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ELIMINATION OF THE FIELD DEGREES OF FREEDOM IN RELATIVISTIC SYSTEM OF POINT-LIKE CHARGES

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Виключення польових ступенів вільности у релятивістичній системі точкових зарядів

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Анотація. Розглянуто гамільтонове формулювання релятивістичної системи заряджених частинок з електромагнетним полем у діраківських миттєвій та фронтовій формах динаміки. Знайдена канонічна реалізація алґебри Пуанкаре у термінах калібрувально-інваріянтних змінних. Розроблена процедура виключення польових ступенів вільности у першому наближенні за константою взаємодії в рамках діраківської теорії в'язей. Одержано ґенератори Пуанкаре у термінах частинкових змінних. Досліджено співвідношення між ґенераторами у миттєвій та фронтових формах. Гамільтоніян у миттєвій формі у слабкорелятивістичному наближенні зведено до гамільтоніяну Дарвіна.

Elimination of the field degrees of freedom in relativistic system of poin-like charges

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Abstract. The Hamiltonian formulation of relativistic system of charged particles plus electromagnetic field in the Dirac's instant and front forms of dynamics is considered. The canonical realization of the Poincaré algebra in the terms of gauge-invariant variables is found. The procedure of elimination of the field degrees of freedom within the framework of the Dirac constraint theory is elaborated up to the first order in the coupling constant. The Poincaré generators in the terms of particle variables are obtained. The relations between the instant form generators and front form ones are examined. The instant form Hamiltonian in the weak-relativistic approximation results in the Darwin Hamiltonian.

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1. Introduction

The field-theoretical description of relativistic system of charged particles is formulated by means of the electromagnetic 4–potential $A_\mu(x)$, $x \in \mathbb{M}_4$, over the Minkowski space–time[1]. Such a system is described by the singular Lagrangian, so the Hamiltonian formulation of dynamics demands the use of the Dirac theory of constraints. Isolating the gauge degrees of freedom and finding a description in the terms of physical variables were initiated by Dirac [1]. A number of papers is devoted to the search of Dirac's observables (physical variables) for electrodynamics [2] and Yang–Mills theory [3]. Such gauge-invariant descriptions can be applied to constructing statistical and quantum mechanics of the "particle+field" system. But often it is desirable to exclude the field degrees of freedom and explore the features of such a system in the terms of particle variables. The elimination of the field is based on substitution of a solution to the field equations into the equations of motion of particles. This procedure has been carried out in the classical [4–6] and quantum [7] domains. It yields a description in the terms of the direct interaction between particles. Since the formal solution of the field equations depends on choice of the Green's function (advanced, retarded or symmetric), physically different theories are obtained.

Usually, exclusion of the field is performed in action integral of the system. In the classical relativistic mechanics, substitution of the formal solution with the symmetric Green's function into the action gives the Fokker–type action [5]. Wheeler–Feynman electrodynamics is an instance of such a theory [8]. Nonlocality of the action leads to serious difficulties in transition to the Hamiltonian description. Possible ways to perform Hamiltonization of the system with nonlocal action by means of approximation methods have been studied in literature [6,9,10]. But we shall consider an alternative way which consists in elimination of the field degrees of freedom *after* transition to the Hamiltonian description [4,11]. It is true that the field equations in the Hamiltonian picture are nonlinear, so the use of the perturbation scheme is required. Also the problem of Green's function lasts there. Fortunately, difference between the advanced, retarded and symmetric Green's functions does not appear in the linear approximation in the coupling constant. Attempt to carry out the similar scheme is presented in work [4], where charge Q_a of ath particle is described by Grassman variables, so that $Q_a^2 = 0$. Clearly, this approach corresponds to the first-order approximation in the coupling constant.

[1]The Minkowski space–time $\mathbb{M}_4$ is endowed with a metric $\|\eta_{\mu\nu}\| = \mathrm{diag}(1,-1,-1,-1)$. The Greek indices $\mu,\nu,\ldots$ run from 0 to 3; the Latin indices from the middle of alphabet, $i,j,k,\ldots$ run from 1 to 3 and both types of indices are subject of the summation convention. The Latin indices from the beginning of alphabet, a,b, label the particles and run from 1 to N. The sum over such indices is indicated explicitly. The velocity of light c is equal to unity.

We are concerned with exclusion of the electromagnetic field degrees of freedom in the classical Hamiltonian picture. In the present paper we start from an action functional S of the classical relativistic system of point-like charges coupled with the electromagnetic field. The system has two kinds of the gauge freedom which are known from Dirac's works. The first is related with arbitrariness in the parametrization of particle world lines (chronometrical freedom). Reduction of this type of freedom requires the choice of a $3+1$ splitting of the Minkowski space–time. We build our formalism into three-dimensional description corresponding to the Dirac's instant and front form of dynamics [12]. The second freedom is generated by the proper gauge transformations of electromagnetic potentials. At first, treating the field and particle variables on equal level, we find the canonical realization of the Poincaré algebra in a given form of dynamics. Then we isolate the gauge degrees of freedom and formulate the dynamics of our system in a gauge-invariant manner. Further, the procedure of elimination of the field is performed in three steps: (i) one finds the solution to the field equations in the first order in the coupling constant; (ii) we transit to new canonical variables which contain the free field corresponding to the solution to homogeneous field equation; (iii) the free field is fixed by imposing additional constraints. These constraints are eliminated by means of the Dirac's method. In such a way, we obtain the canonical realization of the Poincaré algebra in the terms of particle variables. Besides, it is important to demonstrate that the instant and the front form descriptions are related by means of a canonical transformation. The obtained Poincaré generators are studied in weak-relativistic approximation up to the order c^{-2}.

The paper is organized as follows. In section 2 we fix the form of relativistic dynamics of the system of particles plus field and write down the Lagrangian and the conserved quantities. Then, in the instant form of dynamics, we reformulate the system in the terms of canonical variables. We find the constraints produced by the gauge invariance of the action and build the canonical realization of the Poincaré algebra. We eliminate the gauge degrees of freedom by suitable canonical transformations and write down the generators in the terms of gauge-invariant variables. In section 3 we solve the field equations up to the first-order in the coupling constant. Also this section is devoted to the reduction of the field variables. The Poincaré generators depending on the canonical particle variables are obtained. In section 4 we apply step by step our procedure to the system in the front form dynamics and demonstrate special features of the constraint structure. In section 5 we compare the generators corresponding to the instant and front forms of dynamics. In section 6 the obtained instant form generators in approximation up to c^{-2} are considered.

2. Hamiltonian Description in the Instant Form of Dynamics

We consider a system of N point-like charged particles which are described by world lines in the Minkowski space–time $\gamma_a : \tau \mapsto x_a^\mu(\tau)$. An interaction between charges is assumed to be mediated by an electromagnetic field $\tilde{F}_{\mu\nu}(x) = \partial_\mu \tilde{A}_\nu(x) - \partial_\nu \tilde{A}_\mu(x)$ with the electromagnetic potential $\tilde{A}_\mu(x)$; $\partial_\nu \equiv \partial/\partial x^\nu$. Dynamics of the system are completely determined by the following action [13–15]

$$\begin{aligned} S = & - \sum_{a=1}^N \int d\tau_a \left\{ m_a \sqrt{u_a^2(\tau_a)} + e_a u_a^\nu(\tau_a) \tilde{A}_\nu[x_a(\tau_a)] \right\} \\ & - \frac{1}{16\pi} \int d^4x \tilde{F}_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x), \end{aligned} \tag{2.1}$$

where m_a and e_a are the mass and the charge of ath particle, respectively, $u_a^\mu(\tau_a) = dx_a^\mu(\tau_a)/d\tau_a$.

Action (2.1) is manifestly Poincaré-invariant. Its invariance leads to the conservation of symmetric energy–momentum tensor [13,15] which is given by

$$\begin{aligned} \theta^{\mu\nu}(x) = & \sum_{a=1}^N \int m_a \frac{u_a^\mu(\tau_a) u_a^\nu(\tau_a) \delta^4(x - x_a(\tau_a))}{\sqrt{u_a^2(\tau_a)}} d\tau_a \\ & + \frac{1}{4\pi} \left(-\tilde{F}^{\mu\lambda}(x) \tilde{F}^\nu{}_\lambda(x) + \frac{1}{4} \tilde{F}_{\lambda\sigma}(x) \tilde{F}^{\lambda\sigma}(x) \eta^{\mu\nu} \right). \end{aligned} \tag{2.2}$$

Also the action is invariant under two kinds of the gauge transformation. The first consists in an arbitrary parametrization of particle world lines; the second is gauge transformation $\tilde{A}_\mu(x) \mapsto \tilde{A}_\mu(x) + \partial_\mu \Lambda(x)$. The general scheme of exclusion of the gauge freedoms is discussed in Ref. 16. Here we apply the suggested scheme to finding canonical realization of the Poincaré generators in the terms of Dirac's observables in a given form of dynamics.

