta conjugated to y^μ and z^μ, respectively; the function

$$\phi_{\text{free}} = \tfrac{1}{4}P^2 - \tfrac{1}{2}(m_1^2 + m_2^2) + (m_1^2 - m_2^2)\frac{v\cdot z}{P\cdot z} + v^2 \qquad (44)$$

corresponds to the free-particle system,

$$\begin{aligned}
\phi_{\text{int}} \quad = \quad & \frac{\alpha(P^2 - m_1^2 - m_2^2)}{\eta P\cdot z} + \frac{\alpha^2}{\eta P\cdot z}\sum_{a=1}^{2}\frac{m_a^2}{b_a + \alpha} \\
& - 2\beta\left(\frac{b_1 b_2}{\eta P\cdot z} + \alpha\right)
\end{aligned} \qquad (45)$$

describes the interaction, and

$$b_a \equiv \eta\left(\tfrac{1}{2}P\cdot z + (-)^{\bar a}v\cdot z\right), \quad a = 1,2, \quad \bar a \equiv 3 - a. \qquad (46)$$

We note, that the quantities σ_a are related to canonical variables by the equations:

$$\sigma_a = m_a/(b_a + \alpha), \qquad\qquad a = 1,2. \qquad (47)$$

Since σ_a must be positive, the following conditions arise:

$$b_a + \alpha > 0, \qquad\qquad a = 1,2. \qquad (48)$$

They restrict the whole phase space to a physical domain in which the Hamiltonian description is equivalent to the Lagrangian one.

In order to study the dynamics of the present model it is convenient, following [35,36], to transit from the manifestly covariant to three-dimensional Hamiltonian description in the framework of the Bakamjian-Thomas model [45–47]. Within this description ten generators of the Poincaré group P_μ, $J_{\mu\nu}$ as well as the covariant particle positions z_a^μ are the functions of canonical variables $\mathbf{Q}$, $\mathbf{P}$, $\mathbf{r}$, $\mathbf{k}$. The only arbitrary function appearing in expressions for canonical generators is the total mass $|P| = M(\mathbf{r},\mathbf{k})$ of the system which determines its internal dynamics. For the time-asymmetric models this function is defined by the mass-shell equation [35,36] which can be derived from the mass-shell constraint via the following substitution of arguments on the l.-h.s. of (43):

$$P^2 \to M^2, \quad v^2 \to -\mathbf{k}^2, \quad P\cdot z \to \eta M r, \quad v\cdot z \to -\mathbf{k}\cdot\mathbf{r}; \qquad (49)$$

here $r \equiv |\mathbf{r}|$.

Due to the Poincaré-invariance of the description it is sufficient to choose the centre-of-mass (CM) reference frame in which $\mathbf{P} = \mathbf{0}$, $\mathbf{Q} = \mathbf{0}$. Accordingly, $P_0 = M$, $J_{0i} = 0$ $(i = 1,2,3)$, and the components $S_i \equiv \tfrac{1}{2}\varepsilon_i^{\ jk}J_{jk}$ form a 3-vector of the total spin of the system (internal angular momentum) $\mathbf{S} = \mathbf{r}\times\mathbf{k}$ which is the integral of motion. At this point the problem is reduced to the rotation invariant problem of some effective single particle; such a problem is integrable in terms of polar coordinates,

$$\mathbf{r} = r\mathbf{e}_r, \qquad \mathbf{k} = k_r\mathbf{e}_r + S\mathbf{e}_\varphi/r. \qquad (50)$$

Here $S \equiv |\mathbf{S}|$; the unit vectors $\mathbf{e}_r$, $\mathbf{e}_\varphi$ are orthogonal to $\mathbf{S}$, they form together with $\mathbf{S}$ a right-oriented triplet and can be decomposed in terms of Cartesian unit vectors $\mathbf{i}$, $\mathbf{j}$:

$$\mathbf{e}_r = \mathbf{i}\cos\varphi + \mathbf{j}\sin\varphi, \qquad \mathbf{e}_\varphi = -\mathbf{i}\sin\varphi + \mathbf{j}\cos\varphi, \qquad (51)$$

where φ is the polar angle.

The corresponding quadratures read:

$$t - t_0 = \int dr\, \partial k_r(r,M,S)/\partial M, \qquad (52)$$

$$\varphi - \varphi_0 = -\int dr\, \partial k_r(r,M,S)/\partial S, \qquad (53)$$

where $t = \tfrac{1}{2}(z_1^0 + z_2^0)_{\text{CM}}$ is the fixed evolution parameter (unlike the undetermined one τ), and the radial momentum k_r, being the function of r, M, S, is defined by the mass-shell equation written down in terms of these variables,

$$\begin{aligned}
& \phi\left(M^2,\ -\mathbf{k}^2,\ \eta M r,\ -\mathbf{k}\cdot\mathbf{r}\right) \\
& \equiv \quad \phi\left(M^2,\ -k_r^2 - \frac{S^2}{r^2},\ \eta M r,\ -k_r r\right) = 0. \qquad (54)
\end{aligned}$$

The solution of the problem given in terms of canonical variables enables to obtain particle world lines in the Minkowski space using the following formulae [35,36]:

$$z_a^0 = t + \tfrac{1}{2}(-)^{\bar a}\eta r, \qquad (55)$$

$$\mathbf{z}_a = \tfrac{1}{2}(-)^{\bar a}\mathbf{r} + \eta r\frac{\mathbf{k}}{M} \equiv \left(\tfrac{1}{2}(-)^{\bar a} + \eta\frac{k_r}{M}\right)r\mathbf{e}_r + \eta\frac{S}{M}\mathbf{e}_\varphi. \qquad (56)$$

Especially, the vector $\mathbf{z} = \mathbf{z}_1 - \mathbf{z}_2 = \mathbf{r}$ characterizes the relative motion of particles.

