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THE TWO-PARTICLE TIME-ASYMMETRIC MODELS WITH FIELD-TYPE INTERACTION

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Двочастинкові часоасиметричні моделі з взаємодією польового типу

А.Дувіряк

Анотація. Побудовано гамільтонів опис і досліджено динаміку двочастинкових часоасиметричних моделей з взаємодією польового типу для довільної суперпозиції лінійних тензорних полів. Запропоновано часоасиметричну модель з ґравітаційною взаємодією.

The two-body time-asymmetric models with field-type interaction

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Abstract. The Hamiltonian description of the two-particle time-asymmetric models with a field-type interaction for the arbitrary superposition of linear tensor fields is constructed. The dynamics of such models is investigated. The time-asymmetric model with the gravitational interaction is proposed.

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Introduction

The time-asymmetric models appear in literature on few independent ways [1-2;3;4-5], and by now they form integrated approach [6-8] to the *relativistic direct interaction theory* (RDIT). These models are attractive in several aspects. They form rather wide class of exactly integrable relativistic two-particle models, *i.e.*, there exist the sufficient number of integrals of motion which make it possible to reduce the problem to quadratures [3,7]. Besides, the time-asymmetric models can be reformulated equivalently into frameworks of various formalisms of RDIT [3,6,7,9]. The fact of especially physical meaning is that there exists the close relation of these models to the Fokker-type integrals formalism, and by means of the latter – to field-theoretical descriptions of particle interactions. In Refs.[10-12] the class of Fokker-type action integrals is found which corresponds to particle interactions mediated by linear tensor fields of arbitrary rank. It can be the source of two-particle models with time-asymmetric analogs of such interactions (in the cases of massless fields) in which the first particle perceives the retarded field of the second particle while the latter sense the advanced field of the first particle.

In Refs.[8,13] the time-asymmetric models with scalar, vector (see also [3]) and mixed interactions were studied: starting from the Fokker-type action integrals these models were reformulated into the framework of the Hamiltonian description (following the Refs.[6,7]) and corresponding two-body problems were reduced to quadratures and integrated. Here we consider the time-asymmetric interaction which is the superposition of field-type interactions of arbitrary field rank n. The time-asymmetric analog of the gravitational interaction is proposed also.

Among time-asymmetric field-type interactions the only (arbitrary) superposition of scalar and vector interactions permits the exact hamiltonization of corresponding models. Beginning from the rank of field $n = 2$ and going on, the transition to the Hamiltonian description and the construction of quadratures need of complicate algebraic (or transcendental) equations to be solved. Here we overcome these difficulties using the expansion in coupling constant. As a result we obtain in the second approximation the Hamiltonian description of the time-asymmetric model with an arbitrary field-type interaction (including the gravitational interaction) explicitly. The corresponding two-body problem is reduced to quadratures and integrated in the case of finite motion.

1. The time-asymmetric field-type models: original formulation and hamiltonization

Consider the Fokker-type action integral of the following form:

$$\begin{aligned} I \;=\; -&\sum_a m_a \int d\tau_a \sqrt{\dot x_a^2} \;- \\ &\sum_{a<b}\sum \iint d\tau_a d\tau_b \sqrt{\dot x_a^2}\sqrt{\dot x_b^2} f_{ab}(\omega_{ab}) G(x_{ab}). \end{aligned} \tag{1}$$

Here m_a is the rest mass of a-th particle; $x_a^\mu(\tau_a)$ ($\mu = \overline{0,3}$) are the covariant coordinates of the world line of a-th particle parametrized by an arbitrary evolution parameter τ_a; $\dot x_a^\mu(\tau_a) \equiv dx_a^\mu/d\tau_a$; $x_{ab}^\mu \equiv x_a^\mu(\tau_a) - x_b^\mu(\tau_b)$; $G(x_{ab}) = \delta(x_{ab}^2)$ is the symmetrical Green's function of the d'Alambert equation; $\omega_{ab} \equiv \dot x_a \cdot \dot x_b / \sqrt{\dot x_a^2 \dot x_b^2}$. We choose the time-like Minkowski metrics, *i.e.*, $\|\eta_{\mu\nu}\| = diag(+,-,-,-)$, and put the light speed to be unit.