Using the concept of the forms of relativistic dynamics,[12] we reduce the chronometrical freedom. The form of relativistic dynamics is defined by the one-parameter foliation $\Sigma = \{\Sigma_t \mid t \in \mathbb{R}\}$ of the Minkowski space–time with the space-like or isotropic hypersurfaces[2] $\Sigma_t = \{x \in \mathbb{M}_4 \mid x^0 = \varphi(t, \mathbf{x}),\ \mathbf{x} = (x^1, x^2, x^3)\}$. The foliation Σ specifies a certain $3+1$ splitting of the Minkowski space–time $f : \mathbb{M}_4 \to \mathbb{R} \times \Sigma_0$ which is determined by transformation

$$f : (x^0, \mathbf{x}) \mapsto (\varphi(t, \mathbf{x}), \mathbf{x}). \tag{2.3}$$

In our case the geometrical definition of the form of dynamics permits us to perform replacement (2.3) in the action.

The parametric equation of the world line in a given form of dynamics is

$$x^0 = x_a^0(t) = \varphi(t, \mathbf{x}_a(t)) \equiv \varphi_a, \qquad x^i = x_a^i(t). \tag{2.4}$$

The variable t is treated as an evolution parameter of the particle system.

On the other hand, (2.3) induces a transformation of the field variables $A_\mu = \tilde{A}_\mu \circ f^{-1}$, $F_{\mu\nu} = \tilde{F}_{\mu\nu} \circ f^{-1}$.

Let us put

$$\varphi_t(t, \mathbf{x}) \equiv \frac{\partial \varphi(t, \mathbf{x})}{\partial t} > 0, \quad \varphi_i(t, \mathbf{x}) \equiv \frac{\partial \varphi(t, \mathbf{x})}{\partial x^i}. \tag{2.5}$$

Accounting that the Jacobian of transformation (2.3) is equal to $\varphi_t(t, \mathbf{x})$ and introducing notations

$$\begin{aligned} \mathsf{E}_i = A_{0,i} - \left(\dot{A}_i + \dot{A}_0 \varphi_i \right) \varphi_t^{-1}, \quad \mathsf{H}_{ij} = F_{ij} + \varphi_t^{-1} (\dot{A}_i \varphi_j - \dot{A}_j \varphi_i), \\ F_{ij} = \partial_i A_j - \partial_j A_i, \end{aligned} \tag{2.6}$$

we can rewrite action (2.1) into the single-time form

$$S = \int L dt \tag{2.7}$$

with the Lagrangian function

$$\begin{aligned} L = & - \sum_{a=1}^N \left[m_a \sqrt{(D\varphi_a)^2 - \dot{\mathbf{x}}_a^2} + e_a [A_0(t, \mathbf{x}_a) D\varphi_a + A_i(t, \mathbf{x}_a) \dot{x}_a^i] \right] \\ & - \int \frac{\varphi_t}{16\pi} (2\mathsf{E}_i \mathsf{E}^i + \mathsf{H}_{ij} \mathsf{H}^{ij}) d^3x. \end{aligned} \tag{2.8}$$

[2] Generally, the equation of the hypersurface may depend on momenta. In this case another technique is required [3].

Here $\dot{x}_a^i(t) = dx_a^i(t)/dt$ and $D = d/dt = \partial/\partial t + \sum_{a=1}^{N} \dot{x}_a^i \partial/\partial x_a^i$.

The dynamical variables of our problem are the functions $\mathbf{x}_a(t)$, $A^\mu(t, \mathbf{x})$ and their first-order derivatives $\dot{\mathbf{x}}_a(t)$ and $\dot{A}^\mu(t, \mathbf{x})$ with respect to the evolution parameter.

Conservation of energy–momentum tensor (2.2) leads to ten conserved quantities defined on Σ_t:

$$P^\mu = \int dP^\mu, \quad M^{\mu\nu} = \int (x^\mu dP^\nu - x^\nu dP^\mu), \tag{2.9}$$

where dP^μ is given by

$$dP^\mu = [\theta^{\mu 0}(t, \mathbf{x}) - \theta^{\mu i}(t, \mathbf{x})\varphi_i(t, \mathbf{x})]\varphi_t(t, \mathbf{x}) d^3x. \tag{2.10}$$

We intend to consider the two cases of the hypersurface in the Minkowski space–time: space-like and isotropic. We illustrate both cases in the instance of the Dirac's instant and front forms of relativistic dynamics. As shown in Ref. 16, in these forms of dynamics of the "particle+field" system sets of constraints are different. We shall construct the Hamiltonian description and find the canonical realization of the Poincaré algebra in a given form of relativistic dynamics. The particle and field degrees of freedom will be treated on equal rights. Further, we aim to suppress the gauge degrees of freedom and to exclude physical fields with the help of the Dirac theory of constraints.

In the case of the instant form of dynamics we put $x^0 = t$. Then the canonical momenta of our problem are given by

$$p_{ai}(t) = -\frac{\partial L(t)}{\partial \dot{x}_a^i(t)} = \frac{m_a \dot{x}_{ai}(t)}{\sqrt{1 - \dot{\mathbf{x}}_a^2(t)}} + e_a A_i(t, \mathbf{x}_a(t)), \tag{2.11}$$

$$E^i(t, \mathbf{x}) = \frac{\delta L(t)}{\delta \dot{A}_i(t, \mathbf{x})} = \frac{1}{4\pi} \mathsf{E}^i(t, \mathbf{x}), \tag{2.12}$$

$$E^0(t, \mathbf{x}) = \frac{\delta L(t)}{\delta \dot{A}_0(t, \mathbf{x})} = 0. \tag{2.13}$$

The basic Poisson brackets are

$$\{x_a^i(t), p_{bj}(t)\} = -\delta_{ab}\delta_j^i, \quad \{A_\mu(t, \mathbf{x}), E^\nu(t, \mathbf{y})\} = \delta_\mu^\nu \delta^3(\mathbf{x} - \mathbf{y}); \tag{2.14}$$

all other brackets vanish. Equation (2.13) is a primary constraint.

The canonical Hamiltonian of our system is defined as

$$H = -\sum_{a=1}^{N} p_{ai}\dot{x}_a^i + \int E^\mu \dot{A}_\mu d^3x - L. \tag{2.15}$$

The immediate calculations give

$$H = \sum_{a=1}^{N} \left[\sqrt{m_a^2 + [\mathbf{p}_a - e_a \mathbf{A}(\mathbf{x}_a)]^2} + e_a A_0(\mathbf{x}_a) \right] + \int \left(\frac{1}{16\pi} F_{ij}F_{ij} + 2\pi E^i E^i - A_0 \partial_i E^i \right) d^3x. \tag{2.16}$$

Constraint (2.13) reflects the gauge invariance of the action S. The preservation of (2.13) in time produces the only secondary constraint (Gauss law). The obtained set of constraints [2]

$$E^0 \approx 0, \qquad \Gamma \equiv \varrho - \partial_i E^i \approx 0 \tag{2.17}$$

belongs to the first class. Here $\approx$ means "weak equality" in the sense of Dirac and charge density is defined as

$$\varrho(t, \mathbf{x}) = \sum_{a=1}^{N} e_a \delta^3(\mathbf{x} - \mathbf{x}_a(t)). \tag{2.18}$$

The ten Poincaré generators in the terms of the particle and field canonical variables are

$$P^0 = \sum_{a=1}^{N} \sqrt{m_a^2 + [\mathbf{p}_a - e_a \mathbf{A}(\mathbf{x}_a)]^2} + \int \left(\frac{1}{16\pi} F_{ij}F_{ij} + 2\pi E^i E^i \right) d^3x, \tag{2.19}$$

$$P^k = \sum_{a=1}^{N} [p_a^k - e_a A^k(\mathbf{x}_a)] + \int E^l F^{lk} d^3x, \tag{2.20}$$

$$M^{k0} = \sum_{a=1}^{N} x_a^k \sqrt{m_a^2 + [\mathbf{p}_a - e_a \mathbf{A}(\mathbf{x}_a)]^2} + \int \left(\frac{1}{16\pi} F_{ij}F_{ij} + 2\pi E^i E^i \right) x^k d^3x - tP^k, \tag{2.21}$$

$$M^{ik} = \sum_{a=1}^{N} [x_a^i(p_a^k - e_a A^k(\mathbf{x}_a)) - x_a^k(p_a^i - e_a A^i(\mathbf{x}_a))] + \int \left(x^i E^l F^{lk} - x^k E^l F^{li} \right) d^3x. \tag{2.22}$$

We can check directly that they satisfy the commutation relations of the Poincaré algebra in the terms of the Poisson brackets (2.14):

$$\{P^\mu, P^\nu\} = 0, \quad \{P^\mu, M^{\nu\lambda}\} = \eta^{\mu\nu}P^\lambda - \eta^{\mu\lambda}P^\nu,$$

$$\{M^{\mu\nu}, M^{\lambda\sigma}\} = -\eta^{\mu\lambda}M^{\nu\sigma} + \eta^{\nu\lambda}M^{\mu\sigma} - \eta^{\nu\sigma}M^{\mu\lambda} + \eta^{\mu\sigma}M^{\nu\lambda}. \quad (2.23)$$

In our work we are interested in reformulation of dynamics and the Poincaré generators in the terms of canonical particle variables with preservation of commutation relations (2.23).