4.1. Purely confinement model

Hereafter we restrict ourselves to the system of equal rest masses, $m_1 = m_2 \equiv m$. The case $\alpha = 0$ corresponds to purely confinement interaction. The mass-shell equation in this case reads:

$$\frac{S^2}{r^2} + m^2 - \left(1 - 2\frac{\beta r}{M}\right)\left(\tfrac{1}{4}M^2 - k_r^2\right) = 0. \qquad (57)$$

It easy to obtain from (57) the expression for $k_r(r, M, S)$,

$$k_r = \epsilon\sqrt{f(r, M, S)}, \qquad \epsilon = \pm 1, \qquad (58)$$

$$f(r, M, S) = \tfrac{1}{4}M^2 - \frac{m^2 + S^2/r^2}{1 - 2\beta r/M} \geq 0. \qquad (59)$$

Besides, we must take into account the condition:

$$\tfrac{1}{4}M^2 - k_r^2 > 0 \qquad (60)$$

which follows from (48). Then from (58)–(60) we obtain the restriction:

$$0 < r < \tfrac{1}{2}M/\beta. \qquad (61)$$

The quadratures (52), (53) with (58), (59) can be reduced to the elliptic integrals. Here we omit their expressions. The integration is spread over the domain of possible motions (DPM) which is determined by the conditions (59) and (61). In the case $S > 0$ DPM consists of the connected interval $r_1 \leq r \leq r_2$, where r_1, r_2 are positive roots of the equation $f(r, M, S) = 0$. The latter can be presented as the reduced cubic equation with respect to $1/r$:

$$\frac{1}{r^3} - \frac{M^2}{4S^2}\left(1 - \frac{4m^2}{M^2}\right)\frac{1}{r} + \frac{M\beta}{2S^2} = 0. \qquad (62)$$

It has two real positive solutions provided the following condition holds:

$$M \geq M_c(S), \qquad (63)$$

where the function $M_c(S)$ is defined in the implicit form

$$S = \frac{M_c^2}{6\sqrt{3}\beta}\left(1 - \frac{4m^2}{M_c^2}\right)^{3/2}, \qquad M_c \geq 2m. \qquad (64)$$

The equality in (63) corresponds to the case $r_1 = r_2 \equiv r_c$ of circular particle orbits with the distance between particles

$$r_c = \frac{M_c}{3\beta}\left(1 - \frac{4m^2}{M_c^2}\right) \qquad (65)$$

satisfying the set of equations:

$$f(r_c, M_c, S) = 0, \qquad \partial f(r_c, M_c, S)/\partial r_c = 0. \qquad (66)$$

In the limit $S \to 0$ the quadrature (53) yields $\varphi = \varphi_0$, and a particle motion becomes one-dimensional (i.e., in the two-dimensional space-time $\mathbb{M}_2$ parametrized with x^0 and, say, x^1). Besides, $r_1 \to 0$. Thus DPM becomes $0 < r \leq r_2$. The point $r = 0$ corresponds to particle collision. This point is not singular for the quadrature (52) and particle coordinates (55), (56). Thus the motion of particles can be smoothly continued as if they pass through one another.

4.2. General model, $S > 0$

Let us consider the general case $\alpha > 0$, $\beta > 0$. The corresponding mass-shell equation can be written down as follows:

$$\frac{S^2}{r^2}\Delta - \left(\Delta - \frac{\alpha^2}{r^2}\right)\left\{\left(1 - 2\frac{\beta r}{M}\right)\Delta - 2\frac{m^2}{M}\left(\frac{M}{2} + \frac{\alpha}{r}\right)\right\} = 0. \qquad (67)$$

It is quadratic equation with respect to

$$\Delta \equiv \left(\frac{M}{2} + \frac{\alpha}{r}\right)^2 - k_r^2 > 0, \qquad (68)$$

where Δ must be positive because of the conditions (48). As to k_r the equation (67) is biquadratic. Its solution can be presented in the following form:

$$k_r = \epsilon\sqrt{f_{\pm}(r, M, S)}, \qquad \epsilon = \pm 1, \qquad (69)$$

where

$$f_{\pm}(r, M, S) = \tfrac{1}{4}M^2 + M\frac{\alpha}{r} - \frac{h_{\pm}(r, M, S)}{1 - 2\beta r/M}, \qquad (70)$$

$$h_{\pm}(r, M, S) = g(r, M, S) \mp \sqrt{d(r, M, S)} \qquad (71)$$

$$d(r, M, S) = g^2(r, M, S) + \left(1 - 2\frac{\beta r}{M}\right)\frac{\alpha^2 S^2}{r^4}, \qquad (72)$$

$$g(r, M, S) = \frac{m^2}{M}\left(\frac{M}{2} + \frac{\alpha}{r}\right) - \left(1 - 2\frac{\beta r}{M}\right)\frac{\alpha^2}{2r^2} + \frac{S^2}{2r^2}. \qquad (73)$$

Among two solutions $f_\pm$ for k_r^2 we choose that one which is smooth in DPM and reduces to f (59) in the limit $\alpha \to 0$.

DPM is analyzed in Appendix B. In the case $S > 0$ we have $d(r, M, S) > 0$, $r > 0$. Thus both the functions $f_\pm(r, M, S)$ are smooth provided $r \neq \frac{1}{2}M/\beta$, and $f_-(r, M, S)$ reduces to f (59) in the limit $\alpha \to 0$. DPM in this case is determined by inequality $f_-(r, M, S) \geq 0$ provided the condition (68) holds. Similarly to the purely confinement case we obtain $r_1 \leq r \leq r_2$, where r_1, r_2 are positive roots of the equation:

$$\frac{S^2}{r^2}\left(\frac{M}{2} + \frac{\alpha}{r}\right) - \left(\tfrac{1}{4}M^2 + M\frac{\alpha}{r}\right) \times$$
$$\times \left\{\left(1 - 2\frac{\beta r}{M}\right)\left(\frac{M}{2} + \frac{\alpha}{r}\right) - 2\frac{m^2}{M}\right\} = 0 \qquad (74)$$

It can be reduced to a fourth-order algebraic equation which has two
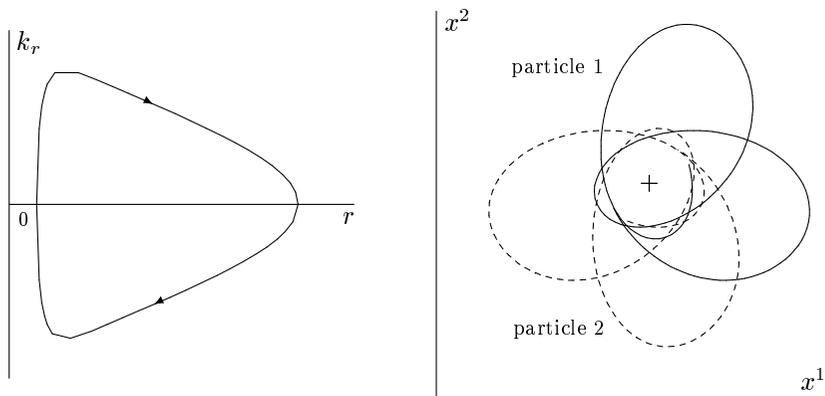
Figure 1. General model, $S > 0$. Typical example of phase (left) and particle (right) trajectories. Arrows show the direction of evolution; $+$ – centre of mass.

real positive solutions provided (63), where the function $M_c(S)$ can be presented in a parametric form,

$$M_c^2(\lambda) = 2m^2\frac{\lambda[4 + 5\lambda + 2\lambda^2 + \nu(1+\lambda)^2(4+3\lambda)]}{(1+\lambda)^3},$$
$$S^2(\lambda) = \alpha^2\frac{\lambda(2+\lambda)^2[1 + \nu(1+\lambda)^2]}{4 + 5\lambda + 2\lambda^2 + \nu(1+\lambda)^2(4+3\lambda)},$$
$$\nu \equiv \frac{\alpha\beta}{m^2}, \qquad 0 < \lambda < \infty. \qquad (75)$$

The condition $M = M_c(S)$ corresponds to circular orbits with $r_c = 2\alpha\lambda/M_c$ to be the distance between particles.

