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Понад-світловий рух нейтрино як наслідок порушення принципу еквівалентності

Роботу отримано 22 березня 2012 р.

Затверджено до друку Вченою радою ІФКС НАН України

Рекомендовано до друку відділом комп'ютерного моделювання багаточастинкових систем

Виготовлено при ІФКС НАН України


**Національна академія наук України**

ІНСТИТУТ
ФІЗИКИ
КОНДЕНСОВАНИХ
СИСТЕМ

ICMP–12–02E

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# SUPERLUMINAL NEUTRINO PHENOMENON AS A RESULT OF THE EQUIVALENCE PRINCIPLE VIOLATION

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ЛЬВІВ

**УДК:** 524.6, 53.01, 53.03, 531.1, 531.3, 531.5, 531.9, 539.1

**PACS:** 04.20.Cv, 13.15.+g, 98.35.-a

**Понад-світловий рух нейтрино як наслідок порушення принципу еквівалентності**

О.Ф.Бацевич, Р.Б.Капустій

**Анотація.** У даній статті показано, що понад-світловий рух нейтрино, який був виявлений у експериментах, проведених колаборацією OPERA, може бути пояснений відсутністю гравітаційної маси нейтрино, та, як наслідок цього, відсутністю взаємодії між нейтрино та гравітаційним полем. На основі даної гіпотези були проведені розрахунки, результати яких повністю співпали з експериментальними даними, отриманими колаборацією OPERA. Також, дані розрахунки передбачають існування анізотропії швидкості руху нейтрино в залежності від напрямку руху Землі відносно Галактики, що в подальшому дозволить провести експериментальну перевірку викладеної гіпотези.

**Superluminal neutrino phenomenon as a result of the equivalence principle violation**

O.F.Batsevych, R.B.Kapustiy

**Abstract.** In this paper it is shown that recently detected superluminal neutrino motion can be explained by the absence of gravitational mass of neutrino, and as a result, an absence of its interaction with a gravitational field. The results theoretically predicted in this paper are in full agreement with the experimental data obtained by OPERA collaboration. The conducted calculations also predict significant anisotropy of the neutrino velocity measurement depending on the direction of the Earth's motion relative to the Galaxy, which allows validation of the obtained results.

**Подається в** Journal of Physics: Condensed Matter
**Submitted to** Journal of Physics: Condensed Matter





## 1. Introduction

On September 22, 2011 OPERA collaboration announced the registration of light exceeding by 0.00248% by muon neutrino [1]. This news has been widely discussed in mass media since this phenomenon went far beyond the limits of modern scientific concepts. The response of the scientific community to this uncanny neutrino behaviour was quite sceptical – a series of articles were released where attempts were made to identify possible errors in the experiment in order to discredit the result [2]. However, two months after the initial announcement, on November 17 a new publication was released by the OPERA collaboration in which the correctness of the conducted experiment was confirmed once again [3].

In February OPERA informed [4] about two possible problems which may affect obtained results. Due to one of them neutrino velocity may be underestimated (the passage of time on the clocks between the arrival of the synchronizing signal has to be interpolated, which probably may not have been done correctly), the other one, on the contrary, may be responsible for overestimate (possible faulty connection between the GPS signal and the OPERA master clock). In this connection it should be reminded that superluminal neutrinos were registered earlier as well, eg. by MINOS collaboration in 2007 [5], and there were no inaccuracies found in these experiments. Therefore existence of superluminal neutrinos seems quite plausible, and most probably will be confirmed by the forthcoming OPERA experiment this year.

Numerous attempts on the field-theory level have been made during last few months to explain neutrino superluminality. Within this approach the phenomenon is explained by neutrino interaction with artificially introduced auxiliary fields [6] emanated by the Earth, or even such construct as a dark matter [7, 8].

However in this article we show how the phenomenon of superluminal neutrino can be explained within General theory of relativity without any additional fields and other far-fetched concepts. The only assumption which will be employed to that end is quite simple, however unobvious from the first glance: we will assume non-equivalence of the gravitational and inertial masses of the neutrino. More precisely, we assume that gravitational mass of neutrino is equal to zero. This means that ubiquitous equivalence principle is violated by the neutrino, allowing us to obtain predictions which coincide perfectly with the experimental data.

