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Перенормування та реакція випромінювання
фотоноподібного заряду

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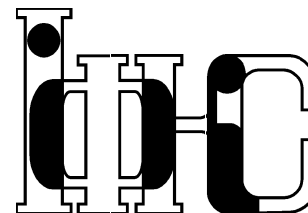
ІНСТИТУТ
ФІЗИКИ
КОНДЕНСОВАНИХ
СИСТЕМ

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Yurij Yaremko

Radiation reaction and renormalization
for a photon-like charged particle

ЛЬВІВ

**Перенормування та реакція випромінювання фотоноподіб-
ного заряду**

Ю.Г.Яремко

Анотація. Пораховані енергія-імпульс та момент кількості руху еле-
ктромагнітного поля безмасового точкового заряду. На відміну від
точкового джерела з ненульовою масою спокою, випромінені енергія-
імпульс та момент кількості руху поля прискореного безмасового за-
ряду необмежено зростають. Тому фотоноподібний заряд може існу-
вати лише в такому зовнішньому полі, яке не змінює його швидкості.
Ефективним рівнянням руху є рівняння на власні вектори та вла-
сні значення тензора напруженості зовнішнього електромагнітного
поля. Якщо це рівняння дозволяє рівномірний і прямолінійний рух,
безмасовий заряд рухається вздовж лінії, паралельної до власного
вектора тензора напруженості. Такий же розв'язок виникає у запро-
понованій Риловим моделі магнетосфери пульсара.

**Radiation reaction and renormalization for a photon-like
charged particle**

Yu.Yaremko

Abstract. Energy-momentum and angular momentum carried by field
of a massless point-like charge are calculated. Unlike the massive case,
the rates of radiated energy-momentum and angular momentum tend to
infinity whenever the source is accelerated. The external electromagnetic
fields which do not change the velocity of the particle admit only its
presence within the interaction area. The effective equation of motion
is the equation on eigenvalues and eigenvectors of the electromagnetic
tensor. The massless charges move along base line determined by the
eigenvectors when the equation of motion possesses uniform solutions.
The same solution arises in Rylov's model of magnetosphere of a pulsar.

1. Introduction

In the paper [1] massless charged particles of spin one or larger are ex-
cluded in quantum electrodynamics by the argument that masslessness,
Lorentz invariance, and electromagnetic coupling, are mutually incom-
patible. Roughly speaking, the interaction with an external electromag-
netic field drastically changes incoming massless particle state, so that
outgoing state does not describe a particle without rest mass. Further [2]
the existence of massless charges is forbidden in general by the condi-
tion that the energy of such particles in the electromagnetic field has no
lower bound. In the present paper we consider the problem of reality of
a massless charge within the realm of classical field theory.

A recent paper by Kazinski and Sharapov [3] considers the problem
of effective equations of motions for a massless charged particle under
the influence of its own electromagnetic field as well as an external one.
The authors apply regularization procedure developed in their previous
paper [4] where the problem of radiation back reaction in classical elec-
trodynamics of a point massive charge arbitrarily moving in flat space-
time of any dimensions is studied. The 5-th order differential equation is
derived [3, eq.(32)] which governs the dynamics of the photon-like charge
in four dimensions. The reduction procedure is developed which allows
to select the solutions of true physical meaning.

Since the concept of a "zero-mass interacting particle" is quite dif-
ferent in quantum and classical theories, it would be more appropriate
to obtain the equation of motion as a limiting case of the well-known
Lorentz-Dirac equation [5]. (It defines the motion of point-like charge
with rest mass m under the influence of an external force as well as its
own electromagnetic field, for a modern review see [6–8].) In [9] the mo-
tion of massive charged particles in a very strong electromagnetic field
is studied. The guiding center approximation [10] is used in the Lorentz-
Dirac equation. In this approximation the particle motion is described
as a combination of forward and oscillatory motions (the field changes
are small during a gyration period). If the gradient of the field potential
is much larger than the rest mass of the particle, the strong radiation
damping suppresses the particle gyration. It is shown [9] that the particle
velocity is directed along one of the eigenvectors of the (external) electro-
magnetic tensor if $m \to 0$ in the rewritten Lorentz-Dirac equation. The
equation on eigenvalues and eigenvectors of the electromagnetic tensor
governs the motion of charges in the massless approximation.

According to [9], the effective equation of motion for this charge does
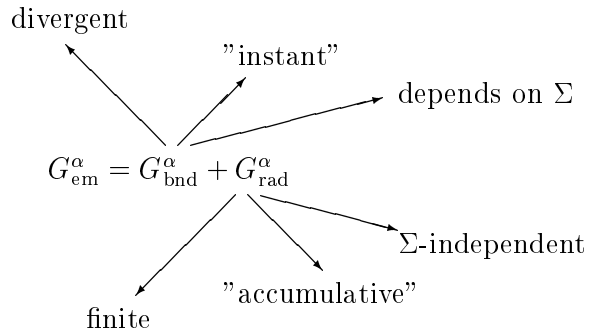not contain derivatives higher than 1. This conclusion is in contradiction

Figure 1. The bound term G^α_{bnd} and the radiative term G^α_{rad} constitute Noether quantity G^α_{em} carried by electromagnetic field. The former diverges while the latter is finite. Bound component depends on instant characteristics of charged particles while the radiative one is accumulated with time. The form of the bound term heavily depends on choosing of an integration surface Σ while the radiative term does not depend on Σ.

with that of Ref. [3] where the radiation back reaction is finite and the 5-th order differential equation determines the evolution of photon-like charge.