Our attempts to express the quadratures (52), (53) with (69)–(73) in terms of known (elementary and special) functions have not met with success. Thus we calculated them with a computer. Nevertheless by means of analytic calculations it can be shown that particle world lines in $\mathbb{M}_4$ are timelike and smooth curves. They represent a bound motion of particles for all values of M allowed by (63), (75), and $S > 0$. The typical example of phase and particle trajectories are shown in figure 1.

4.3. General model, $S = 0$

In the case $S = 0$ we have $d(r, M, 0) = g^2(r, M, 0)$. Since there exists the point $r_0 < \alpha/m$ such that $g(r_0, M, 0) = 0$, the functions $f_\pm(r, M, 0)$ are not smooth. Moreover, in the domain $r < r_0$ the function $f_-(r, M, 0)$ has not the proper form in the limit $\alpha \to 0$. Thus the evolution of particles cannot straightforwardly be continued farther.

We point out that the distance r_0 at which the smoothness of $f_\pm(r, M, 0)$ violates is smaller than α/m. The latter is an analogue of the classical electron radius. In the case of strong interaction the distance r_0 and the Compton length of quarks can be commensurable quantities. Thus the classical description of particle motion at $r < r_0$ may be important for the construction of quantum theory. Especially, this is concerned with the case of S–states. Below we propose the way to continue the particle motion in the domain $r < r_0$. It leads beyond the rigorous treatment of analytical mechanics and therefore cannot be a reliable basis of quantum-mechanical description. But it will be noted that this method arises naturally from the present model itself.

Let us choose in r.h.s. of (69) the function:

$$f_0(r, M) \equiv \begin{cases} f_+(r, M, 0), & r < r_0 \\ f_-(r, M, 0), & r > r_0 \end{cases}$$
$$= \left(\frac{M}{2} + \frac{\alpha}{r}\right)\left\{\frac{M}{2} + \frac{\alpha}{r} - \frac{2m^2/M}{1 - 2\beta r/M}\right\}, \qquad (76)$$

which is smooth provided $r \neq \frac{1}{2}M/\beta$, and reduces to (59) if $\alpha \to 0$. DPM in this case is $0 < r \leq r_2$ while the point $r = 0$ is critical: $\Delta \to \infty$, $r \to 0$. This means that the equivalence between the Lagrangian and Hamiltonian formalisms violates. It can be shown that at the collision one of particles reaches (but not exceeds) the speed of light while another does not. Again, the particle world lines should be somehow continued farther.
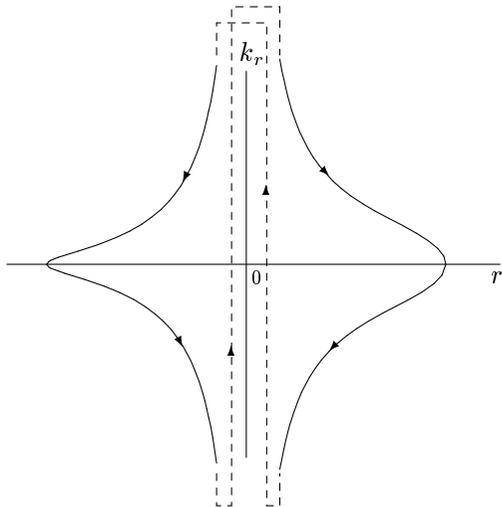
Figure 2. General model, $S = 0$. Typical example of phase trajectory continued in the non-Lagrangian domain $r < 0$. Arrows show the direction of evolution.
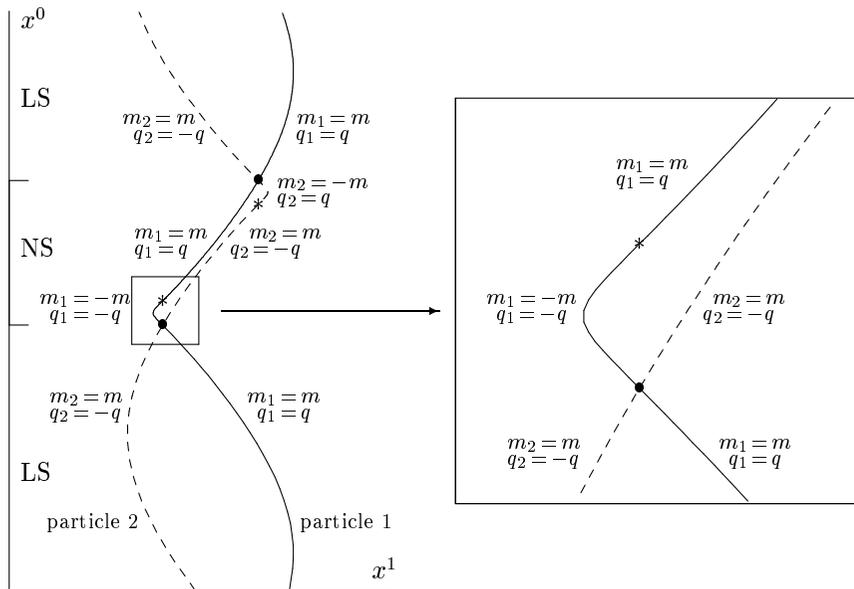

Figure 3. General model, $S = 0$. Typical example of particle world lines. LS – Lagrangian segments; NS – non-Lagrangian segments; $\bullet$ – collision: one of particles reaches the light speed; $*$ – particles reach the light speed.

The existence of a singular collision point is due to the time-asymmetric vector interaction from ST. The confinement interaction does not change qualitatively the behaviour of particles in the neighbourhood of collision. Specific features of the time-asymmetric model with attractive vector interaction in $\mathbb{M}_2$ have been analyzed in [48]. Following this work, in the framework of Hamiltonian description the interesting possibility exists to continue smoothly world lines through the collision point. In our terms it is sufficient for this purpose to suppose formally that after the collision the variable r becomes negative. Then we have $f_0(r, M) \geq 0$, $r \in [-2\alpha/M, 0)$, and the motion of particles can be continued up to the distance $|r| = 2\alpha/M$. At this point which is also critical both particles reach (but not exceed) the speed of light. Again, one can smoothly continue world lines up to the next collision e.t.c. We note that $\Delta < 0$, $r \in (-2\alpha/M, 0)$. Thus the segments of world lines obtained as above do not follow from the Lagrangian description. The resulting world lines combine the Lagrangian and non-Lagrangian segments separated by the collision points. They describe the bound periodic motion of particles. The corresponding phase trajectory and world lines are shown in figure 2 and 3 respectively.