## 2. Equation of free neutrino motion

The equation of free motion [9–11] of an ordinary mass particle having Cartesian coordinates $\boldsymbol{x} = \{x^\alpha\} = \{ct, \vec{x}\}$ in gravitational field is

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0, \tag{1}$$

where affine connection $\Gamma^\alpha_{\mu\nu}$ is the source of the gravitational force acting on the particle.

Equation (1) is written with the assumption that gravitational $m_g$ and inertial $m_I$ masses of the object are the same, which is the essence of the equivalence principle. What should be written instead of (1) if the two masses are not equal? The answer to this question is quite straightforward in the case of a weak field when Newtonian approximation can be used to express the affine connection via the intensity of the field as $\Gamma^i_{00} \sim \frac{\partial\Phi}{\partial x^i}$, where $\Phi$ is the gravitational potential. In this case Eq (1) becomes the well-known Newtonian law of gravitation

$$\frac{d^2\vec{x}}{dt^2} + \frac{\partial\Phi}{\partial\vec{x}} = 0,$$

which follows from the second Newton law $m_I\frac{d^2\vec{x}}{dt^2} = \vec{F}_g$ with the gravitational force $\vec{F}_g = -m_g\frac{\partial\Phi}{\partial\vec{x}}$. If the ratio of the masses $\mu = m_g/m_I$ is not equal to 1, the gravitation reads $\frac{d^2\vec{x}}{dt^2} + \mu\frac{\partial\Phi}{\partial\vec{x}} = 0$, and multiplier $\mu$ appears in front of $\Gamma^\alpha_{\mu\nu}$ in Eq. (1) as well.

Our main statement is that **the gravitational and the inertial masses of neutrino are not equivalent, and the gravitational mass of neutrino is equal to zero**. So further on we will be interested in the special case of $\mu = 0$. Hence, equation (1) for the neutrino should be rewritten as

$$\frac{d^2x^\alpha}{d\tau^2} = 0. \tag{2}$$

The solution of Eq. (2) is a movement with the constant speed, $dx^\alpha/d\tau = \text{const}^\alpha$, or explicitly for the space component:

$$\vec{v} = \frac{d\vec{x}}{dt} = \vec{\text{const}}. \tag{3}$$

It is important to realize that Eq. (3) holds not only in the space outside any local gravitational non-homogeneities, but as well inside the gravitational field, even as strong as one of a black hole.

We remind here that Eq (2) is written for a special case of Cartesian-like coordinates, for which affine connection disappears in the absence of gravitation. Generally, metric tensor $g_{ij}(\boldsymbol{x})$ and affine connection $\Gamma^\alpha_{\mu\nu}(\boldsymbol{x})$ depend on coordinates $\boldsymbol{x}$ not only because of the presence of gravitational potential $\Phi$, but also due to inherent properties of curvilinear coordinates. In such general case the equation of free motion for gravitationally-neutral particle (neutrino) is



$$\frac{d^2x^\alpha}{d\tau^2} + \left\{ {\alpha \atop \mu\nu} \right\}(\boldsymbol{x})\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0, \tag{4}$$

where $\left\{ {\alpha \atop \mu\nu} \right\}(\boldsymbol{x}) = \Gamma^\alpha_{\mu\nu}(\boldsymbol{x})|_{\Phi=0}$ is the affine connection calculated for 'switched off' gravity and consequently in flat space-time.

Equation (4) states that neutrino 'lives' in Minkowski (*flat*) space-time equipped with the metric tensor $\eta_{ij}(\boldsymbol{x}) = g_{ij}(\boldsymbol{x})|_{\Phi=0}$. Hence, space-time interval $ds_0 = 0$ in Minkowski world, which corresponds to propagation with the speed of light, may correspond to $ds < 0$ in gravitationally-affected world, for gravitationally-neutral particle is travelling not along distorted geodesic line, but along a straight short-cut between two points.

## 3. Velocity of neutrino

An observer on the Earth surface is subject to gravitational influences of many massive objects, most important of which are summarized in a Table 1[1] [12–17].

