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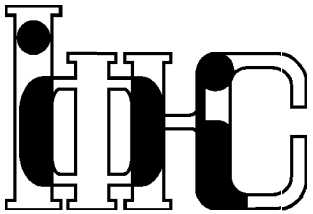

ІНСТИТУТ ФІЗИКИ КОНДЕНСОВАНИХ СИСТЕМ

ICMP–98–14E

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Model of the fermion liquid with a short-range interaction.
The local-field correction function, thermodynamic and dynamic characteristics

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УДК: 537.31; 538.945

PACS: 05.30.Fk

Поправка на локальне поле фермі-системи з короткосяжною взаємодією

Ваврух М., Паславський В.

Анотація. Запропоновано узагальнену модель фермі-системи з короткосяжною взаємодією між частинками, яка описується потенціалом відштовхування Юкави. Досліджено залежність динамічної поправки на локальне поле, енергетичних та динамічних характеристик моделі від параметрів потенціалу. Розраховано область термодинамічної стабільності моделі як функцію радіуса потенціалу взаємодії та густини частинок

Model of the fermion liquid with a short-range interaction. The local-field correction function, thermodynamic and dynamic characteristics

Vavrukh M., Paslavs'kii V.

Abstract. A generalized Fermi liquid model with a short-range interaction potential, which is simulated by the Yukawa repulsion potential is proposed. It is investigated how the dynamic local-field correction and other characteristics of the model depend on the parameters of the interaction potential. The thermodynamical stability region as a function of the interaction range and the particle density are calculated.

Подається в phys. stat. sol. (b)
Submitted to phys. stat. sol. (b)

1. Introduction

The local-field conception holds a prior place in the modern electron liquid theory and the local-field correction function is believed to be the fundamental characteristic of any many-particle system with a local interaction potential. The local-field correction function was studied in detail for the electron liquid case [1, 2]. Up to the present, the investigation of this function for the case of Coulomb repulsion cannot be considered to be completed. We do not know an investigation of the local-field correction function of many-particle systems with a non-Coulomb interaction potential.

Being a functional of the interaction potential, the local-field correction function must possess features, which are inherent to a particular physical system. But the local-field correction function of a degenerate Fermi system also must possess common features caused by a Fermi surface. In this paper we want to investigate the influence of the interaction potential on the local-field correction function and the ground state energy. We shall use a reference system approach [3, 4, 5] for investigating of the model with a non-Coulomb interaction potential between particles, and as a reference system the model of the ideal fermion gas will be used.

We consider the degenerate Fermi system model of N fermions with charge Ae, which are located in a volume V with compensated background charge $-\frac{N}{V}Ae$. We describe the interaction between particles and background charge by the Yukawa repulsion potential

$$V(r_{ij}) = A^2e^2(r_{ij})^{-1}\exp[-\xi r_{ij}/a_0]. \tag{1}$$

Here e is the charge of an electron, a_0 is the Bohr radius. Let us consider the thermodynamical limit, $N, V \to \infty$, $N/V =$ const. Here suggested potential has two parameters, which control the coupling strength and the interaction range. A^2 is an interaction constant and ξ^{-1} defines the effective range of the potential. Such a choice of the potential was caused by our wish to have a look on the features of the electron liquid model with a general position, and we mention a possible its using for the description of nuclear matter. A third parameter of the model is particle mass m.

A dimensionless coupling parameter of this model, defined as the ratio of an average potential energy of the particle to its kinetic one, has the form

$$a = \left(\frac{3}{5}\varepsilon_F\right)^{-1}\frac{N}{V}\int d\mathbf{r}V(\mathbf{r})|g_2^{id}(\mathbf{r}) - 1| \simeq \tag{2}$$

$$\simeq \frac{20\pi A^2e^2}{3\varepsilon_F}\frac{N}{V}\int_0^{R_F} dr\ r\ \exp[-\xi r/a_0] = \frac{10A^2r_s}{\eta^4}m^*f(\xi^*),$$

$$f(\xi^*) = (\xi^*)^{-2}\{1 - [1+\xi^*]\exp(-\xi^*)\}.$$

Here $\varepsilon_F = \frac{\hbar^2k_F^2}{2m}$ is the energy of a free particle on the Fermi surface, $m^* = m/m_0$ (m_0 is the mass of an electron, $g_2^{id}(r)$ is a pair distribution function of the ideal fermion system (without interaction) [6], $R_F = k_F^{-1}$ is a correlation length of the degenerate ideal fermion system, $r_s = (3V/4\pi N)^{1/3}a_0^{-1}$ is the Wigner parameter, $\xi^* = \xi r_s/\eta$, where $\eta = (9\pi/4)^{1/3}$. Taking into account the asymptotic behavior of $f(\xi^*)$, one can see that the coupling parameter α changes from $10A^2r_s\eta^{-4}$(at $\xi^* \to 0$) to $10A^2r_s\eta^{-4}(\xi^*)^{-2}$ (at $\xi^* \gg 1$). The electron liquid model is a limiting case of the model under consideration in the limit $A \to 1, \xi \to 0$. Its coupling parameter r_s is the Wigner parameter. Varying the parameters A, ξ, m^*, we can change the coupling parameter α in a wide region. For example, the weakly non-ideal limit can be obtained in different pathways

$$\begin{aligned} &1) r_s \to 0;\ A, \xi, m^* = \text{const};\\ &2) A \to 0;\ r_s, \xi, m^* = \text{const};\\ &3) \xi \to \infty;\ A, r_s, m^* = \text{const}. \end{aligned} \tag{3}$$

The principle aim of our paper is to investigate the influence of the interaction potential on the local-field correction function and other model characteristics.