First of all let us eliminate the gauge degrees of freedom which are subject of the gauge transformation of electromagnetic potential. Generally, such a reduction consists in decoupling the gauge-varied and gauge-invariant variables by means of suitable Shanmugadhasan transformation [17]. In our problem we have to perform canonization of the first class constraints E^0 and Γ and to determine the canonical basis of Dirac's observables. One immediately sees that A_0, E^0 constitute a pair of conjugated gauge canonical variables. Then it needs to transit from (E^1, E^2, E^3) to (Γ, b^1, b^2), where b^1 and b^2 must be gauge-invariant field canonical momenta, and to find the conjugated variables (Q, a_1, a_2). Let us make the Hodge decomposition [2,14]

$$E^i = E^i_\perp + \partial^i\Delta^{-1}(\Gamma - \varrho) \quad (2.24)$$

with the use of the following projectors

$$P^i{}_j \equiv \delta^i_j + \partial^i\Delta^{-1}\partial_j, \quad \Pi^i{}_\alpha \equiv \delta^i_\alpha - \delta^i_3\frac{\partial_\alpha}{\partial_3}, \quad \alpha = 1, 2. \quad (2.25)$$

The inverse operators to $\Delta = \partial_i\partial_i$ and ∂_3 are defined so that

$$\Delta^{-1}\delta^3(\mathbf{x}) = -\frac{1}{4\pi|\mathbf{x}|},$$

$$\frac{1}{\partial_3}\delta^3(\mathbf{x}) \equiv \left(\frac{\partial}{\partial x^3}\right)^{-1}\delta^3(\mathbf{x}) = \frac{1}{2}\delta(x^1)\delta(x^2)\mathrm{sgn}(x^3). \quad (2.26)$$

Vector $E^i_\perp \equiv P^i{}_jE^j$, whose components are subject of relation $\partial_iE^i_\perp = 0$, can be expressed in the terms of independent variables as follows

$$E^i_\perp = \frac{\Pi^i{}_\alpha b^\alpha}{\sqrt{4\pi}}, \quad b^\alpha = \sqrt{4\pi}E^\alpha, \quad \alpha = 1, 2. \quad (2.27)$$

In order to decouple the gauge and gauge-invariant variables, we perform the canonical transformation

$$((x^i_a, p_{ai}), (A_\mu, E^\mu)) \mapsto ((x^i_a, \pi_{ai}), (a_\alpha, b^\alpha), (Q, \Gamma), (A_0, E^0)) \quad (2.28)$$

determined by the generating functional

$$F = \sum_{a=1}^N x^i_a p_{ai} - \int A_i\left[\frac{\Pi^i{}_\alpha b^\alpha}{\sqrt{4\pi}} + \partial^i\Delta^{-1}(\Gamma - \varrho)\right]d^3x. \quad (2.29)$$

One finds

$$Q = -\frac{\delta F}{\delta\Gamma} = \Delta^{-1}\partial_iA_i, \quad a_\alpha = -\frac{\delta F}{\delta b^\alpha} = \Pi^i{}_\alpha\frac{A_i}{\sqrt{4\pi}}, \quad (2.30)$$

$$\pi_{ai} = \frac{\partial F}{\partial x^i_a} = p_{ai} - e_a\partial_iQ(x_a). \quad (2.31)$$

Note that this transformation changes the particle momenta. From (2.30) we obtain $A_i = A^\perp_i + \partial_iQ$. It turns out that the transverse part $A^\perp_i$ of A_i is related with a_α as

$$A^\perp_i = \sqrt{4\pi}P_i{}^\alpha a_\alpha, \qquad \alpha = 1, 2. \quad (2.32)$$

Therefore the part of the phase space related to the transverse part of the potential and the corresponding momenta is parametrized by means of (a_α, b^α) which constitute the elements of some Darboux basis. Hereafter it is convenient to describe the field by the functions $A^\perp_i$, $E^i_\perp$ of canonical variables a_α and b^α.

Taking into account (2.17), we find after transformation (2.28) that the canonical Hamiltonian become

$$\begin{aligned} H &= \sum_{a=1}^N\sqrt{m_a^2 + [\boldsymbol{\pi}_a - e_a\mathbf{A}_\perp(\mathbf{x}_a)]^2} - 2\pi\int\varrho\Delta^{-1}\varrho d^3x \\ &\quad + \int\left(\frac{1}{16\pi}F^\perp_{ij}F^\perp_{ij} + 2\pi E^i_\perp E^i_\perp\right)d^3x, \end{aligned} \quad (2.33)$$

where $F^\perp_{ij} = \partial_iA^\perp_j - \partial_jA^\perp_i$. As it must be, H does not depend on gauge variables A_0, Q. The second term of H is reformulated without infinite self-energies by means of mass renormalization [14]. It leads to mutual Coulomb interaction.

After canonical transformation (2.28) is performed, we arrive at the following form of Poincaré generators (2.19)–(2.22):

$$P^0 = H, \quad P^k = \sum_{a=1}^N\pi^k_a + \int E^l_\perp\partial^kA^\perp_l d^3x, \quad (2.34)$$

$$\begin{aligned} M^{k0} &= \sum_{a=1}^N x^k_a\sqrt{m_a^2 + [\boldsymbol{\pi}_a - e_a\mathbf{A}_\perp(\mathbf{x}_a)]^2} - 2\pi\int x^k\varrho\Delta^{-1}\varrho d^3x \\ &\quad + \int x^k\left(\frac{1}{16\pi}F^\perp_{ij}F^\perp_{ij} + 2\pi E^i_\perp E^i_\perp + 4\pi E^l_\perp\partial_l\Delta^{-1}\varrho\right)d^3x \\ &\quad - tP^k, \end{aligned} \quad (2.35)$$

$$M^{ik} = \sum_{a=1}^{N}(x_a^i\pi_a^k - x_a^k\pi_a^i) + \int \left(x^i E_\perp^l \partial^k A_l^\perp - x^k E_\perp^l \partial^i A_l^\perp\right) d^3x - \int (A_\perp^i E_\perp^k - A_\perp^k E_\perp^i) d^3x. \quad (2.36)$$

They also are expressed only in the terms of observables. It demonstrates explicitly that gauge-varied variables A_0, E^0, Q and Γ do not influence the dynamical properties of the system.

3. Elimination of the Physical Field Degrees of Freedom

In the previous section we have done reduction of the gauge degrees of freedom which are related with two kinds of the action invariance. The chronometrical freedom has been eliminated by means of fixation $x^0 = \varphi(t, \mathbf{x})$ and introducing evolution parameter t. Let us note that the descriptions of our system with different $\varphi(t, \mathbf{x})$ must be physical equivalent. The gauge freedom of the electromagnetic field has been excluded with the help of transition to the description in the terms of observables. Below we consider another type of reduction which eliminates physical degrees of freedom of the field. As a result, one obtains description of our system in the terms of particle variables. Such a reformulation is especially effective, when the free radiation is not essential.

Our procedure of the field reduction has three steps. (i) It is necessary to find a solution to the field equations which are complicated in the Hamiltonian mechanics. We use the coupling constant expansion coming to the problem of choice of Green's function. The same problem arises in the Lagrangian formalism. In particular, it is known that substitution of the formal solution of the field equations with the symmetric Green's function into the action leads to compensation of the half of interaction by the field part of action. We expect that this fact has to be reflected on the Hamiltonian level. But the advanced, retarded and symmetric solutions coincide in the linear approximation in the coupling constant. The general solution must be a sum of the free field, which satisfies the homogeneous equation, and the solution to the inhomogeneous equation determined by the point-like sources. (ii) We intend to perform a canonical transformation after that the free-field terms become the canonical variables. (iii) We put the free-field variables equal to zero. The obtained canonical second class constraints are eliminated by using respective Dirac bracket which coincides with the particle Poisson bracket. The use of the Dirac bracket allows us to exclude the field

from generators.