In general, the regularization procedure can be performed in two quite different ways: (i) one when Green's functions are used in variational equations of motion; (ii) the other when wave solutions are substituted for field variables in Noether conservation laws (e.g., in energy-momentum carried by electromagnetic field). In [3,4] the first way is realized which is a combination of some heuristic assumptions and calculations by methods of functional analysis. The second way is the integration of the Maxwell energy-momentum density over a space-like surface in Minkowski space.

Teitelboim in [11] classifies the terms which arise due to integration (see figure 1). Within regularization procedure the bound terms are coupled with energy-momentum and angular momentum of "bare" sources, so that already renormalized characteristics G^α_{part} of charged particles are proclaimed to be finite. Noether quantities which are properly conserved become:

$$G^\alpha = G^\alpha_{part} + G^\alpha_{rad}. \tag{1.1}$$

Particle's equations of motion arise from analysis of differential consequences of the conserved quantities (1.1). i.e. from the balance equations

$\dot{G}^\alpha = 0$.

In the present paper we apply the regularization procedure based on Noether conservation laws to the problem of radiation reaction for a massless charge in response to the electromagnetic field.

The paper is organized as follows. In Section 2 we state our notation. In Section 3 we discuss some peculiarities of electromagnetic field generated by a photon-like charge. Contrary to the massive case, the field strengths contain the *far* terms only (these scaled as r^{-1} where r is the retarded distance [6,7]). The term which is scaled as r^{-2} exhausts corresponding *radiative* stress-energy tensor. Volume integration of the Maxwell energy-momentum tensor density gives the flux of radiative momentum (see Section 4). In Section 5 we present our main result — the effective equations of motion for a massless charged particle under the influence of an external force. Since the radiation back reaction diverges when the particle is accelerated, the external device should not change its velocity. Few electromagnetic fields are briefly described in Appendix A which admit the photon-like charges within the interaction area. Finally, in Section 6 a short comment is made about their possible presence in the magnetosphere of a pulsar [12,13].

2. General setting

Let $\mathbb{M}_4$ be Minkowski space with coordinates x^μ and metric tensor $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. We use Heaviside-Lorentz system of units with the velocity of light $c = 1$. Summation over repeated indices is understood throughout the paper; Greek indices run from 0 to 3, and Latin indices from 1 to 3.

We consider a massless point-like particle which carries an electric charge q and moves on a lightlike world line $\gamma : \mathbb{R} \to \mathbb{M}_4$ described by functions $z^\mu(\tau)$, in which τ is an arbitrary parameter. A tangent vector to each point $z^\mu(\tau) \in \gamma$ lies on the future light cone with vertex at this point:

$$\dot{z}^2 = 0. \tag{2.1}$$

(We use an overdot on z to indicate differentiation with respect to the evolution parameter τ.) We let $u^\alpha(\tau) = dz^\alpha/d\tau$ be the 4-velocity, and $a^\alpha(\tau) = du^\alpha/d\tau$ is the 4-acceleration. Initially we take the world line to be arbitrary; our main goal is to find the particle's equation of motion.

Following [3], we deal with an obvious generalization of the standard variational principle for massive charge

$$I = I_{\mathrm{particle}} + I_{\mathrm{int}} + I_{\mathrm{field}}, \tag{2.2}$$

with

$$I_{\text{field}} = -\frac{1}{16\pi} \int d^4x f^{\mu\nu} f_{\mu\nu} \qquad I_{\text{int}} = \int d^4x A_\mu j^\mu. \qquad (2.3)$$

The particle part of variational principle should be consistent with the field and the interaction terms. So, if we require that the renormalized mass be zero, a nonzero bare mass is necessary to absorb a divergent self-energy. Hence the world line of the bare particle should be assumed time-like rather than lightlike. We may also require that the world line be lightlike before renormalization as well as after this procedure. To solve the dilemma we establish the structure of the bound and radiative terms (see figure 1) of energy-momentum and angular momentum carried by electromagnetic field of the photon-like charge.

Having variated (2.3) with respect to potential A_μ, we obtain the Maxwell field equations [3, eq.(14)]

$$\Box A_\mu(x) = -4\pi j_\mu(x) \qquad (2.4)$$

where current density is zero everywhere, except at the particle's position where it is infinite

$$j_\mu(x) = q \int d\tau u_\mu(\tau)\delta[x - z(\tau)] \qquad (2.5)$$

and $\Box := \eta^{\alpha\beta}\partial_\alpha\partial_\beta$ is the wave operator.