From the Table 1 it follows that the strongest gravity field is that of our planet; however, the strongest gravitational potentials is that of our Galaxy, $\Phi_\Gamma = \frac{GM_\Gamma}{R_\Gamma}$, by a few orders of magnitude. Therefore henceforth we will ignore all gravitational potentials, except of $\Phi_\Gamma$. To describe gravitational effects on the Earth's observer, we will use the Schwarzschild metric written in isotropic Cartesian coordinates $\{dt', d\vec{x}\,'\}$ at rest relatively to the Galactic centre (referred henceforth as Galactic-fixed coordinate system, GCS) as follows [18, 19]

$$ds^2 = \frac{\left(1 - \dfrac{\Phi_\Gamma}{2c^2}\right)^2}{\left(1 + \dfrac{\Phi_\Gamma}{2c^2}\right)^2} c^2 dt'^2 - \left(1 + \frac{\Phi_\Gamma}{2c^2}\right)^4 d\vec{x}\,'^2. \tag{5}$$

GCS is asymptotically plain, for significant distances $g_{\alpha\beta} = \eta_{\alpha\beta}$. It is obvious that after 'switching off' gravity the equality $g_{\alpha\beta} = \eta_{\alpha\beta}$ will

[1] The Galactic potential is defined with an assumption that the total mass of the Galaxy is located in its centre.

Table 1. The gravitational influences of the space objects to the observer on the Earth

| **Space objects:** | **Earth** | **Sun** | **Galaxy** | **M31** | **M33** |
|---|---|---|---|---|---|
| Gravitational mass: $M$, $kg$ | $5.974{\cdot}10^{24}$ | $1.9891{\cdot}10^{30}$ | $3.78{\cdot}10^{42}$ | $2.45{\cdot}10^{42}$ | $1.0{\cdot}10^{41}$ |
| Distance to the surface of the Earth: $R$, $m$ | $6.371{\cdot}10^{6}$ | $1.496{\cdot}10^{11}$ | $2.59{\cdot}10^{20}$ ($8.40 kpc$) | $2.43{\cdot}10^{22}$ ($788\ kpc$) | $2.62{\cdot}10^{22}$ ($850\ kpc$) |
| Gravitational potential which creates mass $M$ on the surface of the Earth: $\Phi$, $m^2/s^2$ | $6.258{\cdot}10^{7}$ | $8.874{\cdot}10^{8}$ | $9.74{\cdot}10^{11}$ | $6.73{\cdot}10^{9}$ | $2.55{\cdot}10^{8}$ |
| The intensity of the gravitational field on the surface of the Earth: $g = \Phi/R,\ \ m/s^2$ | 9.822 | $5.932{\cdot}10^{-3}$ | $3.76{\cdot}10^{-9}$ | $2.77{\cdot}10^{-13}$ | $9.72{\cdot}10^{-15}$ |

hold everywhere, and the free motion equation (2) will give

$$\vec{v}\,' = \frac{d\vec{x}\,'}{dt'} = \vec{\text{const}}. \tag{6}$$

Therefore an observer far from the centre of the Galaxy will measure the neutrino velocity $\vec{v}$ to be equal to $\vec{v}_{\nu,E}$ from Eq. (3) for the neutrino emitted on the Earth. This is not a case for all other speed measurements, e.g. even light will be measured to have speed $c_E$ less than $c$.

The inertial Earth's observer coordinate system (referred henceforth as iECS) is moving with the speed

$$\vec{V}_E = \vec{V}_S + \vec{V}_{E/S}\,(t)\,, \tag{7}$$

relative to the GCS due to the measurements in iECS. Here $\vec{V}_S$ is the speed of the Sun's orbital motion around the Galactic centre, $\vec{V}_{E/S}\,(t)$ is the orbital speed of the Earth around the Sun, which has a period of change equal to one year.

To find the neutrino velocity $\vec{V}$ in iECS, we introduce first inertial Galactic-fixed coordinate system (iGCS), which is located at the instantaneous Earth position, but at rest to the Galactic centre. Its coordinates $\{\tau,\vec{\rho}\}$ are related to the Galactic ones by the following *gravitational*



*transformation* $T\,(\Phi_\Gamma)$ :

$$d\tau = \frac{\left(1-\dfrac{\Phi_\Gamma}{2c^2}\right)}{\left(1+\dfrac{\Phi_\Gamma}{2c^2}\right)}dt'; \qquad d\vec{\rho} = \left(1+\frac{\Phi_\Gamma}{2c^2}\right)^2 d\vec{x}\,'. \tag{8}$$

Indeed, from Eq. (5) it follows that coordinates (8) yield locally flat metric $\eta_{\alpha\beta}$

$$ds^2 = c^2 d\tau^2 - d\vec{\rho}^{\,2}, \tag{9}$$

hence they may be regarded as locally-inertial coordinates.