Taking into account relations (2.27), (2.32), let us write down the Hamiltonian equations of motion for $A_i^\perp$ and $E_\perp^i$:

$$\dot{A}_i^\perp = 4\pi E_\perp^i, \quad (3.1)$$

$$\dot{E}_\perp^i = -\sum_{a=1}^{N} e_a \frac{\pi_a^j - e_a A_\perp^j(\mathbf{x}_a)}{\sqrt{m_a^2 + [\boldsymbol{\pi}_a - e_a \mathbf{A}_\perp(\mathbf{x}_a)]^2}} P^i{}_j \delta^3(\mathbf{x} - \mathbf{x}_a) + \frac{\Delta}{4\pi} A_i^\perp. \quad (3.2)$$

Finding solution to these equations requires the use of approximation scheme. Let us rewrite equations (3.1), (3.2) in the following form:

$$\Box_{\text{in}}\, A_i^\perp = 4\pi P_i{}^j j_j, \quad E_\perp^i = \frac{1}{4\pi}\dot{A}_i^\perp. \quad (3.3)$$

with the current density in the linear approximation in the coupling constant:

$$j^i(t, \mathbf{x}) = \sum_{a=1}^{N} e_a v_a^i \delta^3(\mathbf{x} - \mathbf{x}_a(t)). \quad (3.4)$$

Here $v_a^i = \pi_a^i / \sqrt{m_a^2 + \boldsymbol{\pi}_a^2}$ is the free-particle velocity which is time independent in our approximation.

Operator $\Box_{\text{in}} \equiv \partial_t^2 - \Delta$ is defined as the d'Alambertian $\Box \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu$ in the instant form of dynamics ("in"). Generally, the form of d'Alambertian is determined by replacement $(x^0, \mathbf{x}) \mapsto (t, \mathbf{x})$ given by equation of hypersurface in the Minkowski space–time (see Appendix A).

At this point we have come to the linear inhomogeneous field equations which can be solved by means of the Green's function method. The general solution of (3.3) is presented as

$$A_i^\perp(t, \mathbf{x}) = \phi_i^\perp(t, \mathbf{x}) + \mathcal{A}_i^\perp(t, \mathbf{x}), \quad E_\perp^i(t, \mathbf{x}) = \chi_\perp^i(t, \mathbf{x}) + \mathcal{E}_\perp^i(t, \mathbf{x}). \quad (3.5)$$

According to (2.27) and (2.32), $\phi_i^\perp(t, \mathbf{x})$ and $\chi_\perp^i(t, \mathbf{x})$ are connected with independent free field variables $\phi_\alpha(t, \mathbf{x})$, $\chi^\alpha(t, \mathbf{x})$ by means of relations:

$$\phi_i^\perp(t, \mathbf{x}) = (4\pi)^{1/2} P_i{}^\alpha \phi_\alpha(t, \mathbf{x}), \qquad \chi_\perp^i(t, \mathbf{x}) = (4\pi)^{-1/2} \Pi^i{}_\alpha \chi^\alpha(t, \mathbf{x}). \quad (3.6)$$

These functions are the general solutions to the corresponding homogeneous equations:

$$\Box_{\text{in}}\, \phi_i^\perp = 0, \quad \chi_\perp^i = \frac{1}{4\pi}\dot{\phi}_i^\perp. \quad (3.7)$$

We can present solutions $\mathcal{A}_i^\perp$ and $\mathcal{E}_\perp^i$ to inhomogeneous equations as

$$\mathcal{A}_i^\perp = P_i{}^j \mathcal{A}_j, \quad \mathcal{E}_\perp^i = P_j{}^i \mathcal{E}^j. \quad (3.8)$$

Let us calculate functions $\mathcal{A}_j$ and $\mathcal{E}^i$. We can find $\mathcal{A}_i$ by means of symmetric Green's function:

$$\mathcal{A}_i(t,\mathbf{x}) = \sum_{a=1}^{N} e_a v_{ai} W_a(t,\mathbf{x}), \tag{3.9}$$

$$W_a(t,\mathbf{x}) = 4\pi \int G[(t-t')^2 - (\mathbf{x}-\mathbf{x}_a(t'))^2]dt'. \tag{3.10}$$

By using free-particle solutions, integration in (3.10) yields

$$W_a(t,\mathbf{x}) = \left\{[\mathbf{v}_a(\mathbf{x}-\mathbf{x}_a(t))]^2 + (1-\mathbf{v}_a^2)(\mathbf{x}-\mathbf{x}_a(t))^2\right\}^{-1/2}. \tag{3.11}$$

In view of the definition of the field momenta E^i (2.12), it is convenient to define $\mathcal{A}_0(t,\mathbf{x})$ as

$$\mathcal{A}_0(t,\mathbf{x}) = \sum_{a=1}^{N} e_a W_a(t,\mathbf{x}), \tag{3.12}$$

so that

$$\mathcal{E}^i(t,\mathbf{x}) = \frac{1}{4\pi}\left(D_t \mathcal{A}_i(t,\mathbf{x}) - \partial_i \mathcal{A}_0(t,\mathbf{x})\right). \tag{3.13}$$

Here we have introduced

$$D_t \equiv \sum_{a=1}^{N} v_a^i \frac{\partial}{\partial x_a^i}. \tag{3.14}$$

Function $\mathcal{E}_\perp^i$ does not depend on the term $\partial_i\mathcal{A}_0$, because of ${P_j}^i\partial_i = 0$. Since $\dot{\mathcal{A}}_\mu(t,\mathbf{x}) = D_t\mathcal{A}_\mu(t,\mathbf{x})$, we can check directly that the functions $\mathcal{A}_0$ and $\mathcal{A}_i$ satisfy Lorentz gauge condition $D_t\mathcal{A}_0 - \partial_i\mathcal{A}_i = 0$. The transformation properties of $\mathcal{A}_0$, $\mathcal{A}_i$ and $\mathcal{E}^i$ are collected in Appendix B.

Now we deal with the canonical transformation to the new field variables $\phi_i^\perp$, $\chi_\perp^i$ in accordance with relations (3.5). Such a transformation changes the particle variables $(x_a^i, \pi_{ia}) \mapsto (y_a^i, r_{ia})$ as

$$x_a^i = y_a^i + \int \left[\left(\phi_k^\perp + \frac{1}{2}\mathcal{A}_k^\perp\right)\frac{\partial \mathcal{E}_\perp^k}{\partial r_{ai}} - \left(\chi_\perp^k + \frac{1}{2}\mathcal{E}_\perp^k\right)\frac{\partial \mathcal{A}_k^\perp}{\partial r_{ai}}\right] d^3x, \tag{3.15}$$

$$\pi_{ai} = r_{ai} - \int \left[\left(\phi_k^\perp + \frac{1}{2}\mathcal{A}_k^\perp\right)\frac{\partial \mathcal{E}_\perp^k}{\partial y_a^i} - \left(\chi_\perp^k + \frac{1}{2}\mathcal{E}_\perp^k\right)\frac{\partial \mathcal{A}_k^\perp}{\partial y_a^i}\right] d^3x. \tag{3.16}$$

In the considered approximation we have

$$\mathcal{A}_\mu(t,\mathbf{x}) \equiv \mathcal{A}_\mu(\mathbf{x}_a(t), \boldsymbol{\pi}_a; \mathbf{x}) = \mathcal{A}_\mu(\mathbf{y}_a(t), \mathbf{r}_a; \mathbf{x}). \tag{3.17}$$

In the terms of new variables the Hamiltonian is given by

$$\begin{aligned} H &= \sum_{a=1}^{N}\sqrt{m_a^2+\mathbf{r}_a^2} - 2\pi\int \varrho\Delta^{-1}\varrho d^3x + \frac{1}{2}\int j^i\mathcal{A}_i^\perp d^3x \\ &\quad + \int\left[\frac{1}{16\pi}\Phi_{ij}^\perp\Phi_{ij}^\perp + 2\pi\chi_\perp^i\chi_\perp^i\right]d^3x, \end{aligned} \tag{3.18}$$

where $\Phi_{ij}^\perp = \partial_i\phi_j^\perp - \partial_j\phi_i^\perp$.

The next step of our procedure of elimination of the field degrees of freedom consists in fixing the following constraints:

$$\phi_\alpha \approx 0, \quad \chi^\alpha \approx 0, \quad \alpha = 1,2. \tag{3.19}$$

Canonical constraints (3.19) are of the second class, so they can be excluded by means of the Dirac bracket. Therefore, we remain with the particle variables y_a^i, r_{ai} and Poisson commutation relations:

$$\{y_a^i(t), r_{bj}(t)\} = -\delta_{ab}\delta_j^i. \tag{3.20}$$

Now we can put $\phi_\alpha \approx 0$, $\chi^\alpha \approx 0$ in generators before calculating the Dirac brackets.