The components of the momentum 4-vector carried by the electromagnetic field are [6,7]

$$p_{\text{em}}^\nu(\tau) = \int_\Sigma d\sigma_\mu T^{\mu\nu} \qquad (2.6)$$

where $d\sigma_\mu$ is the outward-directed surface element on an arbitrary space-like hypersurface Σ. The angular momentum tensor of the electromagnetic field is written as [6]

$$M_{\text{em}}^{\mu\nu}(\tau) = \int_\Sigma d\sigma_\alpha \left(x^\mu T^{\alpha\nu} - x^\nu T^{\alpha\mu} \right) \qquad (2.7)$$

where

$$T^{\mu\nu} = \frac{1}{4\pi} \left(f^{\mu\lambda} f^\nu{}_\lambda - 1/4\eta^{\mu\nu} f^{\kappa\lambda} f_{\kappa\lambda} \right) \qquad (2.8)$$

is the electromagnetic field's stress-energy tensor.

3. Electromagnetic field of a photon-like charge

Let the past light cone with vertex at an observation point x is punctured by the particle's world line γ at point $z(s)$. The retarded Green function associated with the d'Alembert operator $\Box$ and the charge-current density (2.5) is valuable only. The components of the Liénard-Wiechert potential $\hat{A} = A_\alpha dx^\alpha$ are

$$A_\alpha = q\frac{u_\alpha(s)}{r} \qquad (3.1)$$

where $r = -(R \cdot u)$ is the retarded distance [6,7]; $R^\mu = x^\mu - z^\mu(s)$ is the null vector pointing from $z(s) \in \gamma$ to x. The 4-potential is not defined at points on the ray in the direction of momentary 4-velocity $u(s)$ by reason of the isotropy condition (2.1).

Straightforward computation reveals that $\Box A = 0$ everywhere, except at the particle's position. Indeed, suppose that the observation point P with coordinates x is moved to $P'(x + \delta x)$. This induces a change in the intersection point $z(s)$. The new intersection point is then $z(s + \delta s)$; points $P'(x + \delta x)$ and $z(s + \delta s)$ are still related by null 4-vector $R^\mu = x^\mu + \delta x^\mu - z^\mu(s + \delta s)$. Expanding the relation $R^2 = 0$ to the first order in the displacements, we obtain the differentiation rule

$$\frac{\partial s}{\partial x^\alpha} = -k_\alpha \qquad k^\alpha = \frac{x^\alpha - z^\alpha(s)}{r}. \qquad (3.2)$$

Differentiation of the retarded distance gives

$$\frac{\partial r}{\partial x^\alpha} = -u_\alpha + ra_k k_\alpha \qquad (3.3)$$

where $a_k := (a \cdot k)$ is the component of the acceleration $a(s)$ in the direction of k. We also need the equality

$$\frac{\partial k_\alpha}{\partial x^\beta} = r^{-1}\left(u_\alpha k_\beta + u_\beta k_\alpha + \eta_{\alpha\beta} \right) - a_k k_\alpha k_\beta. \qquad (3.4)$$

Finally we act on (3.1) by the wave operator

$$\Box = \eta_{\alpha\beta}\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial x^\beta} \qquad (3.5)$$

Using (3.2), (3.3) and (3.4), after some algebra we obtain zero.

Because of isotropy condition $(u \cdot u) = 0$ the rules (3.3) and (3.4) are different from their counterparts [7, eqs.(4.7),(4.9)] for massive particle.

Unlike the massive case, the photon-like charge generates the far electromagnetic field $\hat{f} = d\hat{A}$:

$$\hat{f} = q\frac{a \wedge k + a_k u \wedge k}{r} \tag{3.6}$$

Here the dot means the scalar product of two 4-vectors and the symbol $\wedge$ denotes the wedge product. Because of isotropy condition the retarded distance vanishes on the ray in the direction of particle's 4-velocity taken at the instant of emission. The field diverges at all the points of this ray with vertex at the point of emission.

To calculate the stress-energy tensor of the electromagnetic field we substitute the components (3.6) into expression (2.8). Contrary to the massive case [7, eqs.(5.3)-(5.5)], the "photon-like" Maxwell energy-momentum density contains the radiative component only:

$$4\pi T^{\alpha\beta} = \frac{q^2}{r^2}a^2 k^\alpha k^\beta. \tag{3.7}$$

Hence the divergent self-energy which is due to volume integration of the *bound* part of the electromagnetic field's stress-energy tensor [11] does not arise. Unlike the massive case, the photon-like charge does not possess an electromagnetic "cloud" permanently attached to it. The renormalization procedure is not necessary because the photon-like source is not "dressed".

As a consequence, the Brink-Di Vecchia-Howe action term [14, eq.(2)]:

$$I_{\text{particle}} = \frac{1}{2}\int d\tau e(\tau)\dot{z}^2 \tag{3.8}$$

is consistent with the field an interaction terms (2.3). Variation of (3.8) with respect to Lagrange multiplier $e(\tau) \neq 0$ yields the isotropy condition (2.1). The particle part (3.8) of the total action (2.2) describes *already renormalized* massless charge.

The action integral (2.2) being the sum of (2.3) and (3.8) is invariant under arbitrary time and space translations as well as space and mixed spacetime rotations. The Poincaré invariance of (2.2) assures us, via Noether's theorem, of ten conservation laws, i.e. those quantities which do not change with time.