In the case of f_{ab} to be a polynomial (or analytical) function, the interaction of a-th and b-th particles described by the action (1) can be considered (following [10-12]) as a field-type interaction mediated by the finite (infinite) superposition of some massless linear tensor fields. Especially, scalar (rank of field $n = 0$) and vector ($n = 1$) interactions correspond to the function $f_{ab} = g_a g_b \omega_{ab}^n$, where g_a is the charge of a-th particle.

Considering a two-particle system and performing the replacement $G(x_{12}) \to G_\eta(x_{12})$ in (1), where

$$G_\eta(x) = 2\theta(\eta x^0)\delta(x^2), \qquad \eta = \pm 1 \tag{2}$$

is the retarded (advanced) Green's function, one obtain the action integral

$$I = -\sum_{a=1}^{2} m_a \int d\tau_a \sqrt{\dot x_a^2} - \iint d\tau_1 d\tau_2 \sqrt{\dot x_1^2}\sqrt{\dot x_2^2} f(\omega_{12}) G_\eta(x_{12}) \tag{3}$$

which gives rise to the corresponding time-asymmetric model. In order to study the dynamics of this model we transit (following [6,7]) to its Hamiltonian description which is relevant for our purpose.

Once integrating the second term of the action (3) we reduce the latter to a single-time form. So that, we obtain the description of our model in the framework of manifestly covariant Lagrangian formalism with the holonomic constraint $x^2 = 0$, $\eta x^0 > 0$ and the Lagrangian function

$$L = \theta F(\sigma_1, \sigma_2, \omega), \tag{4}$$

where $\theta \equiv \eta\dot{y}\cdot x > 0$, $x \equiv x_1 - x_2$, $y \equiv (x_1 + x_2)/2$, $\sigma_a \equiv \sqrt{\dot{x}_a^2}/\theta$, $\omega \equiv \omega_{12}$,

$$F \equiv \sum_{a=1}^{2} m_a\sigma_a + \sigma_1\sigma_2 f(\omega) \equiv F_f + F_{int} \tag{5}$$

and all variables depend on an arbitrary common evolution parameter τ.

The transition to the manifestly covariant Hamiltonian description with constraints leads to the mass-shell constraint which determines the dynamics of the model and has the following form:

$$\phi(P^2,\ \upsilon^2,\ P\cdot x,\ \upsilon\cdot x) \equiv \phi_f + \phi_{int} = 0. \tag{6}$$

Here $\upsilon_\mu \equiv w_\mu - x_\mu P\cdot w/P\cdot x$; P_μ and w_μ are canonical momenta conjugated to y^μ i x^μ respectively; the function

$$\phi_f = \frac{1}{4}P^2 - \frac{1}{2}(m_1^2 + m_2^2) + (m_1^2 - m_2^2)\frac{\upsilon\cdot x}{P\cdot x} + \upsilon^2 \tag{7}$$

corresponds to the free-particle system, while the explicit form of ϕ_{int} depends in complicate manner on the choice of original Fokker-type action integral [1]. In present case the ϕ_{int} can be written down as follows:

$$\begin{aligned}\phi_{int} \ = \ &-\frac{2m_1m_2}{\eta P\cdot x}(f - (\omega - \lambda)f') - \\ &\frac{m_1^2 b_2 + m_2^2 b_1 - (m_1^2 + m_2^2)f' + 2m_1m_2(f - \omega f')}{\eta P\cdot x\,((b_1 - f')(b_2 - f') - (f - \omega f')^2)} \times \\ &\qquad ((f - \omega f')^2 - f'^2)\,,\end{aligned} \tag{8}$$

where

$$\lambda \equiv \frac{P^2 - m_1^2 - m_2^2}{2m_1m_2}, \tag{9}$$

$$b_a \equiv \eta\left(\frac{1}{2}P\cdot x + (-)^{\bar{a}}\upsilon\cdot x\right), \qquad a = 1,2, \qquad \bar{a} \equiv 3 - a, \tag{10}$$