The Hamiltonian formalism is formed by the Dirac bracket (3.20) and the Hamiltonian:

$$H = \sum_{a=1}^{N}\sqrt{m_a^2+\mathbf{r}_a^2} - 2\pi\int\varrho\Delta^{-1}\varrho d^3x + \frac{1}{2}\int j^i\mathcal{A}_i^\perp d^3x. \tag{3.21}$$

By using relations (B.1)–(B.6), the transformed generators may be rewritten as a sum of particle and free-field terms. Thus, elimination of the field degrees of freedom leads to the result:

$$P^0 = H, \quad P^k = \sum_{a=1}^{N} r_a^k, \quad M^{ik} = \sum_{a=1}^{N}(y_a^i r_a^k - y_a^k r_a^i), \tag{3.22}$$

$$\begin{aligned} M^{k0} &= \sum_{a=1}^{N} y_a^k\sqrt{m_a^2+\mathbf{r}_a^2} - 2\pi\int x^k\varrho\Delta^{-1}\varrho d^3x + \frac{1}{2}\int x^k j^i\mathcal{A}_i^\perp d^3x \\ &\quad -2\pi\int\mathcal{E}_\perp^k\Delta^{-1}\varrho d^3x - tP^k. \end{aligned} \tag{3.23}$$

These generators satisfy the commutation relations of the Poincaré algebra in a given approximation. We immediately see that six generators

(P^k and M^{ik}) do not contain interaction terms. It reflects the general property of the instant form of dynamics [6].

It is possible to exclude ${P^i}_j$ from expressions of H and M^{k0} by means of a canonical transformation:

$$y_a^i = q_a^i + \{F, q_a^i\}, \quad r_{ai} = k_{ai} + \{F, k_{ai}\}, \tag{3.24}$$

$$F = \frac{1}{2}\int \varrho\Delta^{-1}\partial_i\mathcal{A}_i d^3x. \tag{3.25}$$

This transformation preserves the form of P^k and M^{ik}. But the Hamiltonian and boost become

$$H = \sum_{a=1}^N \sqrt{m_a^2 + \mathbf{k}_a^2} + \frac{1}{2}\int (j^i\mathcal{A}_i + \varrho\mathcal{A}_0)d^3x, \tag{3.26}$$

$$M^{k0} = \sum_{a=1}^N q_a^k\sqrt{m_a^2 + \mathbf{k}_a^2} + \frac{1}{2}\int x^k(j^i\mathcal{A}_i + \varrho\mathcal{A}_0)d^3x - tP^k \tag{3.27}$$

It is easy to see that the field part of the Hamiltonian in the particle terms has compensated the half of the interaction.

The generators H and M^{k0} have the following final form

$$H = \sum_{a=1}^N k_a^0 + \frac{1}{2}\sum_{a,b=1}^{N}{}' V_{ab}, \quad M^{k0} = \sum_{a=1}^N q_a^k k_a^0 + \frac{1}{2}\sum_{a,b=1}^{N}{}' q_a^k V_{ab} - tP^k, \tag{3.28}$$

$$k_a^0 = \sqrt{m_a^2 + \mathbf{k}_a^2}, \quad V_{ab} = e_a e_b \frac{\sqrt{m_b^2 + \mathbf{k}_b^2} - \mathbf{k}_a\mathbf{k}_b/\sqrt{m_a^2 + \mathbf{k}_a^2}}{\sqrt{(\mathbf{k}_b\mathbf{q}_{ab})^2 + m_b^2\mathbf{q}_{ab}^2}}, \tag{3.29}$$

where $\mathbf{q}_{ab} = \mathbf{q}_a - \mathbf{q}_b$. The prime over the sum symbol means that $a \neq b$. The terms, which corresponds to self-interaction ($a = b$), can be eliminated by means of the mass renormalization.

According to (3.15), (3.24), the covariant particle positions x_a^i are connected with the canonical variables as

$$x_a^i = q_a^i + \frac{1}{2}\int \left[\mathcal{A}_k \frac{\partial \mathcal{E}^k}{\partial k_{ai}} - \mathcal{E}^k \frac{\partial \mathcal{A}_k}{\partial k_{ai}}\right] d^3x. \tag{3.30}$$

It can be verified directly that in a given approximation the expression (3.30) satisfies the world line condition [18]:

$$\{x_a^i, M^{k0}\} = x_a^k\{x_a^i, H\} - t\delta^{ik}. \tag{3.31}$$

Poisson brackets between particle positions are

$$\{x_a^i, x_b^j\} = \int \left(\frac{\partial \mathcal{A}_k}{\partial k_{bj}}\frac{\partial \mathcal{E}^k}{\partial k_{ai}} - \frac{\partial \mathcal{E}^k}{\partial k_{bj}}\frac{\partial \mathcal{A}_k}{\partial k_{ai}}\right) d^3x. \tag{3.32}$$

It shows that x_a^i cannot be the canonical variable. This fact is in full accordance with the famous no-interaction theorem [18].

4. Description in the Front Form of Dynamics

Here we deal with relativistic system of charged particles coupled with the electromagnetic field on the isotropic hypersurface given by

$$x^0 = t + x^3. \tag{4.1}$$

Application of the elaborated procedure of the elimination of the gauge and physical field degrees of freedom to the front form of dynamics [12] is demonstrated below.

In the Hamiltonian formulation of our system we start with canonical variables $x_a^i(t)$, $A_\mu(t,\mathbf{x})$ and conjugated momenta $p_{ai}(t)$, $E^\mu(t,\mathbf{x})$ which are subject of the first class constraints (2.17). It turns out that additional constraints arise in the front form of dynamics [16]

$$\Omega^\alpha \equiv 4\pi E^\alpha - {F^\alpha}_3 \approx 0, \quad \alpha = 1, 2. \tag{4.2}$$

Let us note some commutation relations between the constraints:

$$\{\Omega^\alpha(t,\mathbf{x}), \Omega^\beta(t,\mathbf{y})\} = -8\pi\delta^{\alpha\beta}\frac{\partial}{\partial x^3}\delta^3(\mathbf{x}-\mathbf{y}), \quad \{\Gamma, \Omega^\alpha\} = 0. \tag{4.3}$$

Therefore the Hamiltonian formalism contains a pair of the first class constraints and the second class constraints $\Omega^\alpha \approx 0$, $\alpha = 1, 2$.

We isolate gauge degrees of freedom like to the instant form of dynamics (see transformation (2.24)–(2.32)). Similarly, we obtain a description in the terms of the physical canonical variables x_a^i, π_{ia}, a_α, b^α.

Now we intend to eliminate the second class constraints:

$$b^\alpha - \sqrt{4\pi}\partial^\alpha\Delta^{-1}\varrho - \partial_3 a_\alpha \approx 0, \quad \alpha = 1, 2. \tag{4.4}$$

Firstly, it is desirable to perform a canonical transformation for the sake of simplicity of the form of (4.4):

$$((x_a^i, \pi_{ai}), (a_\alpha, b^\alpha)) \mapsto ((x_a^i, \tilde{\pi}_{ai}), (a_\alpha, \tilde{b}^\alpha)), \tag{4.5}$$

$$b^\alpha = \tilde{b}^\alpha + \sqrt{4\pi}\partial^\alpha\Delta^{-1}\varrho, \quad \pi_{ai} = \tilde{\pi}_{ai} + \sqrt{4\pi}e_a\partial_i\Delta^{-1}\partial_\alpha a_\alpha(t,\mathbf{x}_a). \tag{4.6}$$

Then the second class constraints become

$$\tilde{b}^{\alpha} - \partial_3 a_{\alpha} \approx 0, \quad \alpha = 1, 2. \tag{4.7}$$

They are non-local and "self-conjugated", so its canonization constitute a meaningful problem which is not studied here. These constraints are eliminated immediately by means of the Dirac bracket. Nonvanishing commutators between variables in the terms of the Dirac bracket are

$$\{x_a^i, \tilde{\pi}_{bj}\}^* = -\delta_{ab}\delta_j^i, \quad \{a_\alpha(t,\mathbf{x}), \tilde{b}^{\beta}(t,\mathbf{y})\}^* = \frac{1}{2}\delta_\alpha^\beta \delta^3(\mathbf{x}-\mathbf{y}),$$
$$\{a_\alpha(t,\mathbf{x}), a_\beta(t,\mathbf{y})\}^* = -\frac{1}{2}\delta_{\alpha\beta}\left(\frac{\partial}{\partial x^3}\right)^{-1}\delta^3(\mathbf{x}-\mathbf{y}),$$
$$\{\tilde{b}^\alpha(t,\mathbf{x}), \tilde{b}^\beta(t,\mathbf{y})\}^* = \frac{1}{2}\delta^{\alpha\beta}\frac{\partial}{\partial x^3}\delta^3(\mathbf{x}-\mathbf{y}). \tag{4.8}$$

We shall consider $a_\alpha(t,\mathbf{x})$ as independent field variables and $\tilde{b}^\alpha(t,\mathbf{x})$ will be treated as the functionals on the potentials. We are not concentrated on the finding Darboux basis, because it is the complicated problem and, moreover, our aim is complete exclusion of the field.