The formal continuation of evolution proposed above permits some reinterpretation in terms of the Lagrangian description. Expressing the quantities θ, σ_a and δ in terms of canonical variables one can examine that some of them have wrong (i.e., negative) sign if $r < 0$, i.e., if particles pass non-Lagrangian segments of world lines. Equivalently, one can keep $r > 0$ changing signs of some constants m_a, α, and β in the Lagrangian (41), (42). In such a manner one can realize that particles move as if each one changes signs of its rest mass and charge, $m_a \rightarrow -m_a$, $q_a \rightarrow -q_a$, once it passes a critical point with the speed of light. We note that at the collision point one of world lines is timelike. Thus the mass and charge of this particle remain unchanged up to the next critical point. As a result, after having passed the non-Lagrangian segment each particle returns its proper values of mass and charge.

5. Semiclassical estimates of Regge trajectory

It is well known that nonrelativistic potential model with the linear potential leads to the Regge trajectory with the unsatisfactory asymptote $M \sim S^{2/3}$. Here we do not propose a quantum version of the present model, but we make the estimates of the Regge trajectory from what follows.

Usually the Regge trajectories in the potential models are calculated

in the oscillator approximation [3]. Then the leading Regge trajectory originates from the classical mechanics: it is close to the curve of circular motions on the (M^2, S)–plane.

In our case this curve follows from the equation (64) for $\alpha = 0$ or from (75) in the general case. The latter has in the ultrarelativistic limit $M_c \to \infty$ the desirable linear asymptote:

$$M_c^2 \approx 6\sqrt{3}\beta S + 6\left(m^2 - 3\alpha\beta\right). \tag{77}$$

It is remarkable that this asymptote is achieved only by taking account of a relativity.
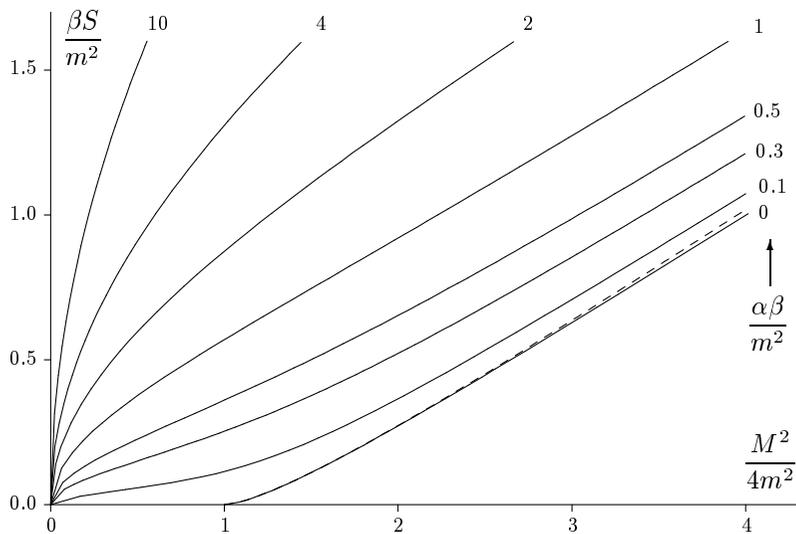
Figure 4. Classical Regge trajectories; ——— from general time-asymmetric model at various rates of parameters; – – – from time-symmetric model.

Let us compare the classical Regge trajectory (64) of purely confinement time-asymmetric model to that which follows from the time-symmetric Fokker-type confinement model with the same parameters $m_1 = m_2 \equiv m$ and β. Considering the circular orbit solution, given in [33][1], for large M one can obtain:

$$M_c^2 \approx 4(1 + \sin\vartheta)\{2\cos\vartheta\,\beta S + m^2\}, \tag{78}$$

where the angle

$$\vartheta \approx 0.7391 \approx 0.2353\pi \tag{79}$$

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[1] Such a solution given in [34] seems to contain a miscount.

is the solution of the transcendental equation

$$\vartheta = \cos\vartheta. \tag{80}$$

The only difference between the asymptotes (77) (with $\alpha = 0$, of course) and (78) consists in slightly different numerical factors at the linear and constant terms

linear terms	constant terms	
$6\sqrt{3} \approx 10.3923$	6	(81)
$8\cos\vartheta\,(1 + \sin\vartheta) \approx 9.8955$	$4(1 + \sin\vartheta) \approx 6.6944$	

Moreover, it turns out that by the substitution $\vartheta = \pi/6$ instead of (79) into the r.h.s. of (78), the latter reduces to (77) (with $\alpha = 0$). In the non-relativistic limit both the time-symmetric and time-asymmetric (purely confinement) models lead to the same relation:

$$M_c - 2m \approx 3\left(\frac{\beta S}{2\sqrt{m}}\right)^{2/3} \tag{82}$$

which is known from the nonrelativistic linear confinement model. Classical Regge trajectories from the general time-asymmetric model as well as one from time-symmetric model are shown in figure 4.

These purely classical results give us the base to consideration of semiclassical quantization of the model. By analogy with WKB approximation method we put

$$S = \hbar(\ell + \tfrac{1}{2}), \qquad \ell = 0, 1, \dots \tag{83}$$

for the quantized internal momentum, and

$$\oint k_r dr = 2\pi\hbar(n_r + \tfrac{1}{2}), \qquad n_r = 0, 1, \dots \tag{84}$$

for radial excitations; the integral runs over the classical phase trajectory.

In the case of purely confinement model we have:

$$\int_{r_1}^{r_2} dr \sqrt{f(r, M, S)} = \pi\hbar(n_r + \tfrac{1}{2}). \tag{85}$$

Using the oscillator approximation [3,6] we expand the function $f(r, M, S)$ (59) about the circular orbit to first nonvanishing orders in $\Delta M \equiv M - M_c$ and $\Delta r \equiv r - r_c$. The result is as follows:

$$f(r, M, S) \approx a^2(M, S) - b^2(S)(\Delta r)^2, \tag{86}$$

where

$$a^2(M,S) \equiv \left.\frac{\partial f(r,M,S)}{\partial M}\right|_c \Delta M = \frac{M_c(M_c^2 + 2m^2)}{(M_c^2 + 8m^2)}\Delta M, \qquad (87)$$

$$b^2(S) \equiv -\left.\frac{\partial^2 f(r,M,S)}{2\,\partial r^2}\right|_c = \frac{27\beta^2 M_c^4}{4(M_c^2 - 4m^2)(M_c^2 + 8m^2)}, \qquad (88)$$

and the function $M_c(S)$ is defined in (64). Then the integral in l.h.s. of (85) is easily calculated:

$$\int\limits_{-a/b}^{a/b} d(\Delta r)\sqrt{a^2 - b^2(\Delta r)^2} = \frac{\pi}{2}\frac{a^2}{b}. \qquad (89)$$

Using (85)–(89) and assuming that ΔM is small compared to M_c we obtain for M^2 the expression:

$$M^2 = M_c^2\left\{1 + \frac{6\sqrt{3}\beta\hbar}{M_c^2 + 2m^2}\sqrt{\frac{M_c^2 + 8m^2}{M_c^2 - 4m^2}}\left(n_r + \tfrac{1}{2}\right)\right\} \qquad (90)$$

which together with (83) and the definition (64) of the function $M_c(S)$ describes the leading (for $n_r = 0$) and daughter (for $n_r = 1, 2, \ldots$) Regge trajectories.