$f'(\omega) \equiv df(\omega)/d\omega$, and ω is related to canonical variables by the set of equations:

$$(b_a - f')\sigma_a - (f - \omega f')\sigma_{\bar{a}} = m_a, \qquad a = 1,2, \qquad \bar{a} \equiv 3 - a, \tag{11}$$

$$b_1^2\sigma_1^2 + b_2^2\sigma_2^2 + 2(b_1b_2\omega - \eta P\cdot x f)\sigma_1\sigma_2 = P^2. \tag{12}$$

[1] The quantities θ, σ_a, ω introduced here and T, A, B, C in [6] are mutually related as follows: $\theta = T$, $\sigma_1 = A$, $\sigma_2 = B$, $\omega = C/(AB)$.

The equations (11) which are linear in σ_a make it possible to express σ_a through b_a and ω,

$$\sigma_a = \frac{(b_{\bar{a}} - f')m_a + (f - \omega f')m_{\bar{a}}}{(b_1 - f')(b_2 - f') - (f - \omega f')^2}. \tag{13}$$

The substitution of (13) into (12) gives for ω an equation which cumbersome form causes the main difficulty of hamiltonization of present model.

In the special case

$$f(\omega) = \alpha + \beta\omega, \tag{14}$$

which corresponds to the arbitrary superposition of scalar and vector interactions with coupling constants α and β, respectively, the ω fall up the right-hand side (r.-h.s.) of Eq.(8) what permits to obtain immediately the mass-shell constraint:

$$\begin{aligned}\phi_f \ - \ &\frac{2m_1m_2}{\eta P\cdot x}(\alpha + \beta\lambda) \ - \\ &\frac{m_1^2(b_2 - \beta) + m_2^2(b_1 - \beta) + 2m_1m_2\alpha}{\eta P\cdot x\,((b_1 - \beta)(b_2 - \beta) - \alpha^2)}(\alpha^2 - \beta^2) \ = \ 0.\end{aligned} \tag{15}$$

Notice that in the case $\alpha = \pm\beta$ the last term in the left-hand side (l.-h.s.) of Eq.(15) vanishes what simplifies to a great extent the dynamics of the model and what makes it to be similar to the dynamics of the nonrelativistic system with Coulombian interaction (see[13]).

In all other (*i.e.*, except the (14)) cases the ϕ_{int} depends essentially on the quantity ω, which, however, can not be expressed explicitly in terms of canonical variables. Other way, one can consider the equation (12) as the mass-shell constraint with the quantities σ_a and ω to be eliminated by means of Eqs.(6)-(8), (13). Although in such a manner we can not achieve a desirable simplification. Indeed, the case of second rank tensor interaction leads to the 5-order algebraic equation (while the $(n \geq 0)$–rank case – to the $(3n - 1)$–order equation) with ω to be found.

In order to overcome this difficulty we use in the next Section the expansions into the power series of a coupling constant.

2. Hamiltonian description in the second approximation of a coupling constant

Hereafter we suppose that the function $f(\omega)$ is of first order of the coupling constant α which is meant to be small, *i.e.*, $f(\omega) \sim O(\alpha)$. Accordingly, ω can be eliminated from the l.-h.s. of Eq.(8) using the successive approximation method. Here we obtain the second approximation in α.