The local-field approximation in the electron liquid theory generalizes expressions, which occur in the random phase approximation, to systems with any value of the coupling parameter r_s. In general, the dynamic local-field correction function $G(x)$ can be defined by the generalization of the two-particle correlation function $\mu_2^{RPA}(x, -x)$

$$\mu_2^{RPA}(x,-x) = \tilde{\mu}_2^0(x,-x)\left\{1 + \frac{V_q}{V}\tilde{\mu}_2^0(x,-x)\right\}^{-1}. \tag{4}$$

Here $\tilde{\mu}_2^0(x,-x)$ is the ideal system pair correlation function [3]. Substituting V_q the Fourier-transform of the interaction potential (1.1) by the effective one $V_q[1-G(x)]$, we obtain the pair correlation function in the form

$$\mu_2(x,-x) = \tilde{\mu}_2^0(x,-x)\left\{1 + \frac{V_q}{V}\tilde{\mu}_2^0(x,-x)[1-G(x)]\right\}^{-1}. \tag{5}$$

which defines the pair distribution function

$$g_2(\mathbf{r}) = 1 + [\beta N(N-1)]^{-1} \sum_{\nu} \sum_{q \neq 0} \mu_2(x,-x) \exp(i\mathbf{qr}), \qquad (6)$$

where β is the reciprocal temperature, $x = (\mathbf{q}, \nu), \mathbf{q}$ is the wave vector, ν is the Bose-Matsubara frequency [7], r is the distance between two particles.

2. Local-Field Correction Function of the Model with a Short-Range Interaction Potential

An integral equation set for the local-field correction function $G(x)$ of the electron liquid model has been obtained in [4]. The positive value of the Fourier-transform of the interaction potential has been applied in this procedure. Therefore the equation set is valid for any potential, which satisfies the condition $V_q > 0$,

$$\begin{aligned} G(x) &= G_1(x) + G_2(x), \\ G_1(x) &= -(2\beta V_q)^{-1} [\tilde{\mu}_2^0(x,-x)]^{-2} \sum_{x_1} [V_0(x_1) - V_{q_1} G_2(x_1)] \\ &\quad \times \tilde{\mu}_{4,1}^0(x,-x,x_1,-x_1), \qquad (7) \\ G_2(x) &= (2\beta V V_q)^{-1} [\tilde{\mu}_2^0(x,-x)]^{-2} \\ &\quad \times \sum_{x_1} V_0(x_1+x)[V_0(x_1) - V_{q_1} G_2(x_1)] \\ &\quad \times \tilde{\mu}_3^0(x,x_1,-x_1-x)\ \tilde{\mu}_3^0(-x,-x_1,x_1+x). \end{aligned}$$

Here $\tilde{\mu}_3^0(...)$ is the three-particle correlation function, $\tilde{\mu}_{4,1}(...)$ is the normal part of the four-particle one for the ideal Fermi system [3], $x_1 = (\mathbf{q}_1, \nu_1)$. They can be represented in terms of elementary functions in the case $T = 0K$ (see [3, 8]). In Eq. (2.1) $V_0(x) = V_q \left[1 + \frac{V_q}{V}\ \tilde{\mu}_2^0(x,-x)\right]^{-1}$ is the screened interaction potential in the random phase approximation. As one can see from (2.1) the ratio $G_2(x) G_1^{-1}(x)$ is proportional to α in a weakly nonideal region (see (1.3)). Therefore we can neglect the component $G_2(x)$ in this region. Representing $G_1(x)$ in integral form, we obtain the following expression:

$$\begin{aligned} G_1^{RPA}(x) &= -(4\pi)^{-1} [q^2 + (\xi^*)^2] I_{2,0}^{-2}(q, \bar{\nu}/q) \\ &\quad \times \int_0^{\infty} dq_1\ q_1^2 \int_0^{\infty} d\bar{\nu}_1 \int_{-1}^{+1} dt\ I_{4,1}(q, q_1; \bar{\nu}/q, \bar{\nu}_1/q_1; t) \qquad (8) \end{aligned}$$

$$\times [q_1^2 + (\xi^*)^2 + 4A^2 m^* r_s (\pi\eta)^{-1}\ I_{2,0}(q_1, \bar{\nu}_1/q_1)]^{-1}.$$

The dimensionless variables $q \equiv |\mathbf{q}|/k_F$, $\bar{\nu} \equiv \nu(2\varepsilon_F)^{-1} m^*$ and functions $I_{2,0}(q, \bar{\nu}/q) = 2\varepsilon_F (3Nm^*)^{-1}\ \tilde{\mu}_2^0(x,-x)$, $I_{4,1}(q, ...; t) = (3N)^{-1}(m^*)^{-3}(2\varepsilon_F)^3\ \tilde{\mu}_{4,1}^0(x,-x,x_1,-x_1)$ have been used (t is the cosine of the angle between the vectors $\mathbf{q}$ and $\mathbf{q}_1$). As one can see $G_1^{RPA}(x)$ becomes the local-field correction function of the Fermi system in the limit $r_s \to 0$ and $A, \xi = \text{const}$, which coincides with the local-field correction function of the ideal degenerate electron gas $G_{id}^{EL}(x)$ in the variables $q, \bar{\nu}$ (Fig. 1). This is a universal function, which does not depend on any parameter. It has the following asymptotic behavior

$$G_{id}^{EL}(x) = \begin{cases} \gamma(\bar{\nu}) q^2 + ... & \text{for } q \ll 1, \\ 1/3 + ... & \text{for } q \gg 1, \end{cases} \qquad (9)$$

where $\gamma(\bar{\nu})$ is a monotonous function of the dimensionless frequency $\bar{\nu}$ [6, 8]. It decreases from $1/4 (at \bar{\nu} = \infty)$.