Action integral (2.2) with I_{part} in form of (3.8) is conformally invariant. This symmetry property is analyzed in Appendix B. It is worth noting that the conformal invariance yields conservation laws, which are functions of energy-momentum and angular momentum conserved quantities.

4. Energy-momentum and angular momentum carried by the electromagnetic field

Volume integration of the radiative energy-momentum density (3.7) over a hyperplane $\Sigma_t = \{x \in \mathbb{M}_4 : x^0 = t\}$ gives the amount of radiated energy-momentum at fixed instant t. An appropriate coordinate system is a very important for the integration. We introduce the set of curvilinear coordinates for flat space-time $\mathbb{M}_4$ involving the observation time t and the retarded time s:

$$x^\alpha = z^\alpha(s) + (t-s)\Omega^\alpha{}_{\alpha'}n^{\alpha'}. \tag{4.1}$$

The null vector $n := (1, \mathbf{n})$ has the components $(1, \cos\varphi\sin\vartheta, \sin\varphi\sin\vartheta, \cos\vartheta)$; ϑ and φ are two polar angles. Matrix space-time components are $\Omega_{0\mu} = \Omega_{\mu 0} = \delta_{\mu 0}$; its space components Ω_{ij} constitute the orthogonal matrix which rotates space axes of the laboratory Lorentz frame until new z-axis is directed along three-vector $\mathbf{v}$. (Particle's 4-velocity has the form $(1, v^i), |\mathbf{v}| = 1$, if parametrization of the world line γ is provided by a disjoint union of hyperplanes Σ_t.) Orthogonal matrix

$$\omega = \begin{pmatrix} \cos\varphi_v & -\sin\varphi_v & 0 \\ \sin\varphi_v & \cos\varphi_v & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\vartheta_v & 0 & \sin\vartheta_v \\ 0 & 1 & 0 \\ -\sin\vartheta_v & 0 & \cos\vartheta_v \end{pmatrix} \tag{4.2}$$

where $v^i = (\cos\varphi_v\sin\vartheta_v, \sin\varphi_v\sin\vartheta_v, \cos\vartheta_v)$ determines the rotation. In terms of curvilinear coordinates $(t, s, \vartheta, \varphi)$ the retarded distance is as follows:

$$r = (t-s)(1 - \cos\vartheta). \tag{4.3}$$

The situation is pictured in figure 2.

So, we construct the global coordinate system centred on the world line of the massless particle. Minkowski space $\mathbb{M}_4$ becomes a disjoint union of hyperplanes $\Sigma_t = \{x \in \mathbb{M}_4 : x^0 = t\}$. A surface Σ_t is a disjoint union of spherical wave fronts

$$S(z(s), t-s) = \{x \in \mathbb{M}_4 : (x^0 - s)^2 = \sum_i (x^i - z^i(s))^2, x^0 = t\} \tag{4.4}$$

which are the intersections of the future light cones with vertices at points $z(s) \in \gamma$ and hyperplane Σ_t. The point $C \in S(z(s), t-s)$ is linked to the point $z(s) \in \gamma$ by a null ray characterized by the angles (φ, ϑ) specifying its direction on the cone.
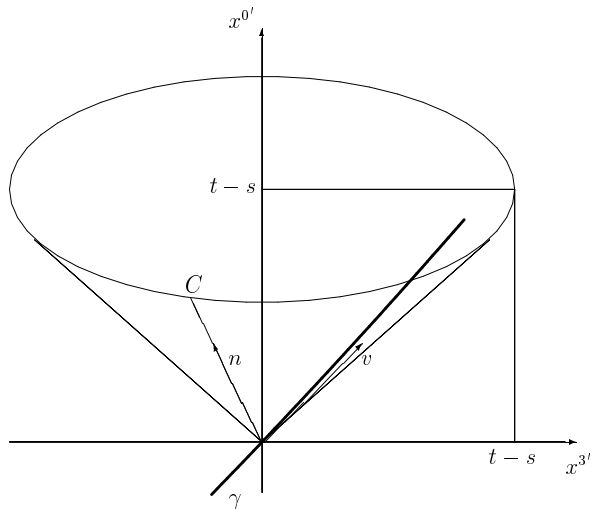
Figure 2. In the particle's momentarily comoving frame the massless charge is placed at the coordinate origin; its 4-velocity is $(1,0,0,1)$. The point $C \in S(0, t-s)$ is linked to the coordinate origin by a null ray characterized by the angles (φ, ϑ). (The null vector $n = (1, \cos\varphi \sin\vartheta, \sin\varphi \sin\vartheta, \cos\vartheta)$ defines this direction.) For a given point C with coordinates $x^{\alpha'} = (t-s)n^{\alpha'}$ the retarded distance is $x^{0'} - x^{3'} = (t-s)(1 - \cos\vartheta)$.

Now we calculate the electromagnetic field momentum

$$p_{\text{em}}^{\mu} = \int_{\Sigma_t} d\sigma_0 T^{0\mu} \tag{4.5}$$

where an integration hypersurface $\Sigma_t = \{x \in \mathbb{M}_4 : x^0 = t\}$ is a surface of constant t.