From (8) it also follows that in iGCS observer will measure the following neutrino velocity $\vec{V}'$:

$$\vec{V}' = \frac{d\vec{\rho}}{d\tau} = \frac{\left(1+\dfrac{\Phi_\Gamma}{2c^2}\right)^3}{\left(1-\dfrac{\Phi_\Gamma}{2c^2}\right)}\frac{d\vec{x}\,'}{dt'} = \frac{\left(1+\dfrac{\Phi_\Gamma}{2c^2}\right)^3}{\left(1-\dfrac{\Phi_\Gamma}{2c^2}\right)}\vec{v}\,'. \tag{10}$$

Neglecting terms of order $c^{-4}$ and higher in Eq. (10), we can write

$$\vec{V}' \approx \left(1+\frac{2\Phi_\Gamma}{c^2}\right)\vec{v}\,'. \tag{11}$$

It is already obvious, that as the result of the gravitational transformation speed $V'$ will be greater than $v\,'$ by the factor

$$\beta \equiv \left(1+\frac{2\Phi_\Gamma}{c^2}\right) > 1. \tag{12}$$

This speed is measured in iGCS, which is related to the Earth observer's iECS coordinates $\{t_E,\vec{r}_E\}$ through the Lorentz boost $\Lambda\left(\vec{V}_E\right)$:

$$dt_E = \frac{d\tau - \frac{\vec{V}_E d\vec{\rho}}{c^2}}{\gamma}; \qquad d\vec{r}_E = \frac{d\vec{\rho}_\parallel - \vec{V}_E d\tau}{\gamma} + d\vec{\rho}_\perp, \tag{13}$$

where $\gamma = \sqrt{1-\vec{V}_E^2/c^2}$. $d\vec{\rho}_\parallel = \frac{d\vec{\rho}\cdot\vec{V}_E}{\vec{V}_E^2}\cdot\vec{V}_E$, and $d\vec{\rho}_\perp = d\vec{\rho} - d\vec{\rho}_\parallel$ are the components of $d\vec{\rho}$, parallel and perpendicular, respectively, to $\vec{V}_E$. From (13) if follows that $\vec{V} = \frac{d\vec{r}_E}{dt_E}$ and $\vec{V}' = \frac{d\vec{\rho}}{d\tau}$ are related via ordinary relativistic speed addition formula

$$\vec{V} = \frac{\vec{V}_\parallel' + \gamma\vec{V}_\perp' - \vec{V}_E}{1-\vec{V}'\cdot\vec{V}_E/c^2}. \tag{14}$$

Taking into account (11), we get expression for the velocity of neutrino $\vec{V}$ measured in iECS through the velocity of neutrino $\vec{v}\,'$ measured in the GCS as follows

$$\vec{V} = \frac{\beta\left(\vec{v}\,'_{\parallel} + \gamma\vec{v}\,'_{\perp}\right) - \vec{V}_E}{1 - \beta\vec{v}\cdot\vec{V}_E/c^2}. \quad (15)$$

Due to relative movement of these reference systems, numerator of (15) has a drift term $-\vec{V}_E$. To avoid it, we must express $\vec{V}$ via velocity of neutrino $\vec{v}$ measured in *(flat)* Earth-fixed coordinate system (ECS) which is following the Earth at all times. ECS is related to GCS via the Lorentz boost $\Lambda\left(-\vec{v}_E\right)$, where, due to gravitational transformation,

$$\vec{v}_E = \frac{\vec{V}_E}{\beta} \approx \left(1 - \frac{2\Phi_\Gamma}{c^2}\right)\vec{V}_E \quad (16)$$

is the Earth's speed measured in GCS. Therefore $\vec{v}\,'$ can be expressed through neutrino velocity $\vec{v}$, measured in ECS, similarly to (14)

$$\vec{v}\,' = \frac{\vec{v}_{\parallel} + \gamma\,'\vec{v}_{\perp} + \vec{v}_E}{1 + \vec{v}\cdot\vec{v}_E/c^2}, \quad (17)$$

where $\gamma\,' = \sqrt{1 - \vec{v}_E^{\,2}/c^2}$.

Substituting (17) into (15), we finally get

$$\vec{V} = \beta\gamma\,'\frac{\gamma\,'\vec{v}_{\parallel} + \gamma\vec{v}_{\perp}}{\gamma^2 + (1-\beta^2)\,\frac{\vec{v}\cdot\vec{v}_E}{c^2}}. \quad (18)$$

This formula contains no drift terms, as it relates speeds of the neutrino in two Earth-fixed reference systems, namely, flat (ECS) and inertial (iECS).