Similarly, in the case of general model we obtain the Regge trajectories determined in the implicit form by the equations

$$M^2 = M_c^2(\lambda)\left\{1 + \frac{\hbar}{\alpha}\,\Phi(\lambda)\,(n_r + \tfrac{1}{2})\right\}, \qquad (91)$$

$$\Phi(\lambda) = \frac{\sqrt{[1+2\lambda+\nu(1+\lambda)^2]\,[3+(1+\nu)(1+\lambda)^2+3\nu(1+\lambda)^4]}}{(1+\lambda)^2\,[1 + \nu(1+\lambda)]\,\sqrt{\lambda(2+\lambda)}}, \qquad (92)$$

and (75), (83). At large ℓ these trajectories reduce to linear ones,

$$M_c^2 \approx 6\sqrt{3}\beta\hbar(\ell + n_r + 1) + 6\left(m^2 - 3\alpha\beta\right), \qquad (93)$$

so that the daughters are parallel to leading trajectory. Moreover, states of unit internal momentum differences form into degenerate towers at a given mass. This tower structure is of interest for the meson spectroscopy, as it is intimated in [6]. The number of relativistic potential models based on single-particle wave equations [4,6–8] as well as two-particle models with oscillator interaction [15,12] lead to degeneracy of $\ell + 2n_r$ type, but
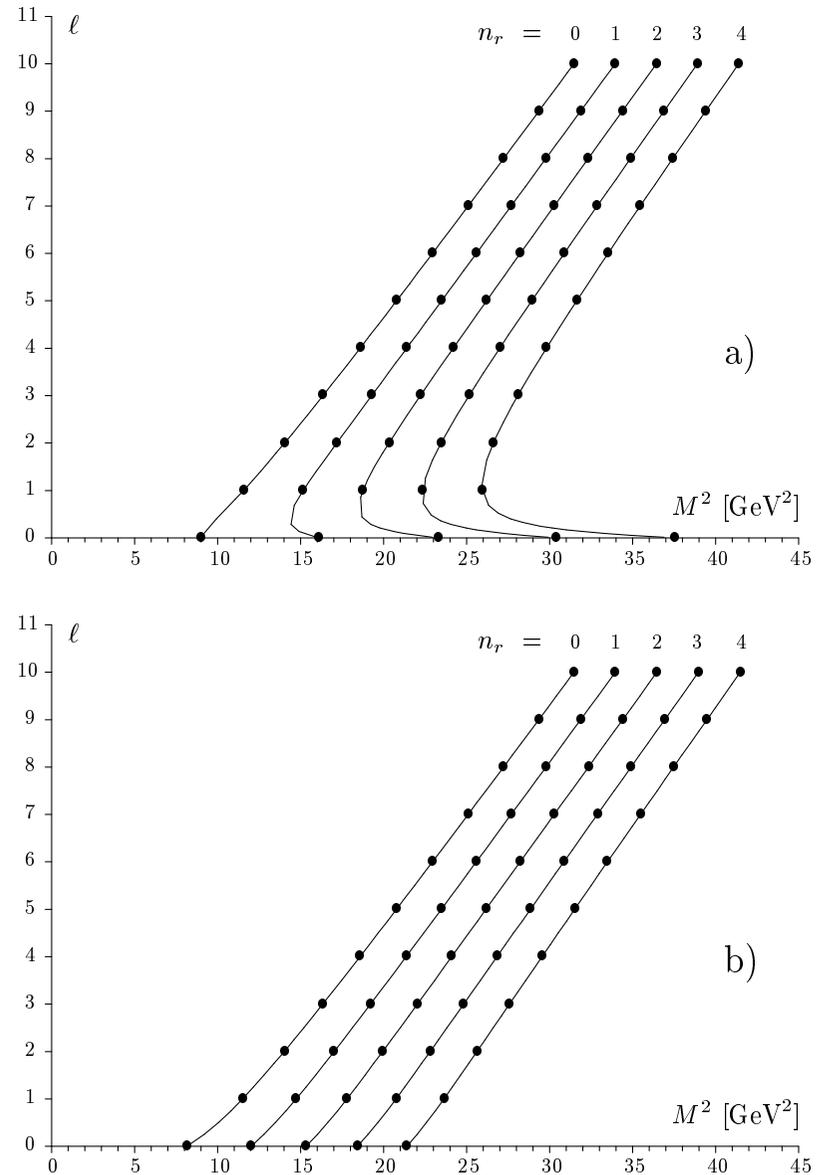