The surface element is given by $d\sigma_0 = \sqrt{-g}\, ds\, d\vartheta\, d\phi$ where

$$\sqrt{-g} = (t-s)^2 \sin\vartheta (1 - \cos\vartheta) \tag{4.6}$$

is the determinant of metric tensor of Minkowski space viewed in curvilinear coordinates (4.1). In these coordinates the components of the electromagnetic field's stress-energy tensor (3.7) have the form:

$$4\pi T^{00} = q^2 \frac{a^2(s)}{(t-s)^2 (1 - \cos\vartheta)^4} \tag{4.7}$$

$$4\pi T^{0i} = q^2 \frac{a^2(s)\omega_{ii'}n^{i'}}{(t-s)^2(1-\cos\vartheta)^4}. \tag{4.8}$$

The angular integration results the radiated energy-momentum:

$$p_{\text{em}}^0 = \frac{q^2}{2} I_0 \int_{-\infty}^t ds\, a^2(s) \qquad p_{\text{em}}^i = \frac{q^2}{2} I_1 \int_{-\infty}^t ds\, a^2(s) v^i(s) \tag{4.9}$$

where factors I_n diverge:

$$I_0 := \int_0^{\pi} d\vartheta \frac{\sin\vartheta}{(1-\cos\vartheta)^3} = -\frac{1}{8} + \lim_{\vartheta \to 0} \frac{1}{2(1-\cos\vartheta)^2} \tag{4.10}$$

$$I_1 := \int_0^{\pi} d\vartheta \frac{\sin\vartheta \cos\vartheta}{(1-\cos\vartheta)^3} = \frac{3}{8} - \lim_{\vartheta \to 0}\left[\frac{1}{1-\cos\vartheta} - \frac{1}{2(1-\cos\vartheta)^2}\right].$$

Similarly, the computation of the electromagnetic field angular momentum which flows across the hyperplane Σ_t gives rise to the divergent quantities:

$$M_{\text{em}}^{0i} = \frac{q^2}{2} I_1 \int_{-\infty}^t ds\, a^2(s) s v^i(s) - \frac{q^2}{2} I_0 \int_{-\infty}^t ds\, a^2(s) z^i(s) \tag{4.11}$$

$$M_{\text{em}}^{ij} = \frac{q^2}{2} I_1 \int_{-\infty}^t ds\, a^2(s) \left[z^i(s)v^j(s) - z^j(s)v^i(s)\right]. \tag{4.12}$$

The energy-momentum (4.9) and the angular momentum (4.11) and (4.12) of electromagnetic field generated by the accelerated photon-like charge tend to infinity in the direction of particle's velocity at the instant of emission. The divergent terms are not bound terms which should be absorbed by corresponding particle characteristics within the renormalization procedure. Indeed, they do not depend on the distance from the particle's world line. Secondly, the energy-momentum and the angular momentum accumulate with time at the observation hyperplane Σ_t (see figure 3). Hence the divergent Noether quantities cannot be referred to an electromagnetic "cloud" which is permanently attached to the charge and is carried along with it.

Changes in energy-momentum and angular momentum radiated by accelerated charge should be balanced by changes in already renormal-
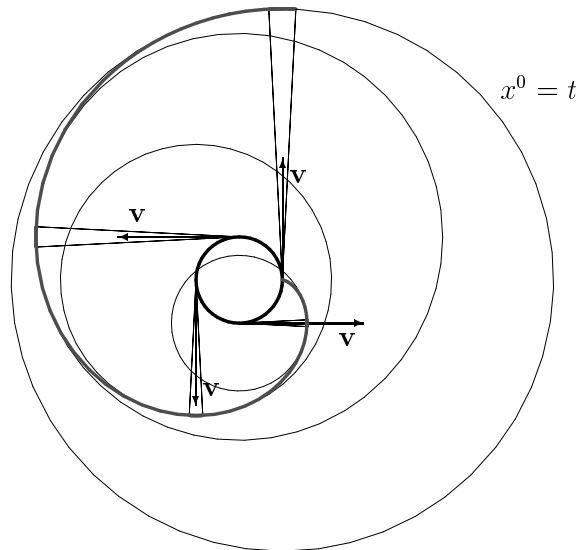
$x^0 = t$

Figure 3. The bold circle pictures the trajectory of a photon-like charge. The others are spherical wave fronts (4.4) viewed in the observation hyperplane $\Sigma_t = \{x \in \mathbb{M}_4 : x^0 = t\}$. The circling photon-like charge radiates infinite rates of energy-momentum and angular momentum in the direction of its velocity $\mathbf{v}$ at the instant of emission. The energy-momentum and angular momentum carried by electromagnetic field of accelerated charge tend to infinity on the spiral curve.

ized 4-momentum and angular momentum of the particle[1]. But the accelerated photon-like charge emits infinite amounts of radiation (see figure 3). To change the velocity of the massless charge the energy is necessary which is too large to be observed. Does it mean that there is no photon-like charges within an interaction area? In the Appendix A we sketch several electromagnetic fields which do not change the velocity of the massless charge.