For the free-particle case which corresponds to zero order approximation ($f=0$), the solution $\overset{0}{\sigma}_a$, $\overset{0}{\omega}$ of the set of equations (12)-(13) is as follows:

$$\overset{0}{\sigma}_a = m_a/b_a, \qquad \overset{0}{\omega} = \lambda. \tag{16}$$

Let us take $\omega = \overset{0}{\omega} + \delta\omega$, where $\delta\omega \sim O(\alpha)$, and calculate the r.-h.s. of Eq.(8) up to α^2. The first term contains the expression in brackets $\sim O(\alpha)$ which can be written down as follows:

$$f(\omega) - (\omega - \lambda)f'(\omega) = f(\lambda + \delta\omega) - \delta\omega f'(\lambda + \delta\omega) \approx f(\lambda) + O(\alpha^3). \tag{17}$$

The second term $\sim O(\alpha^2)$ is equal in this approximation to the main contribution of its expansion series in α. Thus,

$$\phi_{int} = -\frac{2m_1m_2}{\eta P\cdot x} f(\lambda) - \frac{h(\lambda)}{\eta P\cdot x}\left(\frac{m_1^2}{b_1} + \frac{m_2^2}{b_2}\right) + O(\alpha^3), \tag{18}$$

where

$$h(\lambda) \equiv \left((f(\lambda) - \lambda f'(\lambda)\right)^2 - (f'(\lambda))^2 \sim O(\alpha^2). \tag{19}$$

For the linear n-rank tensor field the function $f(\lambda)$ reads:

$$f_n(\lambda) = \alpha T_n(\lambda), \tag{20}$$

where $\alpha = g_1g_2$, and $T_n(\lambda)$ is the Tchebyshev polynomial [12]. Especially, $T_2(\lambda) = 2\lambda^2 - 1$. Taking $\alpha = -\gamma m_1 m_2$, where γ is the gravitational constant, this latter case can be considered as such corresponding to the gravitational interaction. But since the original Fokker action integral like the (1) describes the gravitating particle system correctly in the first approximation only, its time-asymmetric counterpart should be profound not far as in this approximation. In order to construct the more adequate model which takes into account second order effects we refer to the direct gravitational interaction theory.

3. The time-asymmetric model of gravitational interaction

Among approaches to the direct gravitational interaction theory one proposed by Vladimirov and Turygin [14] is meant to be most adequate to the general theory of relativity. This approach is based on the action with multiple Fokker-type integrals which is built up by means of the iteration procedure. In the second approximation of γ the action has the form:

$$\begin{aligned} I \;=\; & -\sum_a m_a \int d\tau_a \sqrt{\dot x_a^2} \;+ \\ & \gamma \sum_{a\,<\,b}\sum m_a m_b \iint d\tau_a d\tau_b \sqrt{\dot x_a^2}\sqrt{\dot x_b^2}\,(2\omega_{ab}^2 - 1)G(x_{ab}) \;+ \\ & \frac{\gamma^2}{2}\sum_a \sum_{b\neq a}\sum_{c\neq a} m_a m_b m_c \iiint d\tau_a d\tau_b d\tau_c \sqrt{\dot x_a^2}\sqrt{\dot x_b^2}\sqrt{\dot x_c^2} \;\times \\ & (2\omega_{ab}^2 - 1)(2\omega_{ac}^2 - 3)G(x_{ab})G(x_{ac}) \end{aligned} \tag{21}$$

(see [15] from which one can easy obtain this formula).

The second order contribution in (21) consists of both sorts of terms: those which correspond to triple interactions and those which describe a self-action via the influence of other particle. Just the latters remain in the two-particle case which we consider below. Now performing the substitution $G(x_{12}) \to G_\eta(x_{12})$ in the first order term and $G(x_{12})G(x_{12'}) \to G_\eta(x_{12})G_\eta(x_{12'})$, $G(x_{12})G(x_{1'2}) \to G_\eta(x_{12})G_\eta(x_{1'2})$ in the second order terms (subscript $a' = 1', 2'$ denotes τ'_a-dependence of corresponding variables) we obtain the action for the time-asymmetric gravitational interaction:

$$\begin{aligned} I \;=\; & -\sum_{a=1}^{2} m_a \int d\tau_a \sqrt{\dot x_a^2} \;+ \\ & \gamma m_1 m_2 \iint d\tau_1 d\tau_2 \sqrt{\dot x_1^2}\sqrt{\dot x_2^2}\,(2\omega_{12}^2 - 1)G_\eta(x_{12}) \;\times \\ & \left\{1 + \frac{\gamma}{2}\left(m_1 \int d\tau'_1 \sqrt{\dot x_{1'}^2}\,(2\omega_{1'2}^2 - 3)G_\eta(x_{1'2}) \;+\right.\right. \\ & \left.\left. m_2 \int d\tau'_2 \sqrt{\dot x_{2'}^2}\,(2\omega_{12'}^2 - 3)G_\eta(x_{12'})\right)\right\}. \end{aligned} \tag{22}$$

The structure of (22) makes it possible (in analogy to the previous cases) to reformulate the description of this model into the framework of the Lagrangian formalism. The corresponding Lagrangian has the form (4)-(5) where, however, the function f depends on σ_a too,

$$f(\sigma_1, \sigma_2, \omega) = -\gamma m_1 m_2 (2\omega^2 - 1)\Big(1 + \frac{\gamma}{2}(2\omega^2 - 3)(m_1\sigma_1 + m_2\sigma_2)\Big). \tag{23}$$

Now we transit to the Hamiltonian description and obtain the mass-shell constraint up to the γ^2-accuracy. Because the σ_a-dependence of f (22) appears in the second order term only, the straightforward substitution of $f(\sigma_1, \sigma_2, \omega)$ into the formulae (18)-(19) is correct. After higher order terms are neglected it turns out that the second order contribution in the first term of (18) is similar (up to a λ-dependent factor) to the second term, thus it can be associated naturally with the latter. The final function ϕ_{int} for the gravitational interaction has the form (18), where

$$f_{gr}(\lambda) = -\gamma m_1 m_2 (2\lambda^2 - 1), \tag{24}$$

$$h_{gr}(\lambda) = -2(\gamma m_1 m_2)^2 (2\lambda^2 + 1). \tag{25}$$

It is essential that the mass-shell constraint (6)-(7), (18) has a common structure for both the linear-field and gravitational interactions. It specifies the sort of interaction by the functions $f(\lambda)$ and $h(\lambda)$ which depend on the integral of motion λ only. This fact permits to integrate the two-body problem considering f and h as arbitrary first and second order functions, respectively, until the final analysis of formulae.

4. Integration of the two-body problem

In order to study the dynamics of the time-asymmetric models it is convenient, following [6,7], to transit from the manifestly covariant to three-dimensional Hamiltonian description in the framework of the Bakamjian-Thomas model [16-18]. Within this description ten generators of the Poincaré group P_μ, $J_{\mu\nu}$ as well as the covariant particle positions x_a^μ are the functions of canonical variables $\mathbf{Q}$, $\mathbf{P}$, $\mathbf{r}$, $\mathbf{k}$. The only arbitrary function entering into expressions for canonical generators is the total mass $|P| = M(\mathbf{r}, \mathbf{k})$ of the system which determines its internal dynamics. For the time-asymmetric models this function is defined by the mass-shell equation [6,7] which can be derived from the mass-shell constraint via the following substitution of arguments on the l.-h.s. of (6):

$$P^2 \to M^2, \quad v^2 \to -\mathbf{k}^2, \quad P\cdot x \to \eta M r, \quad v\cdot x \to -\mathbf{k}\cdot\mathbf{r}; \quad \text{here } r \equiv |\mathbf{r}|. \tag{26}$$