The function $G_1(x)$ depends only on the parameter ξ^* in the limit $A \to 0$, at $r_s, \xi = \text{const}$. Let us use the representation (2.7) from [5] for $\tilde{\mu}_{4,1}^0(x,-x,x_1,-x_1)$ to investigate the asymptotic behavior of $G_1(x)$. We can obtain the following asymptotic $G_{id}(x)$ in the short-wavelength region

$$\begin{aligned} G_{id}(q, \bar{\nu}/q) &= \frac{1}{2}[q^2 + (\xi^*)^2] q^{-2} \{1 - D(\xi^*) + 2D(\xi^*) \qquad (10) \\ &\quad \times \left[\frac{\bar{\nu}}{(\bar{\nu}/q)^2 + q^2/4}\right]^2 \Big\} \end{aligned}$$

at $q \geq 4,\ \bar{\nu} \geq 2q$, where

$$\begin{aligned} D(\xi^*) &= \frac{1}{3} - \frac{3}{4}(\xi^*)^2 + \frac{1}{8}(\xi^*)^4 + (\xi^*)^3\ \arctan\frac{2}{\xi^*} \qquad (11) \\ &\quad - \frac{3}{8}(\xi^*)^4 \left[1 + \frac{1}{12}(\xi^*)^2\right] \ln\left[1 + \frac{4}{(\xi^*)^2}\right], \end{aligned}$$

and

$$\begin{aligned} G_{id}(q;0) &= C_0(\xi^*) + C_2(\xi^*) q^2\ +\ ..., \qquad (12) \\ G_{id}(q; \bar{\nu}/q) &= B_0(\xi^*) + B_2(\xi^*) q^2\ +\ ...,\ \bar{\nu} \gg q, \\ C_0(\xi^*) &= \frac{1}{4}(\xi^*)^2 \left\{1 - \frac{(\xi^*)^2}{4}\ \ln\left[1 + \frac{4}{(\xi^*)^2}\right]\right\}, \qquad (13) \\ B_0 &= \frac{3}{20}(\xi^*)^2 \left\{1 - (\xi^*)^2 + \frac{(\xi^*)^2}{4}[1 + (\xi^*)^2] \ln\left[1 + \frac{4}{(\xi^*)^2}\right]\right\} \end{aligned}$$

in the long-wavelength region. As one can see from (2.4), (2.7) the long-wavelength region asymptote changes from 0 to 1/2, when ξ^* changes from zero to infinity and the short-wave one from 1/3 to 1/2. The function $G_{id}(x)$ changes very essentially near the point $|\mathbf{q}| = 2k_F$. All features of $G_{id}(q,0)$ are depicted in Fig. 2.

As one can see from the asymptotes (2.3), (2.7) the behavior of the local-field correction function is defined by the effective range of the interaction between particles $R_0 = \xi^{-1}a_0$ in the long-wavelength region. The local-field correction function of any short-range potential has a finite value at $q = 0$, which rises, when the parameter ξ^* rise. The asymptote $\gamma(\bar{\nu})q^2 + ...$ occurs only in the Coulomb potential limit ($R_0 = \infty,\ C_0(\xi^*) = 0$). In this way, the local-field correction function of the system with short-range interaction potential is more important than the one of the Coulomb system.

Approximations $G_{id}(x)$ and $G_{id}^{EL}(x)$ correspond to the result of [9], in which, for the first time, the dynamic local-field correction function of the electron-liquid model $G_{id}^{EL}(q,\omega)$ in terms (q,ω), where ω is the Heisenberg frequency, has been investigated. As a function of ω the expression $G_{id}^{EL}(q,\omega)$ has strong singularities. In the terms of (q,ν) the function $G_{id}(x)$ has no singularities. This is convenient for its further use.

Taking into account the existence of extremum of the function $\tilde{\mu}_3^0(x,x_1,-x-x_1)$ and $\tilde{\mu}_{4,1}^0(x,-x,x_1,-x_1)$ near the surface $x = x_1$, we can calculate the local-field correction function in the region of middle and strong nonideality [5]. Using the mean-value theorem of the integral, we obtain the approximate solution of the set (2.1),

$$G_i(x) \approx G_i^{RPA}(x)[1 + G_2^{RPA}(x)]^{-1}, \quad i = 1,2, \tag{14}$$

$G_1^{RPA}(x)$ is computed by (2.2),

$$G_2^{RPA}(x) = [q^2 + (\xi^*)^2] r_s m^* (\pi^2\eta)^{-1} [2I_{2,0}(q,u)]^{-2} A^2 \tag{15}$$
$$\times \int_{-\infty}^{+\infty} du_1 \int_0^{\infty} dq_1 q_1^3 \int_{-1}^{+1} dt P(q_1,u_1) P(q_2,u_2) I_{3,0}^2(q,q_1;u,u_1;t),$$

where $u_1 \equiv \bar{\nu}_1/q_1$, $u \equiv \bar{\nu}/q$,$q_2 = [q^2 + q_1^2 + 2q_1qt]^{1/2}$, $u_2 = q_2^{-1}[\bar{\nu} + \bar{\nu}_1]$, $P(q,u) = [q^2 + (\xi^*)^2 + 4A^2 r_s m^* (\pi\eta)^{-1} I_{2,0}(q,u)]^{-1}$, $I_{3,0}(q,q_1;u,u_1;t) = (2\varepsilon_F)^2 (3N)^{-1} (m^*)^{-2} \tilde{\mu}_3^0(x,x_1,-x-x_1)$. The functions $\tilde{\mu}_3^0(x,x_1,-x-x_1)$ and $\tilde{\mu}_{4,1}^0(x,-x,x_1,-x_1)$ in terms of (q,ν) have no singularities, therefore for the calculation of (2.2) and (2.9) standard numerical methods of integral calculations can be used.