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[1]If the massive charge coupled with electromagnetic field is considered [15,16], the balance equations yield the Lorentz-Dirac equation.

5. Massless charge within an interaction area

According to expression (3.6), non-accelerated photon-like charge does not generate the electromagnetic field. The evolution of the particle beyond an interaction area is determined by the Brink-Di Vecchia-Howe Lagrangian [14]

$$L = \frac{1}{2}e(\tau)\dot{z}^2. \tag{5.1}$$

The particle's 4-momentum $p_{\text{part}}^\mu = e(\tau)\dot{z}^\mu$ does not change with time:

$$\begin{aligned} \dot{p}_{\text{part}}^\mu &= \dot{e}(\tau)\dot{z}^\mu \\ &= 0. \end{aligned} \tag{5.2}$$

Since $\dot{z}^\mu \neq 0$, the Lagrange multiplier e does not depend on τ. We deal with a photon-like particle moving in the $\dot{\mathbf{z}}$-direction with frequency $\omega_0 = e_0\dot{z}^0$, such that its energy-momentum 4-vector can be written $p_{\text{part}}^\mu = (\omega_0, \omega_0\mathbf{k}), |\mathbf{k}| = 1$.

Further in this paper we shall use a disjoint union of hyperplanes $\Sigma_t = \{x \in \mathbb{M}_4 : x^0 = t\}$ for parametrization of the particle world line γ. We define $v^\alpha(t) = dz^\alpha(t)/dt$ as the 4-velocity; 4-acceleration $a^\alpha(t) = dv^\alpha(t)/dt$ looks as $(0, \dot{v}^i)$ in this parametrization. Since γ is degenerate (the condition $a^2 = 0$ at all points $z \in \gamma$ is fulfilled), the 4-acceleration vanishes.

When considering the system under the influence of an external device the change in particle's 4-momentum is balanced by an external force F_{ext}:

$$\begin{aligned} \dot{p}_{\text{part}}^\mu(t) &= \dot{e}(t)v^\mu \\ &= F_{\text{ext}}. \end{aligned} \tag{5.3}$$

(The 4-vector F_{ext} should be orthogonal to the 4-velocity.) This effective equation of motion is supplemented with the condition of absence of radiative damping. In other words, the external device admits a massless charge if and only if the components of null vector of 4-velocity do not change with time despite the influence of the external field. The conclusion is similar to that of Refs. [1,2].

When the photon-like charged particle moves in the external electromagnetic field $\hat{F}$, the Lorentz force balances the change in its 4-momentum:

$$\dot{e}v^\mu = qF^\mu{}_\nu v^\nu. \tag{5.4}$$

It is convenient to decompose $\hat{F}$ into an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$. Equation (5.4) is then rewritten as

$$\dot{e} = q(\mathbf{E} \cdot \mathbf{v}) \tag{5.5}$$
$$\dot{e}\mathbf{v} = q\mathbf{E} + q[\mathbf{v} \times \mathbf{B}]. \tag{5.6}$$

We have the following 4-th order algebraic equation on eigenvalues $\dot{e}$ [10]:

$$\dot{e}^4 + \dot{e}^2 q^2 \left(\mathbf{B}^2 - \mathbf{E}^2\right) - q^4 (\mathbf{B} \cdot \mathbf{E})^2 = 0. \tag{5.7}$$

In general, it possesses two real solutions [9,10]

$$\dot{e}_{\pm} = \pm q \sqrt{\left(\mathbf{E}^2 - \mathbf{B}^2 + \mu\right)/2} \tag{5.8}$$
$$\mu = \sqrt{\left(\mathbf{B}^2 - \mathbf{E}^2\right)^2 + 4(\mathbf{B} \cdot \mathbf{E})^2}.$$

The field admits a photon-like charge if and only if corresponding eigenvectors

$$\mathbf{v}_{\pm} = \frac{[\mathbf{E} \times \mathbf{B}] \pm (\lambda \mathbf{E} + \kappa \nu \mathbf{B})}{\sigma} \qquad \kappa = \mathrm{sgn}[(\mathbf{B} \cdot \mathbf{E})] \tag{5.9}$$

are of constant values. Here

$$\lambda = \sqrt{\left(\mathbf{E}^2 - \mathbf{B}^2 + \mu\right)/2} \quad \nu = \sqrt{\left(\mathbf{B}^2 - \mathbf{E}^2 + \mu\right)/2}$$
$$\sigma = \left(\mathbf{E}^2 + \mathbf{B}^2 + \mu\right)/2. \tag{5.10}$$

The expression (5.9) is obtained in [13, eq.(2.3)] where the model of magnetosphere of a rapidly rotating neutron star (pulsar) is elaborated. It defines the velocity of the massless charged particles which constitute the so-called "dynamical phase" of the gas of ultrarelativistic electrons and positrons moving in a very strong electromagnetic field of the pulsar. In Rylov's model [12,13] the massless charges as a limiting case of massive ones are considered. The reason is that the gradient of star's potential is much larger than the particle's rest energy $m_e c^2$.