In our case use of (7), (18) leads to the mass-shell equation of the form:

$$\frac{1}{4}M^2 - \frac{1}{2}(m_1^2 + m_2^2) - \eta(m_1^2 - m_2^2)\frac{\mathbf{k}\cdot\mathbf{r}}{Mr} - \mathbf{k}^2 - 2m_1 m_2 \frac{f(\lambda)}{Mr} - \frac{h(\lambda)}{Mr}\left(\frac{m_1^2}{\frac{1}{2}Mr - \eta\mathbf{k}\cdot\mathbf{r}} + \frac{m_2^2}{\frac{1}{2}Mr + \eta\mathbf{k}\cdot\mathbf{r}}\right) = O(\alpha^3). \tag{27}$$

Due to the Poincaré-invariance of the description it is sufficient to choose the centre-of-mass (CM) reference frame in which $\mathbf{P} = \mathbf{0}$, $\mathbf{Q} = \mathbf{0}$. Accordingly, $P_o = M$, $J_{0i} = 0$ $(i = 1, 2, 3)$, and the components $S_i \equiv \frac{1}{2}\varepsilon_i{}^{jk} J_{jk}$ form a 3-vector of the total spin of the system (internal angular

momentum) which is the integral of motion. At this point the problem is reduced to the rotating-invariant problem of some effective single particle which is integrable in terms of polar coordinates,

$$\mathbf{r} = r\mathbf{e}_r, \quad \mathbf{k} = k_r \mathbf{e}_r + S\mathbf{e}_\varphi / r. \tag{28}$$

Here $S \equiv |\mathbf{S}|$; the unit vectors $\mathbf{e}_r$, $\mathbf{e}_\varphi$ are orthogonal to $\mathbf{S}$, they form together with $\mathbf{S}$ a right-oriented triplet and can be decomposed in terms of Cartesian unit vectors $\mathbf{i}$, $\mathbf{j}$:

$$\mathbf{e}_r = \mathbf{i}\cos\varphi + \mathbf{j}\sin\varphi, \quad \mathbf{e}_\varphi = -\mathbf{i}\sin\varphi + \mathbf{j}\cos\varphi, \tag{29}$$

where φ is the polar angle.

The corresponding quadratures read:

$$t - t_0 = \int dr\ \partial k_r(r, M, S)/\partial M, \tag{30}$$

$$\varphi - \varphi_0 = -\int dr\ \partial k_r(r, M, S)/\partial S, \tag{31}$$

where $t = \frac{1}{2}(x_1^0 + x_2^0)_{\mathrm{CM}}$ is the fixed evolution parameter (unlike undetermined τ), and the radial momentum k_r being the function of r, M, S is defined by the equation (27) written down in terms of these variables,

$$\frac{1}{4}M^2 - \frac{1}{2}(m_1^2 + m_2^2) - \eta(m_1^2 - m_2^2)\frac{k_r}{M} - k_r^2 - \frac{S^2}{r^2} - 2m_1 m_2 \frac{f(\lambda)}{Mr} - \frac{h(\lambda)}{Mr^2}\left(\frac{m_1^2}{\frac{1}{2}M - \eta k_r} + \frac{m_2^2}{\frac{1}{2}M + \eta k_r}\right) = O(\alpha^3). \tag{32}$$

The solution of the problem given in terms of canonical variables enables to obtain particle world lines in the Minkowski space using the following formulae [6,7]:

$$x_a^0 = t + \frac{1}{2}(-)^{\bar{a}}\eta r, \tag{33}$$

$$\mathbf{x}_a = \left(\frac{1}{2}(-)^{\bar{a}} + \eta\frac{k_r}{M}\right) r\mathbf{e}_r + \eta\frac{S}{M}\mathbf{e}_\varphi. \tag{34}$$

Especially, the vector $\mathbf{x} = \mathbf{r}$ characterizes the relative motion of particles.