Figure 5. Semiclassical Regge trajectories; $m = 1.27\,\mathrm{GeV}$, $\alpha = 0.5$, $\beta = 0.2\,\mathrm{GeV}^2$; a) oscillator approximation; b) numerical solution.
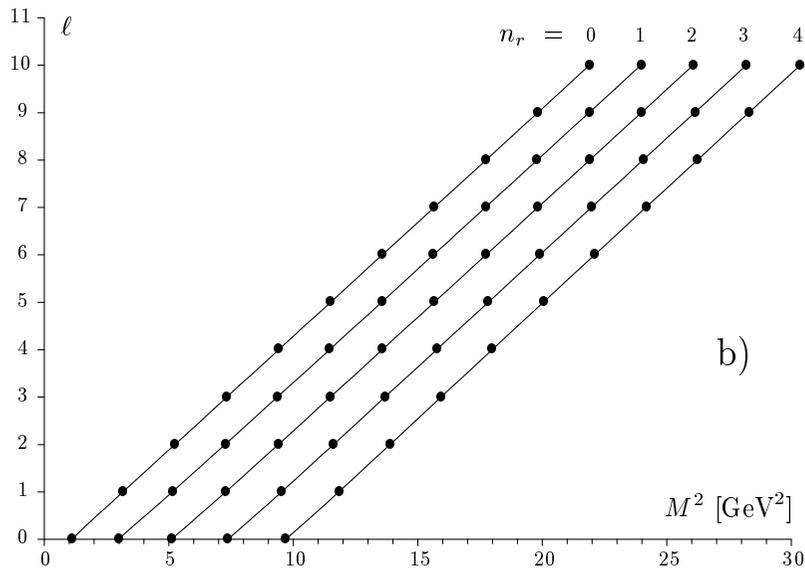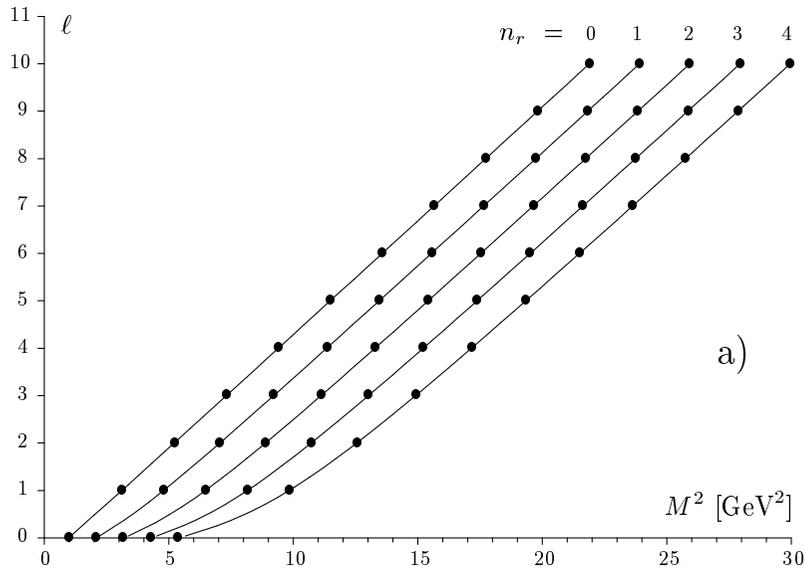
Figure 6. Semiclassical Regge trajectories; $m = 0.005$ GeV, $\alpha = 0.8$, $\beta = 0.2$ GeV^2; a) oscillator approximation; b) numerical solution.

not of $\ell + n_r$ type[2]. The latter cannot be reproduced by single-particle relativistic models with the vector and scalar potentials, as it is shown in [6].

Figures 5 and 6 present two examples of semiclassical Regge trajectories which are characteristic for heavy and light mesons respectively. Trajectories in figures 5a and 6a are calculated in the oscillator approximation which is good for $n_r \ll \ell$. Thus curved segments of daughters at $n_r \geq \ell$ are not sure. This is evident by comparison to trajectories of the same case which are shown in figures 5b and 6b. They are obtained by the numerical solving of the equation (84) where the integral in l.h.s. of (84) runs over phase trajectories of general model. We note that due to (83) phase trajectories in this case correspond to $S > 0$ (see figure 1), and thus they are free of critical points discussed in Subsection 4.3. It is remarkable that numerical solutions for mass spectrum is well described by the asymptotic formula (93) even at small ℓ. This is especially concerned with the case of light mesons (figure 6b).

6. Conclusion

In the present paper we have traced the relation between the nonstandard classical Yang-Mills field theory which arises from the consideration of QCD in the infrared region [30] and the classical relativistic two-particle models with confinement interaction formulated in the framework of Fokker formalism. It is notable that the use of Abelian potentials following from NT provides the confining interaction of particles. The time-symmetric (purely confinement) model turns out known in the literature [33,34] where, although, it was constructed as a priori action-at-a-distance model.

The present time-asymmetric confinement model could be regarded as a classical relativisation of the primitive quarkonium model. It has a number of features which are expected for models of this kind but which usually are not realized together.

1. The model is a self-consistent relativistic two-particle model. It allows the Lagrangian and Hamiltonian formulations. The quantities in terms of which the model is built have clear physical meaning. In the case $S > 0$ solutions of this model lead to smooth timelike particle world lines. If $S = 0$, the collision critical points occur in which particles reach the speed of light. In this case, although, world line can be smoothly continued as well. In both cases a particle motion is bound.

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[2]In the models presented in [7,8] the degeneracy appears if to neglect the spin of quarks.

2. Estimates of Regge trajectory from classical mechanics shows that it has a proper asymptote while the corresponding nonrelativistic potential is linear. This feature is not derivable from nonrelativistic models. The parameters of a linear rise following from the time-symmetric and time-asymmetric models differ from one another by near 5 %. One can hope that other long-range effects which should follow from the forthcoming study of purely retarded, time-symmetric and time-asymmetric confinement interactions differ slightly as well.

The semiclassical consideration leads to the interesting degenerate tower structure of meson spectrum which probably exists in nature.

3. The interpretation of an interaction in terms of some field theory is very important in RDIT. Hopefully, the knowledge of field equations and corresponding variational principle underlying the model allows to include properly into consideration spinning particles and then to construct the quantum-mechanical description.

Appendix

A. Green's functions for field equation of NT.

Let us consider the equation (20) (here we omit unessential Greece indices and the factor κ^2),

$$\Box^2 A(x) = 4\pi J(x). \tag{A.1}$$

It can be recast into the set of two d'Alembert equations,

$$\Box A(x) = B(x), \tag{A.2}$$

$$\Box B(x) = 4\pi J(x). \tag{A.3}$$

Solving them yields the following formal expression for $A(x)$:

$$A = \frac{1}{4\pi} D_\eta * B = \frac{1}{4\pi} D_\eta * D_{\eta'} * J, \tag{A.4}$$

where η and η' can independently take values $+1$, -1, or 0.

Since the convolution of distributions is not guaranteed to be a well defined operation [49], we have to examine all possible combinations of η and η'. Actually, it is sufficient to consider the cases $\eta = \pm 1$, $\eta' = \pm 1$; other cases where $\eta = 0$ or/and $\eta' = 0$ reduce to the previous ones due to the linearity of equations (A.2), (A.3) and the equality $D_0 = \frac{1}{2}(D_+ + D_-)$.

Let us write down the expression (A.4) in the explicit form,

$$A(x) = \frac{1}{4\pi} \int d^4 y \int d^4 z \, D_\eta(y) D_{\eta'}(z) J(x - y - z). \tag{A.5}$$

Representing D_η in the form

$$D_\eta(x) = 2\Theta(\eta x^0)\delta(x^2) = \delta(x^0 - \eta|\mathbf{x}|)/|\mathbf{x}|, \qquad \eta = \pm 1, \tag{A.6}$$

where $\mathbf{x} \equiv \{x^i, i = 1, 2, 3\}$, yields for (A.5) the expression

$$A(x) = \frac{1}{4\pi} \int d^3 y \int d^3 z \, \frac{J(x^0 - \eta|\mathbf{y}| - \eta'|\mathbf{z}|, \mathbf{x} - \mathbf{y} - \mathbf{z})}{|\mathbf{y}|\,|\mathbf{z}|}, \tag{A.7}$$

or, in terms of new variables $\mathbf{u} = \mathbf{y} + \mathbf{z}$, $\mathbf{v} = \mathbf{y} - \mathbf{z}$,

$$
\begin{aligned}
A(x) &= \frac{1}{8\pi} \int d^3 u \int d^3 v \times \\
&\times \frac{J(x^0 - \frac{1}{2}\eta|\mathbf{u} + \mathbf{v}| - \frac{1}{2}\eta'|\mathbf{u} - \mathbf{v}|, \mathbf{x} - \mathbf{u})}{|\mathbf{u} + \mathbf{v}|\,|\mathbf{u} - \mathbf{v}|}.
\end{aligned} \tag{A.8}
$$