6. Conclusions

Our consideration is founded on the Maxwell equations with point-like source which governs the propagation of the electromagnetic field produced by a photon-like charge. Unlike the massive case, it generates the *far* electromagnetic field which does not yield to divergent Coulomb-like self-energy. Hence the world line is null before renormalization as well as after this procedure. We choose Brink-Di Vecchia-Howe action [14] for a

bare particle moving on the world line which is proclaimed then to be lightlike.

A surprising feature of the study of the radiation back reaction in dynamics of the photon-like charge is that the Larmor term diverges whenever the charge is accelerated. Since the emitted radiation detaches itself from the charge and leads an independent existence, it cannot be absorbed within a renormalization procedure.

Inspection of the energy-momentum and angular momentum carried by the electromagnetic field of a photon-like charge reveals the reason why it is not yet detected (if it exists). Noninteracting massless charges do manifest themselves in no way. Any external electromagnetic field (including that generated by a detecting device) will attempt to change the velocity of the charged particle. Whenever the effort will be successful, the radiation reaction will increase extremely. In general, this circumstance forbids the presence of the photon-like charges within the interaction area.

Nevertheless, there exists the electromagnetic fields which do not change the velocities of the massless charged particles. For example, superposition of plane waves propagating along some base line admits the massless charges moving analogously. (But any disturbance annuls such a "loyalty".) It is worth noting that the quantum mechanical results [1,2] are in favour the conception that the external field distinguishes the directions of non-accelerating motions of photon-like charges (if they exist).

To survive photon-like charges need an extremely strong field of specific configuration, as that of the rotating neutron star (pulsar). In [12,13] the model of the pulsar magnetosphere is elaborated. It involves the so-called *dynamical phase* which consists of the massless charged particles moving with speed of light along some base line determined by the electromagnetic field of the star[2]. It is worth noting that the expression for the particles' velocity [13, eq.(2.2)] coincides with the solution (5.9) of the "massless" equations of motion derived in the present paper.

Equation (5.4) on eigenvalues and eigenvectors of the electromagnetic tensor governs the motion of charges in zero-mass approximation. This conclusion is in contradiction with that of Ref. [3] where the radiation back reaction is finite and the 5-th order differential equation determines the evolution of photon-like charge. The reason is that regularization approach to the radiation back reaction (smoothing the behaviour of the Lorentz force in the immediate vicinity of the particle's world line), em-

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[2]The massless approximation is meant where the gradient of star's potential is much larger than electron's rest energy.

ployed by Kazinski and Sharapov, is not valid in the case of the photon-like charged particle and its field. Indeed, the field diverges not only at point of world line but at all points of the ray in the direction of particle's 4-velocity taken at the instant of emission. The ray singularity is stronger that δ-like singularity of Green's function involved in [3] in the self-force expression. Hence integration over world line does not yield a finite part of the self force.

Conformal invariance of our particle plus (external) field system reinforce our conviction that the back-reaction force vanishes. Indeed, the appropriate renormalization procedure should preserve this symmetry property while the Brink-Di Vecchia-Howe action term does not contain a parameter to be renormalized. Therefore, the photon-like charge must not radiate.

Acknowledgments

Appendix A

Plane wave

In case of a plane wave moving in the positive $z-$direction, the electric and magnetic fields are related to each other as follows:

$$E_x = B_y \qquad E_y = -B_x \qquad E_z = B_z = 0. \qquad (A.1)$$

Since $\mathbf{B}^2 - \mathbf{E}^2$ as well as $(\mathbf{B} \cdot \mathbf{E})$ vanish, the eigenvalues' equation (5.7) get simplified:

$$\dot{e}^4 = 0. \qquad (A.2)$$

The eigenvector corresponding to the fourthly degenerate eigenvalue $\dot{e} = 0$ is defined by [9]

$$\mathbf{v} = \frac{[\mathbf{E} \times \mathbf{B}]}{\mathbf{B}^2} = \mathbf{n}_z. \qquad (A.3)$$

Hence the plane wave admits massless charges moving along z-line in the positive direction. Their frequencies do not change with time.

Uniform static electric field

When $\mathbf{B} = 0$ the equation (5.7) becomes

$$\dot{e}^4 - \dot{e}^2 q^2 \mathbf{E}^2 = 0. \qquad (A.4)$$

If $\dot{e} = 0$ then $\mathbf{E}$ vanishes (see equations (5.5) and (5.6)) and the charge's velocity is completely arbitrary (free particle).

The others are $\dot{e}_+ = q|\mathbf{E}|$ and $\dot{e}_- = -q|\mathbf{E}|$. The photon-like charge moves in the direction $\mathbf{n}_E = \mathbf{E}/|\mathbf{E}|$ or in the opposite one. Its 4-momentum

$$p_{\text{part}}^0 = \omega_0 \pm q|\mathbf{E}|t \qquad p_{\text{part}}^i = \pm\omega_0 n_E^i + qE^i t \qquad (A.5)$$

heavily depends on the time.