Formula (18) is the one which have sought for as it relates the Earth-measured velocity of neutrino $\vec{V}$ to the "flat" velocity of neutrino $\vec{v}$, which is ordinary velocity and can for example be expressed through the experimentally measured neutrino energy $E$ as

$$v = c\sqrt{1 - \frac{m^2c^4}{E^2}}. \quad (19)$$

An interesting peculiarity of formula (18), which may be a basis for future experimental validation of our model, is the dependence of the measured velocity on the direction at which neutrino was emitted relatively to the Earth's velocity.



## 4. Alternative derivation

Derivation of the formula (18) was based on the sequence of transitions between coordinate systems ECS → GCS → iGCS → iECS. In terms of the Lorentz boosts $\Lambda(\cdot)$ and *gravitational transformation* $T\left(\Phi_\Gamma\right)$ defined by (8), this may be summarized by

$$\boldsymbol{V} = \Lambda\left(\beta\vec{v}_E\right)T\left(\Phi_\Gamma\right)\Lambda\left(-\vec{v}_E\right)\cdot\boldsymbol{v}, \quad (20)$$

where all calculation complexities arise due to the fact that $T\left(\Phi_\Gamma\right)$ and $\Lambda(\cdot)$ are not commuting with each other.

The impossibility to make gravitational transformation on spot is in the fact that Schwarzschild metric is given in GCS which is at rest relatively to the Galactic centre, while an observer on the Earth is moving. Let us now rewrite the Schwarzschild interval (5) in terms of the ECS coordinates $\{t, \vec{x}\}$ related to the GCS coordinates $\{t', \vec{x}\,'\}$ by the Lorentz boost $\Lambda\left(-\vec{v}_E\right)$:

$$dt' = \frac{dt + \frac{\vec{v}_E d\vec{x}}{c^2}}{\gamma}; \qquad d\vec{x}\,' = \frac{d\vec{x}_{\parallel} + \vec{v}_E dt}{\gamma} + d\vec{x}_{\perp}. \quad (21)$$

Substituting these expressions in (5) we get with the accuracy of $O\left(\Phi_\Gamma/c^2\right)$ the following:

$$ds^2 = \frac{c^2}{\beta}\left(\frac{\gamma dt + \frac{\vec{v}_E d\vec{x}\left(1-\beta^2\right)}{c^2\gamma}}{\gamma\,'}\right)^2 - \beta\left(\frac{\gamma\,''^2}{\gamma\,'^2}d\vec{x}_{\parallel}^{\,2} + d\vec{x}_{\perp}^{\,2}\right), \quad (22)$$

where $\gamma\,'' = \sqrt{1 - \frac{\vec{v}_E^{\,2}}{c^2\beta^2}}$.

Expression (22) defines space-time distortion created by a point mass moving with the speed $-\vec{v}_E$ about the origin, therefore it may be interpreted as a Galactic potential acting on the Earth's observer. Eq. (22) was in purpose written in a form with explicitly extracted squared factor near $c^2$, which immediately allows us to perform transition to inertial coordinate system iECS with coordinates $\{t_E, \vec{r}_E\}$ as follows

$$dt_E = \beta^{-\frac{1}{2}}\cdot\frac{\gamma dt + \frac{\vec{v}_E\cdot d\vec{x}\left(1-\beta^2\right)}{c^2\gamma}}{\gamma\,'}; \qquad d\vec{r}_E = \beta^{\frac{1}{2}}\cdot\left(\frac{\gamma\,''}{\gamma\,'}d\vec{x}_{\parallel} + d\vec{x}_{\perp}\right). \quad (23)$$

And for neutrino velocity $\vec{V} = \frac{d\vec{r}_E}{dt_E}$ we obtain

$$\vec{V} = \beta\gamma\,'\frac{\frac{\gamma''\gamma}{\gamma\,'}\vec{v}_{\parallel} + \gamma\vec{v}_{\perp}}{\gamma^2 + (1-\beta^2)\,\frac{\vec{v}\cdot\vec{v}_E}{c^2}}. \quad (24)$$

This formula is equivalent to (18) with accuracy of $O\left(c^{-5}\right)$, as during simplification of expression for the interval (22) we freely neglected terms of order $O\left(\Phi_\Gamma^2/c^4\right)$ and higher.