The results of the local-field correction function calculation in the approximation (2.2), (2.8), (2.9) are depicted in Fig. 3, 4. When the parameter A changes,$G(x)$ changes weakly in the region $o \le |\mathbf{q}| \le 2k_F$, but a strong dependence is noticed in the region $|\mathbf{q}| > 2k_F$. The increase of the local-field correction function with increasing A is similar to the dependence of the electron liquid local-field correction function on the parameter r_s [7]. The dependence of $G(x)$ on the parameter ξ at $A, r_s, \bar{\nu} = \text{const}$ is presented in Fig. 3. As one can see $G(x)$ is weakly dependent on the parameter ξ in the short-wavelength region. The range R_0 has a strong influence on the local-field correction function in the region of small and medium wave vectors. The dependence of $G(x)$ on the parameter r_s at $\xi = \text{const}$ is depicted in Fig. 4. The behavior of $G(x)$ is very similar to the electron liquid local-field correction function in the region $q > 2k_F$. Comparing $G^{EL}(x)$ with $G(x)$ of our model, we notice that in the region $q \le 2k_F$, $G^{EL}(x)$ and $G(x)$ are very different. On the whole, $G(x)$ has the following asymptotic behavior

$$G(x) \to \begin{cases} a(\xi^*, r_s, \bar{\nu})[q^2 + (\xi^*)^2] + \ ... & \text{at } q \ll 1, \\ G_\infty(A, r_s, \xi^*) + \ ... & \text{at } q \gg 1. \end{cases} \tag{16}$$

3. Ground State Energy of the Model

As is known, the local-field correction function defines the integral and local characteristics of a system with a local two-particle interaction. In accordance with (1.5), the ground state energy expression has the form

$$E = E_0 + \lim_{\beta\to\infty} \sum_{q\neq 0} \sum_{\nu} \int_0^1 \frac{d\lambda}{\lambda}\ V_q^\lambda\ \tilde{\mu}_2^0(x,-x) \tag{17}$$
$$\times \left\{ 1 + \frac{V_q^\lambda}{V}\ \tilde{\mu}_2^0(x,-x)\ [1 - G_\lambda(x)] \right\}^{-1},$$

where V_q^λ and $G_\lambda(x)$ depend on the coupling parameter λA^2, E_0 is the ground state energy of the ideal system. Extracting the ideal correlation contribution, we can represent the total energy in the usual dimensionless form : $E = N \cdot \text{Ry}\ \varepsilon(r_s, A, \xi)$,

$$\varepsilon(r_s, A, \xi) = \varepsilon_0(r_s) + \varepsilon_{HF}(r_s, A, \xi) + \varepsilon_c(r_s, A, \xi). \tag{18}$$

Here $\varepsilon_0(r_s) = 3/5\eta^2 r_s^{-2} (m^*)^{-1}$ is the ideal system energy at $T = 0K$ in Ry per particle,

$$\varepsilon_{HF}(r_s, A, \xi) = -2A\eta(\pi r_s)^{-1} \Big[\frac{3}{4} - \xi^* \arctan \frac{2}{\xi^*} \tag{19}$$

$$+\frac{(\xi^*)^2}{8}\left\{\left(3+\frac{(\xi^*)^2}{4}\right)\ln\left(1+\frac{4}{(\xi^*)^2}\right)-1\right\}\Big]$$

is the Hartree-Fock energy contribution,

$$\varepsilon_c(r_s,A,\xi)=-\frac{24}{\pi^3}A^4\frac{(m^*)^2}{r_s^2}\int\limits_0^\infty dq\ q^3\int\limits_0^\infty du\int\limits_0^{r_s}dr_s^{'}\ r_s^{'}\ I^2_{2,0}(q,u)$$
$$\times\ [1-G(q,u)][q^2+(\xi^*)^2]^{-1} \qquad (20)$$
$$\times\ \left\{q^2+(\xi^*)^2+\frac{4A^2r_s^{'}m^*}{\pi\eta}\ I_{2,0}(q,u)[1-G(q,u)]\right\}^{-1}$$

is the correlation energy. The function $G(q,u)$ depends on the parameter $r_s^{'}\equiv\lambda r_s$ in (3.4) (but $\xi^*\equiv\xi r_s/\eta$). The Hartree-Fock energy contribution has the following asymptote

$$\varepsilon_{HF}(r_s,A,\xi)\to-\frac{2A^2\eta}{\pi r_s}\left\{\begin{array}{ll}\frac{3}{4} & \text{at } \xi\to 0,\\ \frac{1}{3(\xi^*)^2} & \text{at } \xi\to\infty.\end{array}\right. \qquad (21)$$

The correlation energy ε_c has been computed by a numerical method in a wide region of its parameters. The dependence of the total energy on the parameter r_s and ξ is depicted in Fig. 5. The total energy increases with the rise of the parameter ξ at fixed value of r_s. The stability region also decreases, when the total energy is negative. The curve

$$\varepsilon(r_s,1,\xi)\ =\ 0 \qquad (22)$$

is depicted in Fig. 6 as a solid line (curve 1). The shaded part of the picture corresponds to $\varepsilon(r_s,l,\xi)>0$ and the unshaded part (below curve 1) corresponds to $\varepsilon(r_s,l,\xi)<0$. The solid line defines the particle equilibrium density in the system with minimum total energy (at a given magnitude of the parameter ξ) (curve 2)

$$\frac{d}{dr_s}\ \varepsilon(r_s,1,\xi)\ =\ 0. \qquad (23)$$

As one can see a critical value ξ_c exists. The total energy of the system can not become negative at any density, when $\xi>\xi_c$. Therefore the correlation effects can not provide the equilibrium of a quantum system at small value of the interaction range, contrary to the case of the electron liquid.