The physical evolution of the system after exclusion of the gauge degrees of freedom and accounting the second class constraints is generated by the following Hamiltonian:

$$H = \int \mathcal{H} d^3x, \tag{4.9}$$

where the Hamiltonian density is

$$\mathcal{H} = \frac{1}{2}\sum_{a=1}^{N}\left[\tilde{\pi}_{a3} + \frac{(\tilde{\pi}_{a\alpha} - \sqrt{4\pi}e_a a_\alpha)^2 + m_a^2}{\tilde{\pi}_{a3}}\right]\delta^3(\mathbf{x}-\mathbf{x}_a)$$
$$+\frac{1}{2}\left(\partial_i a_\alpha \partial_i a_\alpha - \partial_\alpha a_\beta \partial_\beta a_\alpha\right) + \frac{1}{2}\left(\frac{\sqrt{4\pi}}{\partial_3}\varrho - \partial_\alpha a_\alpha\right)^2 . \tag{4.10}$$

We remark that dependence of H on the field variables is quadratic. This fact has used in Ref. 16 to calculating classical partition function, when the particle and field variables are treated on equal rights.

The Poincaré generators after decoupling of the gauge and gauge-invariant degrees of freedom have the form

$$P^0 = H, \quad P^k = \sum_{a=1}^{N}\tilde{\pi}_a^k + \int \tilde{b}^\alpha \partial^k a_\alpha d^3x + \delta_3^k H, \tag{4.11}$$

$$M^{k0} = -\sum_{a=1}^{N} x_a^3 \tilde{\pi}_a^k - \int x^3 \tilde{b}^\alpha \partial^k a_\alpha d^3x + \int (x^k - \delta_3^k x^3)\mathcal{H} d^3x$$
$$-\eta^{k\alpha}\int a_\alpha \left(\Pi_\beta^3 \tilde{b}^\beta + \frac{\sqrt{4\pi}}{\partial_3}\varrho\right) d^3x - tP^k, \tag{4.12}$$

$$M^{ik} = \sum_{a=1}^{N}(x_a^i\tilde{\pi}_a^k - x_a^k\tilde{\pi}_a^i) + \int (x^i \tilde{b}^\alpha \partial^k a_\alpha - x^k \tilde{b}^\alpha \partial^i a_\alpha) d^3x$$
$$+\int d^3x a_\alpha \left[\eta^{k\alpha}\left(\Pi_\beta^i \tilde{b}^\beta + \delta_3^i \frac{\sqrt{4\pi}}{\partial_3}\varrho\right) - \eta^{i\alpha}\left(\Pi_\beta^k \tilde{b}^\beta + \delta_3^k\frac{\sqrt{4\pi}}{\partial_3}\varrho\right)\right]$$
$$+\int (x^i\delta_3^k - x^k\delta_3^i)\mathcal{H} d^3x. \tag{4.13}$$

They satisfy the commutation relations of the Poincaré algebra (2.23) in the terms of the Dirac brackets.

Now we are concentrated on the elimination of the field degrees of freedom. Let us first find a solution of the Hamiltonian field equations which are written as

$$\Box_{\mathrm{fr}}\, a_\alpha = \sqrt{4\pi}\sum_{a=1}^{N} e_a\left(\frac{\tilde{\pi}_{a\alpha} - \sqrt{4\pi}e_a a_\alpha(t,\mathbf{x}_a)}{\tilde{\pi}_{a3}} - \frac{\partial_\alpha}{\partial_3}\right)\delta^3(\mathbf{x}-\mathbf{x}_a), \tag{4.14}$$

$$\Box_{\mathrm{fr}} = 2\partial_t\partial_3 - \Delta. \tag{4.15}$$

The d'Alambertian $\Box_{\mathrm{fr}}$ in the front form of dynamics is the first-order differential operator with respect to the evolution parameter.

In the linear approximation we have equations

$$\Box_{\mathrm{fr}}\, a_\alpha = \sqrt{4\pi}\left(j_\alpha - \frac{\partial_\alpha}{\partial_3}\varrho\right). \tag{4.16}$$

with the following current density

$$j^i(t,\mathbf{x}) = \sum_{a=1}^{N} e_a v_a^i \delta^3(\mathbf{x} - \mathbf{x}_a(t)). \tag{4.17}$$

Here $v_a^i = (\tilde{\pi}_a^i + \delta_3^i h_a)/\tilde{\pi}_{a3} = \mathrm{const}$, $h_a = (m_a^2 + \tilde{\boldsymbol{\pi}}_a^2)/2\tilde{\pi}_{a3}$.

By using the Green's function method, the general solution of the inhomogeneous equation (4.16) can be presented in the form

$$a_\alpha = \phi_\alpha + \mathsf{a}_\alpha, \tag{4.18}$$

$$\mathsf{a}_\alpha = \alpha_\alpha - \frac{\partial_\alpha}{\partial_3}\alpha_0, \tag{4.19}$$

where ϕ_α is the general solution to homogeneous equation $\Box_{\rm fr}\phi_\alpha = 0$. The solution to inhomogeneous equation, namely, a_α depending on particle variables can be written by means of the following functions:

$$\alpha_i(t,\mathbf{x}) = (4\pi)^{-1/2}\sum_{a=1}^{N} e_a v_{ai} W_a(t,\mathbf{x}),$$
$$\alpha_0(t,\mathbf{x}) = (4\pi)^{-1/2}\sum_{a=1}^{N} e_a W_a(t,\mathbf{x}). \qquad (4.20)$$

Here we have introduced

$$W_a(t,\mathbf{x}) \equiv 4\pi\int G[(t-t'+x^3-x_a^3(t'))^2-(\mathbf{x}-\mathbf{x}_a(t'))^2]dt' \qquad (4.21)$$

containing symmetric Green's function G. Accounting free-particle equations, the integration leads to the following expression:

$$W_a(t,\mathbf{x}) = \left\{[x^3-x_a^3(t)-v_a^\alpha(x^\alpha-x_a^\alpha(t))]^2+\gamma_a^{-2}(x^\alpha-x_a^\alpha(t))^2\right\}^{-1/2}, \qquad (4.22)$$

where $\gamma_a^{-2}\equiv 1+2v_a^3-(v_a^\alpha)^2$.

It is easy to check that the functions α_μ satisfy Lorentz gauge condition in the linear approximation in the coupling constant.

According to (4.7), let us introduce the following functions:

$$\mathsf{b}^\alpha(t,\mathbf{x}) = \partial_3\mathsf{a}_\alpha(t,\mathbf{x}), \quad \beta^\mu(t,\mathbf{x}) = \partial_3\alpha_\mu(t,\mathbf{x}). \qquad (4.23)$$

Some useful transformation properties of the obtained functions are shown in Appendix B.

Transition to the new field variables is done by means of the following transformation:

$$a_\alpha(t,\mathbf{x}) = \phi_\alpha(t,\mathbf{x}) + \mathsf{a}_\alpha(t,\mathbf{x}), \quad \tilde{b}^\alpha(t,\mathbf{x}) = \chi^\alpha(t,\mathbf{x}) + \mathsf{b}^\alpha(t,\mathbf{x}), \qquad (4.24)$$

$$x_a^i = y_a^i + \int\left[\left(\phi_\alpha+\frac{1}{2}\mathsf{a}_\alpha\right)\frac{\partial\mathsf{b}^\alpha}{\partial r_{ai}} - \left(\chi^\alpha+\frac{1}{2}\mathsf{b}^\alpha\right)\frac{\partial\mathsf{a}_\alpha}{\partial r_{ai}}\right]d^3x, \qquad (4.25)$$

$$\tilde{\pi}_{ai} = r_{ai} - \int\left[\left(\phi_\alpha+\frac{1}{2}\mathsf{a}_\alpha\right)\frac{\partial\mathsf{b}^\alpha}{\partial y_a^i} - \left(\chi^\alpha+\frac{1}{2}\mathsf{b}^\alpha\right)\frac{\partial\mathsf{a}_\alpha}{\partial y_a^i}\right]d^3x, \qquad (4.26)$$

which preserves commutation relations in the terms of Dirac brackets. We must remember that $\phi_\alpha(t,\mathbf{x})$ and $\chi^\alpha(t,\mathbf{x})$ are related by constraints $\chi^\alpha - \partial_3\phi_\alpha \approx 0$.

The Hamiltonian density after the performed transformation is

$$\mathcal{H} = \sum_{a=1}^{N}\frac{\mathbf{r}_a^2+m_a^2}{2r_{a3}}\delta^3(\mathbf{x}-\mathbf{y}_a) + \frac{1}{2}\frac{\sqrt{4\pi}}{\partial_3}\varrho\frac{\sqrt{4\pi}}{\partial_3}\varrho - \frac{\sqrt{4\pi}}{2}\left(j_\alpha - \frac{\partial_\alpha}{\partial_3}\varrho\right)\mathsf{a}_\alpha$$
$$+\frac{1}{2}\left[\partial_i\phi_\alpha\partial_i\phi_\alpha - \partial_\alpha\phi_\beta\partial_\beta\phi_\alpha + (\partial_\alpha\phi_\alpha)^2\right]. \qquad (4.27)$$

In analogy with the scheme of reduction in the instant form, the free field degrees of freedom are eliminated by fixing a set of constraints

$$\phi_\alpha(t,\mathbf{x}) \approx 0, \quad \chi^\alpha(t,\mathbf{x}) \approx 0 \qquad (4.28)$$

and introducing the respective new Dirac bracket.