Let us consider the quadrature (31). In terms of denotations

$$u \equiv 1/r, \quad q \equiv k_r + \eta\frac{m_1^2 - m_2^2}{2M}. \tag{35}$$

it can be written down as follows:

$$\varphi - \varphi_0 = \int dr\ \frac{\partial\phi/\partial S}{\partial\phi/\partial k_r}\bigg|_{\phi=0} = \int \frac{2S\,du}{\partial\phi/\partial k_r|_{\phi=0}}, \tag{36}$$

where

$$\phi = \varepsilon - q^2 - S^2u^2 - 2\frac{m_1m_2}{M}f(\lambda)u - \frac{h(\lambda)}{M}\left(\frac{m_1^2}{M_1-\eta q} + \frac{m_2^2}{M_2+\eta q}\right)u^2 = O(\alpha^3), \tag{37}$$

$$\frac{\partial\phi}{\partial k_r} = -2q - \eta\frac{h(\lambda)}{M}\left(\frac{m_1^2}{(M_1-\eta q)^2} - \frac{m_2^2}{(M_2+\eta q)^2}\right)u^2 + O(\alpha^3); \tag{38}$$

$$\varepsilon \equiv \frac{1}{4}M^2 - \frac{1}{2}(m_1^2+m_2^2) + \frac{(m_1^2-m_2^2)^2}{4M^2} = \left(\frac{m_1m_2}{M}\right)^2(\lambda^2-1), \tag{39}$$

$$M_a \equiv \frac{M}{2}\left(1 + \frac{m_a^2 - m_{\bar a}^2}{M^2}\right). \tag{40}$$

In order to calculate the integrant of (36) in an explicit form the q should be eliminated from the r.-h.s. of (38) by means of the equation (37). Using the successive approximation method we do it in such a way to express the quadrature (36) in terms of elementary functions and to spread the integration over whole the domain of possible motions (DPM) – including returning points. The latters obey the conditions

$$\phi = 0, \qquad \partial\phi/\partial k_r = 0, \tag{41}$$

which yield the following value of q:

$$q = -\eta\frac{h(\lambda)}{2M}\left(\frac{m_1^2}{M_1^2} - \frac{m_2^2}{M_2^2}\right)u^2 + O(\alpha^3) \sim O(\alpha^2), \tag{42}$$

and which lead to the square equation for u,

$$\Pi_2(u) \equiv \phi\,|_{q=O(\alpha^2)} = \varepsilon - 2\frac{m_1m_2}{M}f(\lambda)u - \hat S^2u^2 = O(\alpha^3), \tag{43}$$

where

$$\hat S^2 \equiv S^2 + \frac{h(\lambda)}{M}\left(\frac{m_1^2}{M_1} + \frac{m_2^2}{M_2}\right). \tag{44}$$

Now we search the expression for q in whole DPM. Let $\overset{1}{q} = q + \delta q$ where

$$\overset{1}{q} = \pm R \equiv \pm\sqrt{\Pi_2(u)} \tag{45}$$

satisfies the equation (37) in the first order approximation. The above expression for $\overset{1}{q}$ enables to represent the integrant of (36) in the simple calculative form without lost the accuracy at returning points. The resulting expression for δq is as follows:

$$\delta q = -\eta\frac{h(\lambda)}{2M}\left(\frac{m_1^2}{M_1(M_1\mp\eta R)} - \frac{m_2^2}{M_2(M_2\pm\eta R)}\right)u^2 + O(\alpha^3). \tag{46}$$

Using (45)–(46) in (38) we write down explicitly the quadrature (36),

$$\varphi - \varphi_0 = \mp\int\frac{Sdu}{R}\left\{1 - \frac{h(\lambda)}{M}\left(\frac{m_1^2}{M_1(M_1\mp\eta R)^2} + \frac{m_2^2}{M_2(M_2\pm\eta R)^2}\right)u^2 + O(\alpha^3)\right\}, \tag{47}$$

which can be evidently exppressed in terms of elementary functions.

Similarly one can obtain rather cumbersome expression for the quadrature (30) which we ommit here.