In the limit $\xi\to 0$ formula (3.2) defines ground state energy of the electron liquid model.

At the case of the model which is consist of neutral fermions with interaction potential (1.1) expression (3.2) must be supplemented by the term

$$\delta\ \varepsilon(r_s,A,\xi)=\frac{N}{2V}\ V_{q=0}\ \left(\frac{e^2}{2a_0}\right)^{-1}=\frac{4}{3\pi}\ A^2\eta^3\xi^{-2}r_s^{-3}. \qquad (24)$$

This term is significant at the region of small values of the parameters ξ and r_s. It influence on the ground state energy vanishes in the region $r_s,\ \xi\to 0$.

4. Plasmon Excitation Spectrum

As is known, the local-field correction function makes it possible for calculation of the dynamic characteristics. One of them is a spectrum of the plasmon excitations $\omega\equiv\omega(\mathbf{q})$. It can be found as a solution of the equation

$$\varepsilon(\mathbf{q},\hbar\omega)\ =\ 0, \qquad (25)$$

where $\varepsilon(\mathbf{q},\hbar\omega)$ is a real part of the dielectric response function, ω is a Heisenberg frequency (see [9]). Transition for the thermodynamic perturbation theory to dynamic one be done when one replaces Matsubara frequencies ν on $i\hbar\omega-\delta(\delta\to+0)$ in all characteristics. As a result $\tilde\mu_2^0(x,-x)$ and $G(x)$ transform to $\tilde\mu_2^0(\mathbf{q},\hbar\omega)$ and $G(\mathbf{q},\hbar\omega)$ correspondently, and (4.1) can be written in the form

$$\varepsilon(\mathbf{q},\hbar\omega)=1+\frac{V_q}{V}\ \tilde\mu_2^0(\mathbf{q},\hbar\omega) \qquad (26)$$
$$\times\ \left[1-\frac{V_q}{V}\ \tilde\mu_0^2(\mathbf{q},\hbar\omega)\ G(\mathbf{q},\hbar\omega)\right]^{-1}.$$

On the basis of (4.2) one can obtain equation for plasmon excitations

$$1+\frac{V_q}{V}\ \tilde\mu_2^0(\mathbf{q},\hbar\omega)[1-G(\mathbf{q},\hbar\omega)]\ =\ 0. \qquad (27)$$

$\tilde\mu_2^0(\mathbf{q},\hbar\omega)$ was calculated in [7, 10]

$$\tilde\mu_2^0(\mathbf{q},\hbar\omega)=\frac{3N}{2\varepsilon_F}\ f(q,\tilde\omega), \qquad (28)$$
$$f(q,\tilde\omega)=\frac{1}{2}\left\{1+\frac{1}{2q}\sum_{\sigma=\pm1}[1-t_\sigma^2,q,\tilde\omega)]\ln\frac{1+t_\sigma(q,\tilde\omega)}{1-t_\sigma(q,\tilde\omega)}\right\},$$

where $t_\sigma(q,\tilde{\omega}) = \frac{1}{2}\Big(q + \frac{\sigma}{q}\tilde{\omega}\Big)$, $\tilde{\omega} = \hbar\omega\varepsilon_F^{-1}$, $q = |\mathbf{q}|k_F^{-1}$. One may obtain asymptotic behavior of the $f(q,\tilde{\omega})$ in the region $q \to 0$:

$$f_0(c) = \lim_{q\to 0}\ f(q, \varepsilon_F c q) = 1 + \frac{c}{4}\ \ln\left|\frac{c-2}{c+2}\right|. \tag{29}$$

The constant c can be found as a solution of the equation

$$f_0(c) = -\frac{\pi}{4}\ \frac{r_s}{\eta}\Big(\frac{\xi}{A}\Big)^2\ (1 - G_0(c)), \tag{30}$$

where

$$G_0(c) = \lim_{q\to 0}\ G(q, c\varepsilon_F q). \tag{31}$$

As you can see from (4.6), the constant c in the random phase approximation can change in the region $c_0 \le c < \infty$, where $c_0 \simeq 1,67...$ is a solution of the equation $f_0(c) = 0$. Branch of the solution $c > 2$ corresponds to "deformed" plasmons. At the $c \gg 2$ in the limit of the small wave vector $(0 \le q \le \xi r_s/\eta)$ one may obtain:

$$\hbar\omega_q \simeq \hbar\omega_0 \cdot Aq \cdot \eta(\xi r_s)^{-1}, \tag{32}$$

where $\omega_0 = (4\pi e^2 N/mV)^{1/2}$ is a plasmon frequency for the model of the electron liquid model.

Using asymptotic behavior of the $f(q,\tilde{\omega})$ in the region of the large value of the ω and the medium value of the q

$$f(q,\tilde{\omega}) = -\frac{4}{3}\ q^2\{\tilde{\omega}^2 - q^4\}^{-1} \cdot \left\{1 + \frac{12}{5}\ q^2[\tilde{\omega}^2 - q^4]^{-1}\ +\ ...\right\}, \tag{33}$$

we obtain in the random phase approximation spectrum for collective excitations in the region $\xi < 1$ at $q \ge \xi r_s/\eta$

$$\hbar\omega_q \simeq \hbar\omega_q^{EL} \cdot Aq[q^2 + (\xi r_s/\eta)^2]^{-1/2}. \tag{34}$$

Here

$$\hbar\omega_q^{EL} \simeq \hbar\{\omega_0 + 2\alpha\omega_F q^2\} \tag{35}$$

is a plasmon excitation spectrum for the model of the electron liquid, α is a dispersion coefficient, which is equal $3\omega_F(5\omega_0)^{-1}$; $\omega_F \equiv \varepsilon_F/\hbar$ in the random phase approximation.