Thus, the Hamiltonian formalism in the terms of y_a^i and r_{ai} is formed by the Dirac bracket and the Hamiltonian (4.9) with density

$$\mathcal{H} = \sum_{a=1}^{N}\frac{\mathbf{r}_a^2+m_a^2}{2r_{a3}}\delta^3(\mathbf{x}-\mathbf{y}_a) + \frac{1}{2}\frac{\sqrt{4\pi}}{\partial_3}\varrho\frac{\sqrt{4\pi}}{\partial_3}\varrho - \frac{\sqrt{4\pi}}{2}\left(j_\alpha - \frac{\partial_\alpha}{\partial_3}\varrho\right)\mathsf{a}_\alpha. \qquad (4.29)$$

Accounting transformation properties (B.9)–(B.14), application of the procedure of the field elimination to the Poincaré generators gives:

$$P^0 = H, \quad P^k = \sum_{a=1}^{N} r_a^k + \delta_3^k H, \qquad (4.30)$$
$$M^{k0} = -\sum_{a=1}^{N} y_a^3 r_a^k + \frac{\eta^{k\alpha}}{2}\int\mathsf{a}_\alpha\frac{\sqrt{4\pi}}{\partial_3}\varrho d^3x$$
$$+\int(x^k-\delta_3^k x^3)\mathcal{H}d^3x - tP^k, \qquad (4.31)$$
$$M^{ik} = \sum_{a=1}^{N}(y_a^i r_a^k - y_a^k r_a^i) + \int(x^i\delta_3^k - x^k\delta_3^i)\mathcal{H}d^3x$$
$$+\frac{\sqrt{4\pi}}{2}\int\left(\delta_\alpha^i\delta_3^k - \delta_\alpha^k\delta_3^i\right)\mathsf{a}_\alpha\frac{1}{\partial_3}\varrho d^3x. \qquad (4.32)$$

These generators act on the particle phase space and satisfy the commutation relations of the Poincaré algebra in a given approximation.

In order to avoid the expressions containing $1/\partial_3$, we carry out a canonical transformation:

$$y_a^i = q_a^i + \{F, q_a^i\}, \quad r_{ai} = k_{ai} + \{F, k_{ai}\}, \qquad (4.33)$$

$$F = \frac{1}{2}\int \alpha_0 \frac{\sqrt{4\pi}}{\partial_3}\varrho d^3x. \tag{4.34}$$

Then the Hamiltonian density becomes

$$\mathcal{H} = \sum_{a=1}^{N} \frac{\mathbf{k}_a^2 + m_a^2}{2k_{a3}}\delta^3(\mathbf{x} - \mathbf{q}_a) + \frac{\sqrt{4\pi}}{2}[j^i\alpha_i + (\varrho + j^3)(\alpha_0 + \alpha^3)]. \tag{4.35}$$

The Hamiltonian of our system can be written as follows

$$H = \sum_{a=1}^{N} h_a + \frac{1}{2}\sum_{a,b=1}^{N}{}' V_{ab}, \quad h_a = \frac{m_a^2 + \mathbf{k}_a^2}{2k_{a3}}, \tag{4.36}$$

$$V_{ab} = \frac{e_a e_b[1 + v_a^3 + v_b^3 - v_a^\alpha v_b^\alpha]}{\sqrt{[q_{ab}^3 - v_b^\alpha q_{ab}^\alpha]^2 + \gamma_b^{-2}(q_{ab}^\alpha)^2}}, \quad v_a^i = \frac{k_a^i + \delta_3^i h_a}{k_{a3}}. \tag{4.37}$$

It is assumed that self-action terms are reduced by means of mass renormalization.

The immediate calculations give us the final form of the Poincaré generators

$$P^0 = H, \quad P^k = \sum_{a=1}^{N} k_a^k + \delta_3^k H, \tag{4.38}$$

$$\begin{aligned} M^{k0} &= -\sum_{a=1}^{N} q_a^3 k_a^k + \sum_{a=1}^{N}(q_a^k - \delta_3^k q_a^3)h_a \\ &\quad + \frac{1}{2}\sum_{a,b=1}^{N}{}'(q_a^k - \delta_3^k q_a^3)V_{ab} - tP^k, \end{aligned} \tag{4.39}$$

$$\begin{aligned} M^{ik} &= \sum_{a=1}^{N}(q_a^i k_a^k - q_a^k k_a^i) + \sum_{a=1}^{N}(q_a^i\delta_3^k - q_a^k\delta_3^i)h_a \\ &\quad + \frac{1}{2}\sum_{a,b=1}^{N}{}'(q_a^i\delta_3^k - q_a^k\delta_3^i)V_{ab}. \end{aligned} \tag{4.40}$$

One can extract six interaction free generators: $P^k - \delta_3^k P^0$ and $M^{ik} + \delta_3^i M^{k0} - \delta_3^k M^{i0}$. Besides, if $t = 0$, M^{30} does not contain the interaction term. In next section we shall demonstrate explicitly the canonical transformation which connects the canonical realizations of the Poincaré algebra in the instant and front forms of dynamics.

In the case of a two-particle system in two-dimensional space–time, when $q_a^\alpha = 0$ and $k_{a\alpha} = 0$, nonvanishing generators P^0, P^3, and M^{30} agree with the results of Ref. 19 for vector interaction in the linear approximation in the coupling constant. Comparing these results, we must take into account that the front form of dynamics in Ref. 19 is defined as $x^0 = t - x$.

From (4.25), (4.28), (4.33) we obtain relation between the covariant particle positions x_a^i and the canonical variables:

$$x_a^i = q_a^i + \int\left[\alpha_j\frac{\partial\beta^j}{\partial k_{ai}} - (\alpha_0 + \alpha^3)\frac{\partial(\beta^0 + \beta_3)}{\partial k_{ai}}\right]d^3x. \tag{4.41}$$

We can check directly that expression (4.40) satisfies the world line condition.

Taking into account (4.23), the Poisson brackets between particle positions are

$$\{x_a^i, x_b^j\} = 2\int\left[\frac{\partial\alpha_k}{\partial k_{bj}}\frac{\partial\beta^k}{\partial k_{ai}} - \frac{\partial(\alpha_0 + \alpha^3)}{\partial k_{bj}}\frac{\partial(\beta^0 + \beta_3)}{\partial k_{ai}}\right]d^3x. \tag{4.42}$$

These expressions show that the relation between x_a^i and q_a^i cannot correspond to the canonical transformation. However, it may be illustrated that in the case of the one-dimensional space the covariant coordinates and canonical variables coincide, i.e. $x_a = q_a$, and $\{x_a, x_b\} = 0$ (see Refs. 20, 21).

5. Relation Between the Instant and Front Forms

Here we aim to show that the obtained Hamiltonian descriptions in the instant and front forms of relativistic dynamics are equivalent. It will be done by means of finding a canonical transformation which relates the expressions in the instant and front forms. It is convenient for the following study to denote the particle canonical variables of the instant form as (x_a^i, p_{ia}). By using inhomogeneous equations for functions $\mathcal{A}_\mu$ in the terms of particle variables (see (3.3))

$$\Box_{\text{in}}\,\mathcal{A}_0 = 4\pi\varrho, \quad \Box_{\text{in}}\mathcal{A}_i = 4\pi j_i, \tag{5.1}$$

let us rewrite the instant form generators (3.26), (3.27) as follows

$$P^k = \sum_{a=1}^{N} p_a^k, \quad M^{ik} = \sum_{a=1}^{N}(x_a^i p_a^k - x_a^k p_a^i), \tag{5.2}$$

$$P^0=\sum_{a=1}^N p_a^0+\int \mathcal{W}d^3x, \quad M^{k0}=\sum_{a=1}^N x_a^k p_a^0+\int x^k\mathcal{W}d^3x-tP^k, \tag{5.3}$$

$$p_a^0=\sqrt{m_a^2+\mathbf{p}_a^2}, \quad \mathcal{W}=\frac{1}{8\pi}(\mathcal{A}_0\,\Box_{\rm in}\,\mathcal{A}_0-\mathcal{A}_i\,\Box_{\rm in}\,\mathcal{A}_i). \tag{5.4}$$

Similarly, we can reexpress the front form Hamiltonian density as

$$\mathcal{H}=\sum_{a=1}^N h_a\delta^3(\mathbf{x}-\mathbf{q}_a)+\mathcal{V}, \quad h_a=\frac{\mathbf{k}_a^2+m_a^2}{2k_{a3}}, \tag{5.5}$$

which contains the following interaction term

$$\mathcal{V}\equiv\frac{1}{2}[(\alpha_0+\alpha^3)\,\Box_{\rm fr}\,(\alpha_0+\alpha^3)-\alpha_i\,\Box_{\rm fr}\,\alpha_i]. \tag{5.6}$$

We first perform a canonical transformation $(x_a^i,p_{ai})\mapsto(q_a^i,k_{ai})$:

$$x_a^i=q_a^i-q_a^3\frac{k_a^i+\delta_3^i h_a}{h_a}, \quad p_a^i=k_a^i+\delta_3^i h_a, \tag{5.7}$$

which transits immediately instant form generators without interaction into the front form ones. Then we can derive that

$$\frac{\partial}{\partial x_a^i}=\delta_i^3 v_a^j\frac{\partial}{\partial q_a^j}+\frac{\partial}{\partial q_a^i}, \tag{5.8}$$

$$D_t=\sum_{a=1}^N u_a^i\frac{\partial}{\partial x_a^i}=\sum_{a=1}^N v_a^i\frac{\partial}{\partial q_a^i}, \tag{5.9}$$

where $u_a^i=p_a^i/\sqrt{m_a^2+\mathbf{p}_a^2}$ and $v_a^i=(k_a^i+\delta_3^i h_a)/k_{3a}$.