Let us calculate the internal integral over $d^3 v$ in r.h.s. of (A.8). Expressing Cartesian coordinates v_1, v_2, v_3 of $\mathbf{v}$ in terms of ellipsoidal coordinates,

$$
\begin{aligned}
v_1 &= |\mathbf{u}|\sqrt{(\sigma^2 - 1)(1 - \tau^2)} \cos\varphi, \\
v_2 &= |\mathbf{u}|\sqrt{(\sigma^2 - 1)(1 - \tau^2)} \sin\varphi, \\
v_3 &= |\mathbf{u}|\sigma\tau, \\
&\quad \sigma \geq 1 \geq \tau \geq -1, \qquad 0 \leq \varphi < 2\pi,
\end{aligned} \tag{A.9}
$$

we obtain

$$
\begin{aligned}
A(x) &= \frac{1}{8\pi} \int d^3 u \, |\mathbf{u}| \int\limits_1^\infty d\sigma \int\limits_{-1}^1 d\tau \int\limits_0^{2\pi} d\varphi \times \\
&\times J\left(x^0 - \eta|\mathbf{u}| \times \left\{ \begin{array}{ll} \sigma, & \eta' = \eta \\ \tau, & \eta' = -\eta \end{array} \right\}, \mathbf{x} - \mathbf{u} \right).
\end{aligned} \tag{A.10}
$$

If $\eta' = -\eta$ this integral diverges due the factor $\int_1^\infty d\sigma$. In the case $\eta' = \eta$ it reads:

$$A(x) = \frac{1}{2} \int d^3 u \, |\mathbf{u}| \int\limits_1^\infty d\sigma J(x^0 - \eta|\mathbf{u}|\sigma, \mathbf{x} - \mathbf{u}). \tag{A.11}$$

Using the change of the variable $\sigma \to u^0 = \eta|\mathbf{u}|\sigma$ we obtain the final expression for $A(x)$:

$$A(x) = \tfrac{1}{2} \int d^4 u \, \Theta(\eta u^0) \Theta(u^2) J(x-u). \tag{A.12}$$

It follows from (A.12) that fundamental solutions to the equation (A.1) are:

$$E_\eta(x) = \tfrac{1}{2}\Theta(\eta x^0)\Theta(x^2), \qquad \eta = \pm 1. \tag{A.13}$$

Since their supports are the interiors of future– and past-oriented light cones, these distributions are the retarded and the advanced Green's functions of the equations (A.1). The time-symmetric Green's function is constructed by linearity,

$$E_0(x) = \tfrac{1}{2}(E_+(x) + E_-(x)) = \tfrac{1}{4}\Theta(x^2). \tag{A.14}$$

Eqs. (A.13), (A.14) are unified in eq. (21).

We note that some complex fundamental solution to (A.1) is obtained by means of another technique in [49],

$$E_c(x) = \frac{\pm i}{4\pi} \ln(x^2 \pm i0). \tag{A.15}$$

Its real part,

$$\Re E_c(x) = \tfrac{1}{4}(\Theta(x^2) - 1), \tag{A.16}$$

coincides with the symmetric Green's function (A.15) up to a constant (which is the solution of homogeneous equation). This solution can be considered as the analogue of the Feynman propagator in QED,

$$D_c(x) = \frac{\pm i}{\pi(x^2 \pm i0)}, \tag{A.17}$$

real part of which is the symmetric Green's function of d'Alembert equation.

B. Analysis of DPM for general model.

Let us introduce the dimensionless positive quantities:

$$\xi = \frac{2\alpha}{Mr}, \quad \mu = \frac{M}{2m}, \quad \sigma = \frac{S}{\alpha}, \quad \nu = \frac{\alpha\beta}{m^2}, \tag{B.1}$$

and functions:

$$\bar{f}_\pm(\xi,\mu,\sigma) \;=\; 1 + 2\xi - \frac{\xi}{\xi - \nu/\mu^2}\bar{h}_\pm(\xi,\mu,\sigma), \tag{B.2}$$

$$\bar{h}_\pm(\xi,\mu,\sigma) \;=\; \bar{g}_-(\xi,\mu,\sigma) \mp \sqrt{\bar{d}(\xi,\mu,\sigma)}, \tag{B.3}$$

$$\bar{d}(\xi,\mu,\sigma) \;=\; \bar{g}_-^2(\xi,\mu,\sigma) + \sigma^2\left(\xi - \frac{\nu}{\mu^2}\right)\xi^3, \tag{B.4}$$

$$\bar{g}_\pm(\xi,\mu,\sigma) \;=\; \frac{1}{2}\left[\frac{1}{\mu^2}(1+\xi) \pm \left(\xi - \frac{\nu}{\mu^2}\right)\xi + \sigma^2\xi^2\right], \tag{B.5}$$

which are related to (70)–(73) as follows:

$$\bar{f}_\pm(\xi,\mu,\sigma) \;=\; \frac{4}{M^2} f_\pm(r,M,S) \tag{B.6}$$

$$\bar{h}_\pm(\xi,\mu,\sigma) \;=\; \frac{4}{M^2} h_\pm(r,M,S) \tag{B.7}$$

$$\bar{d}(\xi,\mu,\sigma) \;=\; \frac{16}{M^4} d(r,M,S) \tag{B.8}$$

$$\bar{g}_-(\xi,\mu,\sigma) \;=\; \frac{4}{M^2} g(r,M,S) \tag{B.9}$$

Then DPM is determined by conditions:

$$\bar{f}_-(\xi,\mu,\sigma) \geq 0, \tag{B.10}$$

$$\bar{\Delta} \equiv (1+\xi)^2 - \bar{f}_-(\xi,\mu,\sigma) > 0. \tag{B.11}$$

Although ξ is positive by definition, it is useful to consider the functions (B.2)–(B.5) of ξ for $\xi \in \mathbb{R}$.