These solutions were obtained by numerical method. The local-field correction function $G(x)$ was taken in the form $G_{id}(q,\hbar\omega)$, which corresponds to [9] under replacing the Coulomb potential V_q on the Yukawa

one:

$$G_{id}(q,\hbar\omega) = [q^2 + (\xi r_s/\eta)^2]f^{-2}(q,\tilde{\omega}) \int_{-1}^{+1}\int dz_1\ dz_2 \tag{36}$$
$$\times \sum_{\sigma_1,\sigma_2} \sigma_1,\sigma_2 \Phi_{q,\tilde{\omega}}\Big(z_1 + \frac{\sigma_1}{2}q;\ z_2 + \frac{\sigma_2}{2}q;\ 1 - z_1^2;\ 1 - z_2^2\Big);$$
$$\Phi_{q,\tilde{\omega}}(a;b;w,v) = \frac{q^2}{8}(a-b)J(s^2;w,v)\Big(b^2q^2 - \frac{\tilde{\omega}^2}{4}\Big)^{-1}$$
$$\times \Big(a^2q^2 - \frac{\tilde{\omega}^2}{4}\Big)^{-2}\Big[q^2a^2b + \frac{\tilde{\omega}^2}{4}(2a+b)\Big].$$

The function $J(s^2,w,v)$ has the some structure as $J(s^2,w,v)$ from [9]

$$J(s^2,w,v) = \frac{1}{2}\{W(s^2,w,v) - w - v - s^2\}$$
$$+\ w\ \ln\{(2s^2)^{-1}[W(s^2,w,v) + s^2 + v - w]\} \tag{37}$$
$$+\ v\ \ln\{(2s^2)^{-1}[W(s^2,w,v) + s^2 + w - v]\},$$

but $s^2 = (a-b)^2 + (\xi r_s/\eta)^2$. Dependence of the plasmon excitation spectrum on the parameters ξ and r_s is depicted on the Fig. 7A, 7B. As one can see, spectrum of the collective excitations for the generalized model and the electron liquid model have different behavior in the longwavelength region and have the some one in the shortwavelength region.

5. Using the Model System with a Short-Range Interaction Potential as a Reference System for Description of the Electron Liquid Model

Model with a short-range interaction can be used as a reference system for the description of the electron liquid model in the region of strong nonideality. The main idea of such a method is the precise description of the short-range correction and the approximative description of the long-range correlation in a wide region of the parameter r_s. For that let us write the Coulomb potential as a sum of two terms

$$\frac{e^2}{r} = V_S(r) + V_L(r), \tag{38}$$
$$V_S(r) = \frac{e^2}{r}\exp(-\xi r/a_0), \quad V_L(r) = \frac{e^2}{r}\{1 - \exp(-\xi r/a_0)\}.$$

The model system with interaction potential $V_S(r)$ is used as a reference system. Using approximative description of the polarization function as in [4], long-range component $V_L(r)$ can be considered within the frame of the perturbation theory. The ground state energy of the electron liquid model can be written in the form

$$E = E_R + \lim_{\beta\to\infty} \frac{1}{2\beta V} \sum_{q\neq 0} \sum_{\nu} V_L(q) \int_0^1 d\lambda \ \mu_2(x, -x|\lambda), \qquad (39)$$

where E_R is a ground state energy of a reference system, $V_L(q)$ is the Fourier-transform of the potential $V_L(\mathbf{r})$ and $\mu_2(x, -x|\lambda)$ is the two-particle correlation function, which describes long-range part of the Coulomb potential. The function $\mu_2(x, -x|\lambda)$ can be described by polarization function, which is computed in the post-RPA [4]:

$$\begin{aligned}
&\mu_2(x, -x|\lambda) = M(x|\lambda) \ \{1 + V^{-1}V_L(q)\lambda \ M(x|\lambda)\}^{-1}, \\
&M(x|\lambda) = \mu_2^R(x, -x) + \mathcal{M}_{2,1}(x|\lambda) + \mathcal{M}_{2,2}(x|\lambda), \qquad (40)\\
&\mathcal{M}_{2,1}(x|\lambda) = -(2\beta V)^{-1} \sum_{x_1} v_L(x_1|\lambda)\mu_4^R(x, -x, x_1, -x_1); \\
&\mathcal{M}_{2,2}(x|\lambda) = (2\beta V^2)^{-1} \sum_{x_1} v_L(x_1|\lambda)v_L(x + x_1|\lambda) \\
&\quad \times [\mu_3^R(x, x_1, x - x_1)]^2.
\end{aligned}$$

Here $v_L(x|\lambda)$ is a screened potential in RPA:

$$\begin{aligned}
v_L(x|\lambda) &= \lambda V_L(q) \ \varepsilon_L^{-1}(x|\lambda), \qquad (41)\\
\varepsilon_L(x|\lambda) &= 1 + \lambda V^{-1}V_L(q) \ \tilde{\mu}_2^R(x, -x),
\end{aligned}$$

The n-particle correlation function of a reference system has the form

$$\begin{aligned}
&\tilde{\mu}_2^R(x, -x) = \tilde{\mu}_2^0(x, -x) \ \varepsilon_s^{-1}(x), \\
&\varepsilon_s(x) = 1 + V^{-1}V_s(q) \ [1 - G(x)] \ \tilde{\mu}_2^0(x, -x), \qquad (42)\\
&\tilde{\mu}_n^R(x_1, ..., x_n) = \tilde{\mu}_n^0(x_1, ..., x_n) \ \prod_{i=1}^{n} \ \varepsilon_s^{-1}(x_i), \quad n = 3, 4,
\end{aligned}$$

where $G(x)$ is a local-field correction function of a reference system, and $\tilde{\mu}_n^0(x_1, ..., \alpha_n)$ is the n-particle dynamic correlation function of the ideal electron gas [3].