## 5. Comparison with the experimental data

Since experiments of scientists of OPERA collaboration in which they measured $V > c$ referred to muon neutrino, then in this paper we will speak about exactly this type of neutrino [3]. However, most likely, that all conclusions of the article will be also fully fair for other types of neutrino. In experiments it was measured that light covered distance of $S = 731\,278.0\, m$ in $t_c = 2\,439\,280.9\, ns$, while neutrino was ahead of light by

$$\Delta t = 57.8 \pm 7.8\,(\text{stat.}) \pm {}^{8.3}_{5.9}\,(\text{sys.})\ \ ns. \tag{25}$$

The relative deviation of neutrino velocity from the velocity of light, defined as

$$\delta V \equiv \frac{V-c}{c}, \tag{26}$$

is

$$\delta V_{\text{OPERA}} = \left(2.37 \pm 0.32\,(\text{stat.}) \pm {}^{0.34}_{0.24}\,(\text{sys.})\right)\cdot 10^{-5}. \tag{27}$$

Experimental measurements were split on two datasets with the neutrino energies of $E_1 = 13.9$ GeV and $E_2 = 42.9$ GeV. Experiments detected no dependence the muon neutrino velocity on its energy, what indicates that the (inertial) mass of the neutrino is very small.

There are many estimates of the upper limit of the muon neutrino rest mass; now we will use the maximal one [20] $m_I(\nu_\mu)/c^2 = E_0 = 2.2$ MeV. For the experimentally detected neutrino energies deviation of the neutrino velocity $v$ (in the flat coordinate system ECS), from the speed of light given by $\delta v^{\text{lim}} = 1 - \sqrt{1-(E_0/E)^2}$, will be

$$\begin{aligned} &\delta v^{\text{lim}}(13.9\text{ GeV}) < 1.25\cdot 10^{-8};\\ &\delta v^{\text{lim}}(42.9\text{ GeV}) < 1.31\cdot 10^{-9}. \end{aligned} \tag{28}$$

These numbers are much less then the measured light speeds exceed (27) and we will neglect (in ECS) the difference of the neutrino velocity from $c$ without any loss of precision. So, henceforth we put

$$v \approx v^{\text{lim}} = c. \tag{29}$$

The above experimental studies were not the first to indicate that the neutrino velocity can exceed the speed of light. In particular, the similar



results were obtained back in 2007 by a group of scientists from MINOS collaboration [5]. However, in all the previous studies the accuracy of the obtained results was much smaller; therefore one could not definitely state the fact that the speed of light was exceeded by neutrino.

Taking into account (29), we can expand the neutrino velocity (18) into series and neglecting terms of order $O\left(c^{-3}\right)$ and higher, we get

$$V = \left(1 + \frac{2\Phi_\Gamma}{c^2} + \frac{4\Phi_\Gamma V_E}{c^3}\cos\alpha\right)\cdot c. \tag{30}$$

Consequently,

$$\delta V = \frac{2\Phi_\Gamma}{c^2} + \frac{4\Phi_\Gamma V_E}{c^3}\cos\alpha. \tag{31}$$

Putting in formula (30) the most recent estimates for numerical values $V_E = 2.55\cdot 10^5$ m/s and $\Phi_\Gamma = 9.74\cdot 10^{11}$ m$^2$/s$^2$, we obtain the following expression for the absolute value of the neutrino velocity

$$V = \left(1 + 2.16744\cdot 10^{-5} + 3.69\cdot 10^{-8}\cos\alpha\right)\cdot c. \tag{32}$$

So, the experimentally measured value of $\delta V$ must belong to the interval

$$\delta V = 2.16744\cdot 10^{-5} \pm 3.69\cdot 10^{-8}, \tag{33}$$

where the second term is the purely anisotropic part; maximal or minimal values of $\delta V$ are reached when the neutrino is emitted strictly collinear forward or backward, respectively, to the Earth's speed relative to the Galaxy.

An amazing coincidence of our estimate (33) with the experimental value (27) is the strong confirmation of our hypothesis. Another possible way of verification of the given hypothesis would be an experimental approval of the neutrino velocity anisotropy, but this certainly would demand increasing of the accuracy of measurements by two orders of magnitude.