First of all we consider the condition (B.10). It follows from (B.5) and (B.4) that

$$\bar{g}_-(\xi) > 0, \qquad \xi \in [0, \nu/\mu^2], \tag{B.12}$$

$$\bar{d}(\xi) > \bar{g}_-^2(\xi), \qquad \xi \in (-\infty, 0) \cup (\nu/\mu^2, \infty), \tag{B.13}$$

$$\bar{d}(\xi) = \bar{g}_-^2(\xi), \qquad \xi = 0, \nu/\mu^2, \tag{B.14}$$

$$\bar{d}(\xi) < \bar{g}_-^2(\xi), \qquad \xi \in (0, \nu/\mu^2). \tag{B.15}$$

Thus from (B.12)–(B.14) we obtain

$$\bar{d}(\xi) > 0, \qquad \xi \in (-\infty, 0] \cup [\nu/\mu^2, \infty). \tag{B.16}$$

Using the equivalent form of $\bar{d}$,

$$\bar{d}(\xi,\mu,\sigma) = \bar{g}_+^2(\xi,\mu,\sigma) - \frac{1}{\mu^2}(1+\xi)\left(\xi - \frac{\nu}{\mu^2}\right)\xi, \tag{B.17}$$

we obviously have

$$\bar{d}(\xi) > \bar{g}_+^2(\xi), \qquad \xi \in (-\infty, -1) \cup (0, \nu/\mu^2), \qquad (B.18)$$
$$\bar{d}(\xi) = \bar{g}_+^2(\xi), \qquad \xi = -1, 0, \nu/\mu^2, \qquad (B.19)$$
$$\bar{d}(\xi) < \bar{g}_+^2(\xi), \qquad \xi \in (-1, 0) \cup (\nu/\mu^2, \infty), \qquad (B.20)$$

and then, from (B.16) and (B.18),

$$\bar{d}(\xi) > 0, \qquad \xi \in \mathbb{R}. \qquad (B.21)$$

Hence the functions $\bar{h}_\pm(\xi)$, $\bar{f}_\pm(\xi)$ (as well as $h_\pm(r)$, $f_\pm(r)$) are real.

Taking into account (B.3), (B.12)–(B.15), and (B.21) one obtains

$$\bar{h}_+(\xi) > 0, \qquad \xi \in (0, \nu/\mu^2), \qquad (B.22)$$
$$\bar{h}_+(\xi) = 0, \qquad \xi = 0, \nu/\mu^2, \qquad (B.23)$$
$$\bar{h}_+(\xi) < 0, \qquad \xi \in (-\infty, 0) \cup (\nu/\mu^2, \infty) \qquad (B.24)$$

Thus, using (B.2) and (B.22)–(B.24) we conclude that

$$\bar{f}_+(\xi) > 0, \qquad \xi \in [-1/2, \infty). \qquad (B.25)$$

We note that the function $\bar{f}_+(\xi)$ is smooth at $\xi = \nu/\mu^2$.

In order to clarify the behaviour of the function $\bar{f}_-(\xi)$ for $\xi > 0$ let us consider the function $\bar{f}_+\bar{f}_-$. It can be presented in the form:

$$\bar{f}_+(\xi)\bar{f}_-(\xi) = \frac{1+\xi}{\xi - \nu/\mu^2}\Pi(\xi), \qquad (B.26)$$

where

$$\Pi(\xi) \equiv (1 + 2\xi)\left[(1+\xi)\left(\xi - \frac{\nu}{\mu^2}\right) - \frac{1}{\mu^2}\xi\right] - \sigma^2(1+\xi)\xi^3 \qquad (B.27)$$

is 4th-order polynomial (in terms of original quantities it is written down in l.h.s. of (74)). It is evident that

$$\Pi(\xi) < 0 \qquad \text{for } |\xi| \text{ large.} \qquad (B.28)$$

Moreover, it is easy to examine that

$$\Pi(\xi) < 0, \qquad \xi \in (-\infty, -1] \cup [0, \nu/\mu^2], \qquad (B.29)$$
$$\Pi(-\tfrac{1}{2}) > 0. \qquad (B.30)$$

Thus $\Pi(\xi)$ has two negative roots, ξ_1 and ξ_2, $-1 < \xi_1 < -1/2 < \xi_2 < 0$, which exist at arbitrary (positive) values of μ, σ, and ν. The number of

positive roots, should they exist, is not more than two. Let us note that one can choose the sufficiently large value of σ such that $\Pi(\xi) > 0$, $\xi > 0$. On the other hand, at $S = 0$ there exists $\xi_+ > 0$ such that $\Pi(\xi) > 0$, $\xi > \xi_+$. Thus, given μ and ν, two other roots, ξ_3 and ξ_4, are positive for sufficiently small values of σ, and $\nu/\mu^2 < \xi_3 < \xi_4$. In this case we have:

$$\Pi(\xi) > 0, \qquad \xi \in (\xi_1, \xi_2) \cup (\xi_3, \xi_4), \qquad (B.31)$$
$$\Pi(\xi) < 0, \qquad \xi \in (-\infty, \xi_1) \cup (\xi_2, \xi_3) \cup (\xi_4, \infty). \qquad (B.32)$$

Hereafter we restrict all functions on $\xi > 0$. Using (B.26), (B.31), (B.32), and (B.25) one concludes that

$$\bar{f}_-(\xi) \geq 0, \qquad \xi \in (0, \nu/\mu^2) \cup [\xi_3, \xi_4], \qquad (B.33)$$
$$\bar{f}_-(\xi) < 0, \qquad \xi \in (\nu/\mu^2, \xi_3) \cup (\xi_4, \infty), \qquad (B.34)$$

and this function has a pole at $\xi = \nu/\mu^2$.

Now let us consider the condition (B.11) which can be presented in the following form:

$$\bar{\Delta} = \frac{\xi}{\xi - \nu/\mu^2}\left[\bar{g}_+(\xi, \mu, \sigma) + \sqrt{\bar{d}(\xi, \mu, \sigma)}\right] > 0. \qquad (B.35)$$

Using (B.18), (B.20) and the evident inequality

$$\bar{g}_+(\xi) > 0, \qquad \xi \in (\nu/\mu^2, \infty), \qquad (B.36)$$

one concludes that

$$\bar{\Delta} > 0, \qquad \xi \in (\nu/\mu^2, \infty), \qquad (B.37)$$
$$\bar{\Delta} < 0, \qquad \xi \in (0, \nu/\mu^2). \qquad (B.38)$$

Finally, taking into account (B.33), (B.34), (B.37), and (B.38), we find DPM: $\xi_3 \leq \xi \leq \xi_4$, i.e., $r_1 \leq r \leq r_2$, where $r_1 = 2\alpha/(M\xi_4)$, $r_2 = 2\alpha/(M\xi_3)$. It disappear if $\xi_3 \to \xi_4$. The degenerated case $\xi_3 = \xi_4 \equiv \xi_c$, where ξ_c satisfies the set of equations:

$$\Pi(\xi_c) = 0, \qquad \Pi'(\xi_c) = 0, \qquad (B.39)$$

corresponds to circular orbit motion. The set (B.39) in this case can be considered as a relation between M and S which is presented by (75), where $\lambda = 1/\xi_c$.

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