Ground state energy can be written in the dimensionless form

$$\begin{aligned}
E &= N \cdot \ \mathrm{Ry} \ \cdot \varepsilon(r_s), \qquad (43)\\
\varepsilon(r_s) &= \varepsilon_0(r_s) + \varepsilon_{HF}(r_s) + \varepsilon_c(r_s).
\end{aligned}$$

The result of the calculation of the ground state energy is represented in the Table 1. The numerical calculation has been done at $\xi_0 = k_F a_0 = \eta r_s^{-1}$. As one can see from Table 1 the correlation energy of the electron liquid model, calculated on the basis (5.1), and the results obtained by Monte-Carlo method and interpolated in [11], are very similar. Our result is more exact than result of [3], where post-RPA approximation for the polarization function was used, and a reference system was taken ideal electron gas. For comparison we refer to result of [12], where correlation energy was obtained by the static local-field correlation function.

Table 1.
Correlation energy for the electron liquid model ($-10^3\varepsilon_c(r_s)$)

r_s	1	2	3	4	5	6	10
RPA	157.6	123.6	105.5	93.6	84.9	78.2	61.3
$PRPA$	119.7	89.3	72.2	62.1	54.2	48.0	32.6
MC [11]	120.0	89.6	73.8	63.6	56.3	50.7	37.1
[12]	117.4	86.9	71.1	61.0	53.8	48.3	35.0
* * *	120.0	89.2	73.2	62.8	55.7	50.1	36.2

6. Conclusion

The local-field approximation is one of the most significant achievement in the electron liquid theory during the last decades and the local-field correction function is one of many universal characteristics of the model. The generalized Fermi system model with the interaction described by Yukawa repulsion potential has been proposed. On the basis of this model the dependence of the local-field correction function on the interaction range of the potential R_0 has been investigated. As one can see from Fig. 2-4, the local-field correction function is such a model has a behavior different from the electron liquid model. Especially the region $0 < q < 2k_F$ is important, where the Fourier-transform of the potential V_q is large. On the basis of the computation we may assert that the local-field correction function in the model with short-range interaction potential is always larger than in the model with Coulomb potential in the region of small and medium values of the $\mathbf{q}$. This fact shows the relative importance of the short-range part of the interaction potential

for systems with a finite value of the interaction range.

As is known, the electron liquid model is stable, has negative total energy in the region $r_s \geq 2.0$, and has the equilibrium density at $r_s^0 = 4.1825...$, which corresponds to the minimum of the total energy (-0.15533 Ry per electron). As one can see from Fig. 5 the total energy of the generalized model in the ground state is smaller than that for the electron-liquid model for any value of the parameter ξ at the same value of the parameter r_s. Every value of r_s in the generalized model has been proved to correspond to a critical range of interaction $R_c(r_s) = a_0 \xi_c^{-1}(r_s)$. If the potential range is less than R_c, the correlation effects which have the range $R_F = k_F^{-1}$ cannot provide the stability of the model and a negative value of the total energy. Qualitative conclusions concerning the influence of the range of the interaction potential between particles on the character of the local-field correction function and the stability are true not only for the Yukawa potential model but for any Fermi system with arbitrary short-range potential.

The consideration of the represented model, at first, has heuristic meaning, and secondly, the model with a short-range interaction potential can be used as a reference system for the description of neutral fermion systems at $\xi \neq 0$. In this case the term $V(\mathbf{q} = 0)$ must be added to the expression of the total energy (3.2), and its coupling parameter will have the form

$$\alpha^{'} = \left(\frac{3}{5}\,\varepsilon_F\right)^{-1} \frac{N}{V} \int d\mathbf{r}^{'}\; V(\mathbf{r})\; g_2^{id}(\mathbf{r}^{'}) = \frac{10A^2}{\eta^4}\; r_s m^* f^{'}(\xi^*),$$

$$f^{'}(\xi^*) = \{(\xi^*)^{-2} 2 - (1+\xi^*)\;\exp\,(-\xi^*)\}.$$

Model under consideration has interesting behavior of the spectrum of the plasmon excitations. The effective method for describing of the electron liquid in the strong nonideality region is using the model with a short-range interaction as a reference system.

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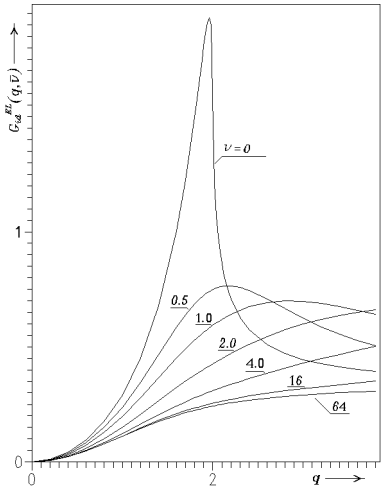

Figure 1. Dynamic local-field correction function of the electron liquid model ($\xi \to 0,\ A = 1,\ G_{id}^{EL}(q;\nu)$ (2.2) at $r_s \to 0$.