Constant magnetic field

When considering the magnetic field of constant value, the equation (5.7) looks as follows

$$\dot{e}^4 + \dot{e}^2 q^2 \mathbf{B}^2 = 0. \qquad (A.6)$$

The only real solution is the doubly degenerate trivial eigenvalue. Since $[\mathbf{v} \times \mathbf{B}] = 0$, massless charges move along the base line determined by $\mathbf{B}$. The magnetic field does not change their 4-momenta.

Orthogonal constant electric and magnetic fields

Since $(\mathbf{B} \cdot \mathbf{E}) = 0$, the basic equation (5.7) becomes

$$\dot{e}^4 + \dot{e}^2 q^2 \left(\mathbf{B}^2 - \mathbf{E}^2\right) = 0. \qquad (A.7)$$

Two unit three-vectors $\mathbf{v}$ which satisfy the *force-free approximation* [13, eq.(1.5)]

$$\mathbf{E} + [\mathbf{v} \times \mathbf{B}] = 0 \qquad (A.8)$$

correspond to the doubly degenerate eigenvalue $\dot{e} = 0$. After some algebra we arrive at

$$\mathbf{v}_\pm = \frac{[\mathbf{E} \times \mathbf{B}] \pm \mathbf{B}\sqrt{\mathbf{B}^2 - \mathbf{E}^2}}{\mathbf{B}^2}. \qquad (A.9)$$

The condition $(\mathbf{B} \cdot \mathbf{E}) = 0$ supplemented with the inequality $|\mathbf{E}| < |\mathbf{B}|$ defines the *capture surface* in Rylov's model of pulsar magnetosphere [13]. Massive particles (electrons and positrons) are captured in the immediate vicinity of this surface. Their kinetic energies vanish; they constitute the so-called *statical phase*. Nevertheless, the photon-like charges move across the capture surface with the velocity (A.9). The region of pulsar magnetosphere where there are the dynamical phase and the statical phase is called *leaky capture region* in Refs. [12,13].

If $|\mathbf{E}| > |\mathbf{B}|$, then two eigenvalues $\dot{e} = \pm\sqrt{\mathbf{E}^2 - \mathbf{B}^2}$ are valuable. Corresponding eigenvectors are

$$\mathbf{v}_\pm = \frac{[\mathbf{E} \times \mathbf{B}] \pm \mathbf{E}\sqrt{\mathbf{E}^2 - \mathbf{B}^2}}{\mathbf{E}^2}. \qquad (A.10)$$

Having integrated $\dot{e}$ over time variable, we are sure that the 4-momenta of photon-like charges moving with the velocities (A.10) depend on time t:

$$p^{\mu}_{\text{part}} = \left[\omega_0 \pm \sqrt{\mathbf{E}^2 - \mathbf{B}^2}\, t\right] \mathbf{v}_{\pm}. \qquad (A.11)$$

Appendix B. Conformal invariance of the effective equation of motion

According to [17,18], conformal group $\mathcal{C}(1,3)$ consists of Poincaré transformations (time and space translations, space and mixed space-time rotations), dilatations

$$x^{'\mu} = e^{\theta} x^{\mu} \qquad (B.1)$$

and conformal transformations

$$x^{'\mu} = \frac{x^{\mu} - b^{\mu}(x \cdot x)}{D}, \qquad D = 1 - 2(x \cdot b) + (x \cdot x)(b \cdot b). \qquad (B.2)$$

(The scalar θ and 4-vector b are group parameters.)

The components of electromagnetic field are transformed as follows:

$$F_{\alpha\beta} = e^{2\theta} F'_{\alpha\beta}, \qquad F_{\alpha\beta} = F'_{\mu\nu} \Omega^{\mu}{}_{\alpha} \Omega^{\nu}{}_{\beta} \qquad (B.3)$$

where matrix

$$\Omega^{\mu}{}_{\alpha} : \quad = \quad \frac{\partial x^{'\mu}}{\partial x^{\alpha}} = D^{-1} \lambda^{\mu}{}_{\beta}(x'') \lambda^{\beta}{}_{\alpha}(x), \quad x'' = \frac{x}{(x \cdot x)} - b,$$

$$\lambda^{\beta}{}_{\alpha}(x) \quad = \quad \delta^{\beta}{}_{\alpha} - \frac{2x^{\beta} x_{\alpha}}{(x \cdot x)} \qquad (B.4)$$

satisfies the condition

$$\eta_{\mu\nu} \Omega^{\mu}{}_{\alpha} \Omega^{\nu}{}_{\beta} = D^{-2} \eta_{\alpha\beta}. \qquad (B.5)$$

Since

$$\dot{z}^{'\mu} = e^{\theta} \dot{z}^{\mu} \qquad \dot{z}^{'\mu} = \Omega^{\mu}{}_{\alpha} \dot{z}^{\alpha} \qquad (B.6)$$

the Lagrange multiplier $e(\tau)$ involved in the Brink-Di Vecchia-Howe action term (3.8) transforms as

$$e(\tau) = e^{2\theta} e'(\tau), \qquad e(\tau) = D^{-2} e'(\tau). \qquad (B.7)$$

Direct calculation shows, that the effective equation of motion (5.4) is invariant with respect to dilatation (B.1) and conformal transformation (B.2).

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