Hereafter we limit ourselves by finite motions and integrate (47). DPM in this case is bounded by two returning points which are the roots of the equation (43),

$$u_{1,2} = a \pm b, \tag{48}$$

where

$$a \equiv \frac{m_1m_2}{M\hat S^2}f(\lambda), \qquad b \equiv \sqrt{a^2 + \varepsilon/\hat S^2}. \tag{49}$$

They must be real and positive what yields the conditions:

$$f(\lambda) < 0 \tag{50}$$

and

$$-\left(\frac{m_1m_2}{M\hat S}f(\lambda)\right)^2 < \varepsilon < 0. \tag{51}$$

The latter means that $\varepsilon \sim O(\alpha^2)$ what simplifies the calculation of the quadrature (47). Indeed, in this case $b \sim |a| \sim O(\alpha)$, $u^2 \sim O(\alpha^2)$ (because $u_1 < u < u_2$), and the u^2-proportional term can be neglected. Hence the quadrature can be calculated elementarily using the substitution:

$$u = |a| + b\cos\psi \tag{52}$$

and it leads (at the certain choice of an integration constant) to the following equality:

$$\psi = \frac{\hat S}{S}\varphi \equiv (1-\delta)\varphi. \tag{53}$$

The quantity δ in the taken approximation gives the simple form,

$$\delta = 1 - \frac{\hat S}{S} = 1 - \sqrt{1 + \frac{h(\lambda)}{MS^2}\left(\frac{m_1^2}{M_1} + \frac{m_2^2}{M_2}\right)} \approx -\frac{h(\lambda)}{2MS^2}\left(\frac{m_1^2}{M_1} + \frac{m_2^2}{M_2}\right) \approx -\frac{h(1)}{2S^2}, \tag{54}$$

since one can put $M \approx m_1 + m_2$, $M_a \approx m_a$, $\lambda \approx 1$ in second order terms.

The equation of relative motion trajectory follows immediately from (52)-(53):

$$1/r \;=\; |a| \;+\; b\cos\left((1-\delta)\varphi\right) \qquad (b <|a|). \tag{55}$$

It describe an ellipse which precesses with the perihelion advance

$$\Delta\varphi \;=\; 2\pi\delta \;=\; -\pi h(1)/S^2. \tag{56}$$

In the case of the linear tensor interaction of arbitrary rank n the perihelion advance $\Delta\varphi$ can be calculated by means of the formulae (19)–(20) and the equalities $T_n(1) = 1,\ T_n'(1) = n^2$:

$$\Delta\varphi \;=\; \pi(2n^2-1)(g_1g_2/S)^2, \tag{57}$$

and for the gravitational interaction – using (24):

$$\Delta\varphi \;=\; 6\pi(\gamma m_1 m_2/S)^2. \tag{58}$$

Finally, we note that spatial particle trajectories calculated by means of (34) turn out to be more intricate than the relative trajectory what is the typical feature of time-asymmetric models.

Conclusion

The specific feature of the formalism of Fokker-type action integrals is that the relativistic particle systems described within its framework possess infinite number of degrees of freedom. In order to make the dynamics of such systems tractable mechanically we are forced somehow to cut off extra degrees of freedom. The time-asymmetric models can be considered as a possible way leading to the description of systems with field-type interaction in usual terms of the analytical mechanics.

The results presented here substantiate physical grounds of these models. Namely, the values of perihelion advance fit those obtained in frameworks of various quasirelativistic approaches to RDIT (see [19] and [20] for the electromagnetic (i.e., vector) and gravitational interactions, and [21,12,22] from which it is easy to derive values of $\Delta\varphi$ for other field-type interactions). Furthermore, unlike these latter approaches the time-asymmetric models possess the exact Poincaré invariance even in approximations in a coupling constant. This fact admits the analysis of essentially relativistic particle motions which occur in the scattering problem just in the first-order approximation, while in the bounded states problem – in the exact consideration (for scalar and vector interactions [8,3]) or in higher-order approximations.

The Hamiltonian description of time-asymmetric models outlines a way leading to the quantum mechanics of particle systems with a field interaction which is alternative to the Bethe-Salpeter equations and quasipotential approach.

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