There are other factors which can make $\delta V$ variable. One of them is the seasonal movement of the Earth $\vec{V}_{E/S}(t)$, Eq. (7). Taking for simplicity that the orbit of the Earth around the Sun is circular and the angle between the Earth orbital plane and the vector $\vec{V}_S$ is $\theta = 60°$ [17], it is easy to find that the absolute value of $\vec{V}_E$ is given by

$$V_E^2 = V_S^2 + V_{E/S}^2 + 2V_S V_{E/S}\cos\omega\cos\theta, \tag{34}$$

where $\omega \in [0,\ 2\pi]$ is the angle of the annual motion of the Earth around the Sun, $V_S = 2.54\cdot 10^5\ m/s$ [16], and the average $V_{E/S} = 2.978\,\cdot$

$10^4\ m/s$ [17]. The maximal and minimal values of $V_E$ then will be

$$V_{E,\text{min/max}} = \sqrt{V_S^2 + V_{E/S}^2 \pm 2V_S V_{E/S}\cos\theta}, \tag{35}$$

$$V_{E,\text{min}} = 2.40 \cdot 10^5\ m/s; \qquad V_{E,\text{max}} = 2.70 \cdot 10^5\ m/s. \tag{36}$$

Performing numerical calculations of $\delta V$ for $V_{E,\text{min/max}}$, we obtain the correction to $\delta V$

$$\Delta\delta V_{\mathbf{seasonal}} = \pm\ 2.2 \cdot 10^{-9}. \tag{37}$$

The final expression which incorporates all corrections may be presented as

$$\delta V = 2.16744 \cdot 10^{-5} \pm \left(3.69 \cdot 10^{-8}\right)_{\mathbf{directional}} \pm \left(2.2 \cdot 10^{-9}\right)_{\mathbf{seasonal}}. \tag{38}$$

## 6. Conclusion and discussion

Our attempt to explain superluminal neutrino velocity measured in OPERA experiment was made by assuming that the gravitational neutrino mass is equal to zero, which makes neutrino movement insensitive to gravitational space-time distortion. Formulas (18) and (24) were obtained, relating neutrino velocity $V\,(> c)$ measured by an observer on the Earth (laboratory) with neutrino velocity $v\,(\leq c)$ which it would have if it were an ordinary particle. Taking into account that the neutrino velocity for experimentally measured energies is very close to the speed of light in the *flat* coordinate system, we set $v \approx c$, giving possible error not higher than $10^{-8}$. The theoretical estimate of relative neutrino velocity deviation from the speed of light $\delta V$ gives positive value (38), which coincides perfectly with OPERA result (27).

This coincidence is even more remarkable if we take into account that the gravitational potential of the Galaxy and other variables which are involved in formula (18) are known with a substantial inaccuracy, as well as other simplifying assumptions, such as neglecting the gravitational influence of all objects on the Earth with the exception of the Galaxy, approximation of the metric tensor as if it were generated by the point mass located in the centre of Galaxy etc.

Another confirmation to correctness of our model is an astronomical registration of neutrino and light emission from the supernova outburst SN 1987A [21,22] in Large Magellanic Cloud, which showed a 4-hour neutrino outrun. If neutrino were travelling with a constant speed measured by OPERA all the time, the outrun would be 4 years. Due to our model neutrino is not an ordinary tachyon, which is always faster than light, so the smaller the neutrino's energy, the smaller the neutrino's velocity. It is well known that the energy of the detected neutrino from SN 1987A was 3 order of magnitude smaller then in OPERA experiments. Thus, in the inter-galactic space where the metric is more flat, the neutrino travels with a velocity not greater than the speed of light. Therefore the registered light delay was only 4 hours rather than 4 years.

Justification of neutrino superluminal velocity made in this paper has far-reaching consequences (superluminality itself has them on the first place). It may be a hint to validation alternate theories of gravitation, such as for example, N.Rosen's bi-metric theory [23,24]. "Legalization" of superluminal neutrino means revision of quantum field theory principles as well. Indeed, superluminality of a neutrino implies violation of causal relationships, which was an allowance, previously made only for virtual particles.

Despite the aforementioned explanations of existing experimental facts provided by our model, one can finally confirm its correctness only by experimental detecting of the anisotropy of neutrino velocity. This will certainly demand increase of measurement accuracy by two orders of magnitude. Detecting of seasonal velocity fluctuations, also predicted by our model, will need even higher precision.



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**ABSTRACTED/INDEXED IN:** Chemical Abstract Service, Current Contents/Physical, Chemical&Earth Sciences; ISI Science Citation Index-Expanded, ISI Alerting Services; INSPEC; “Referatyvnyj Zhurnal”; “Dzherelo”.



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