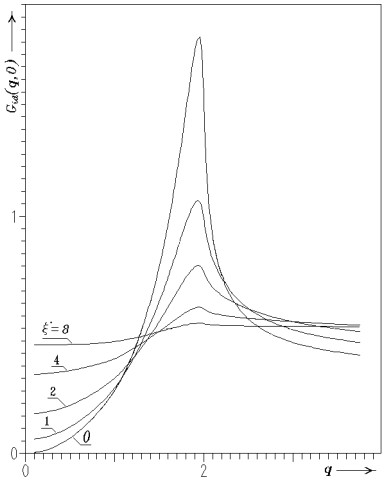

Figure 2. Static local-field correction function $G_{id}(q;0)$ (2.2) in dependence on the parameter ξ^*.

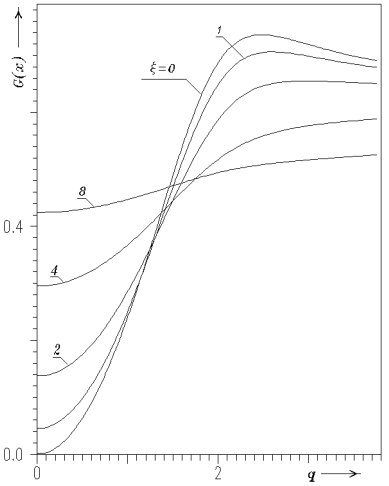

Figure 3. Dynamic local-field correction function $G(x)$ in dependence on the parameter ξ in the approximation (2.8), (2.9) at $A = 1,\ r_s = 1,\ u = 1$.

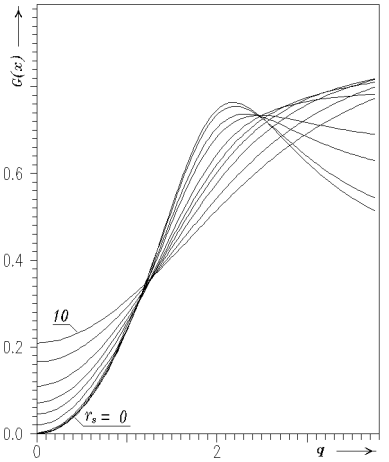

Figure 4. Dynamic local-field correction function $G(x)$ in dependence on the parameter r_s in the approximation (2.8), (2.9) (r_s has the values 0.0; 0.1; 0.5; 1.0; 2.0; 3.0; 4.0; 5.0; 7.0; 10.0) at $A = 1.0,\ \xi = 0.25,\ u = 1.0$.

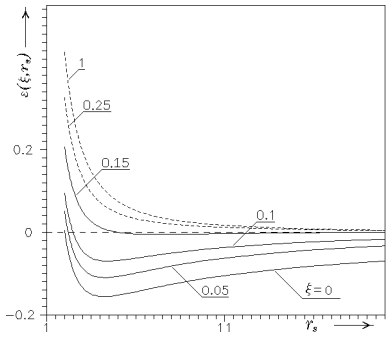

Figure 5. Dependence of the total energy of the generalized model on the parameter ξ. The calculations are based on (2.8), (2.9), (3.1), (3.4).

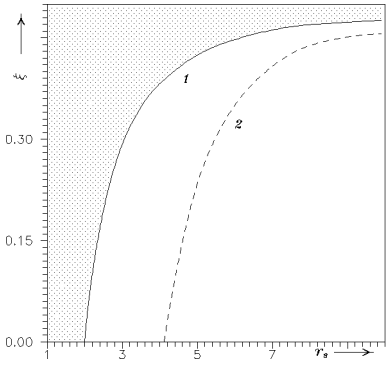

Figure 6. Stability and existence region of the generalized model as a function of the parameters r_s and ξ at $A = 1$. Curve 1 is the solution of (3.6), curve 2 is the solution of (3.7).

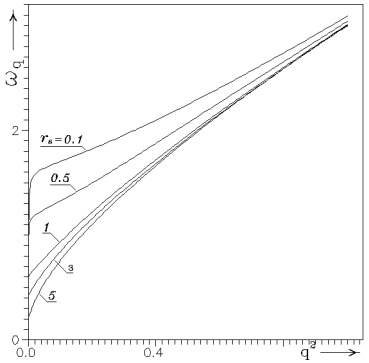

Figure 7. Dependence of the spectrum of the collective excitations ω_q on the parameter r_s at the fixed value of $\xi = 0.01$. (r_s has the values 0.1; 0.5; 1.0; 3.0; 5.0). The calculations are based on (4.2), (4.12).

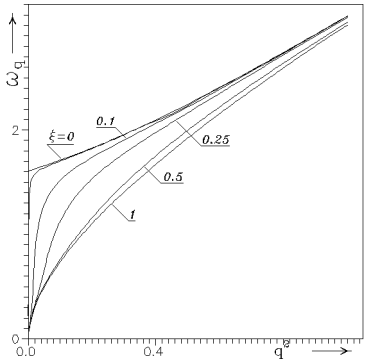

Figure 8. Dependence of the spectrum of the collective excitations ω_q on the parameter ξ at the fixed value of $r_s = 5.0$. (ξ has the values 0.0; 0.05; 0.1; 0.25; 0.5; 1.0). The calculations are based on (4.2), (4.12).

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Поправка на локальне поле фермі-системи з короткосяжною взаємодією

Роботу отримано 12 червня 1998 р.

Затверджено до друку Вченою радою ІФКС НАН України

Рекомендовано до друку семінаром відділу теорії модельних спінових систем

Виготовлено при ІФКС НАН України