Since $\dot{\mathcal{A}}_\mu(t,\mathbf{x})=D_t\mathcal{A}_\mu(t,\mathbf{x})$, and $\mathcal{A}_\mu(t,\mathbf{x})$ depends on $\mathbf{x}_a-\mathbf{x}$ (see (3.10)), operator $\Box_{\rm in}$ may be rewritten in the terms of partial derivatives with respect to the particle variables in a given approximation as

$$\begin{aligned}\Box_{\rm in} &= \sum_{a=1}^N\left[\left(u_a^i\frac{\partial}{\partial x_a^i}\right)^2-\frac{\partial^2}{\partial x_a^i\partial x_a^i}\right]\\ &= \sum_{a=1}^N\left[-2v_a^i\frac{\partial^2}{\partial q_a^i\partial q_a^3}-\frac{\partial^2}{\partial q_a^i\partial q_a^i}\right]=\Box_{\rm fr}.\end{aligned} \tag{5.10}$$

If we consider $\mathcal{A}_\mu(t,\mathbf{x})$ and $\alpha_\mu(t,\mathbf{x})$ at the point $\mathbf{x}=0$, then we immediately obtain the following relations:

$$\mathcal{A}_0(\mathbf{x}_a,\mathbf{p}_a;\mathbf{0})=\sqrt{4\pi}[\alpha_0(\mathbf{q}_a,\mathbf{k}_a;\mathbf{0})+\alpha^3(\mathbf{q}_a,\mathbf{k}_a;\mathbf{0})], \tag{5.11}$$

$$\mathcal{A}_i(\mathbf{x}_a,\mathbf{p}_a;\mathbf{0})=\sqrt{4\pi}\alpha_i(\mathbf{q}_a,\mathbf{k}_a;\mathbf{0}). \tag{5.12}$$

Function $\mathcal{A}_\mu(t,\mathbf{x})$ can be recovered from $\mathcal{A}_\mu(t,\mathbf{0})$ by replacement $\mathbf{x}_a\mapsto\mathbf{x}_a-\mathbf{x}$ which is generated by the translation operator:

$$\mathcal{A}_\mu(t,\mathbf{x})=\mathcal{A}_\mu(\mathbf{x}_a(t),\mathbf{p}_a;\mathbf{x})=\exp\left(-\sum_{a=1}^N x^i\frac{\partial}{\partial x_a^i}\right)\mathcal{A}_\mu(\mathbf{x}_a(t),\mathbf{p}_a;\mathbf{0}). \tag{5.13}$$

According (5.8), the exponent is rewritten as

$$\sum_{a=1}^N x^i\frac{\partial}{\partial x_a^i}=x^3D_t+\sum_{a=1}^N x^i\frac{\partial}{\partial q_a^i}. \tag{5.14}$$

The second term in r.h.s. will produce translation of a function depending on q_a^i. Taking this into account, we have

$$\mathcal{A}_0(t,\mathbf{x})=\sqrt{4\pi}\exp(-x^3D_t)[\alpha_0(t,\mathbf{x})+\alpha^3(t,\mathbf{x})], \tag{5.15}$$

$$\mathcal{A}_i(t,\mathbf{x})=\sqrt{4\pi}\exp(-x^3D_t)\alpha_i(t,\mathbf{x}). \tag{5.16}$$

Reexpression of the instant form interaction between charged particles in the terms of the new canonical variables results in

$$\mathcal{W}=\exp(-x^3D_t)\mathcal{V}. \tag{5.17}$$

Then we obtain

$$P^0 = \sum_{a=1}^N h_a+\int\exp(-x^3D_t)\mathcal{V}d^3x, \tag{5.18}$$

$$P^k = \sum_{a=1}^N k_a^k+\delta_3^k\sum_{a=1}^N h_a, \tag{5.19}$$

$$\begin{aligned}M^{k0} &= -\sum_{a=1}^N q_a^3k_a^k+\sum_{a=1}^N(q_a^k-\delta_3^kq_a^3)h_a\\ &\quad+\int x^k\exp(-x^3D_t)\mathcal{V}d^3x-tP^k,\end{aligned} \tag{5.20}$$

$$M^{ik} = \sum_{a=1}^N(q_a^ik_a^k-q_a^kk_a^i)+\sum_{a=1}^N(q_a^i\delta_3^k-q_a^k\delta_3^i)h_a. \tag{5.21}$$

Now it may be shown that difference between the generators in the instant ("in") and front ("fr") form is presented as

$$G_{\rm in}-G_{\rm fr}=\{F,G_{\rm in}\}, \tag{5.22}$$

where

$$F = \int (\exp(-x^3 D_t) - 1)\mathcal{F} d^3x, \qquad D_t\mathcal{F} = \mathcal{V}. \tag{5.23}$$

Therefore, we prove that the generators, which correspond to the instant and front form of relativistic dynamics, are connected by a canonical transformation.

6. The Weak-Relativistic Approximation

Let us consider the instant form generators in approximation up to the order c^{-2}. We shall first explore the Hamiltonian and its transformation to the Darwin Hamiltonian. We rewrites (3.28) in the explicit form with c

$$\begin{aligned} H &= \sum_{a=1}^{N} m_a c^2 \sqrt{1+\frac{\mathbf{k}_a^2}{m_a^2c^2}} + \frac{1}{2}\sum_{a,b=1}^{N}{}' \frac{e_a e_b}{\sqrt{\mathbf{q}_{ab}^2 + (\mathbf{k}_a\mathbf{q}_{ab})^2/m_a^2c^2}} \\ &\times \left(\sqrt{1+\frac{\mathbf{k}_a^2}{m_a^2c^2}} - \frac{\mathbf{k}_a\mathbf{k}_b/m_am_bc^2}{\sqrt{1+\mathbf{k}_b^2/m_b^2c^2}}\right). \end{aligned} \tag{6.1}$$

Expansion of the Hamiltonian up to c^{-2}-order yields

$$\begin{aligned} H &= \sum_{a=1}^{N}\left(m_ac^2 + \frac{\mathbf{k}_a^2}{2m_a} - \frac{\mathbf{k}_a^4}{8m_a^3c^2}\right) + \frac{1}{2}\sum_{a,b=1}^{N}{}' \frac{e_ae_b}{|\mathbf{q}_{ab}|} \\ &\times \left(1 - \frac{(\mathbf{k}_a\mathbf{q}_{ab})^2}{2m_a^2c^2\mathbf{q}_{ab}^2}\right)\left(1+\frac{\mathbf{k}_a^2}{2m_a^2c^2} - \frac{\mathbf{k}_a\mathbf{k}_b}{m_am_bc^2}\right). \end{aligned} \tag{6.2}$$

We observe, therefore, that the Darwin Hamiltonian [22]

$$H_{\mathrm{Dw}} = H_0 - \sum_{a=1}^{N}\frac{\mathbf{k}_a^4}{8m_a^3c^2} - \sum_{a>b}\frac{e_ae_b}{2c^2m_am_b|\mathbf{q}_{ab}|}\left[\mathbf{k}_a\mathbf{k}_b + \frac{(\mathbf{k}_a\mathbf{q}_{ab})(\mathbf{k}_b\mathbf{q}_{ab})}{\mathbf{q}_{ab}^2}\right] \tag{6.3}$$

and H are related as

$$H = H_{\mathrm{Dw}} + \{\Lambda, H_0\}, \tag{6.4}$$

where

$$\begin{aligned} H_0 &= \sum_{a=1}^{N}\left(m_ac^2 + \frac{\mathbf{k}_a^2}{2m_a}\right) + \sum_{a>b}\frac{e_ae_b}{|\mathbf{q}_{ab}|}, \\ \Lambda &= \frac{1}{4c^2}\sum_{a>b}\frac{e_ae_b}{|\mathbf{q}_{ab}|}\left[\mathbf{q}_{ab}\left(\frac{\mathbf{k}_a}{m_a} - \frac{\mathbf{k}_b}{m_b}\right)\right]. \end{aligned} \tag{6.5}$$

We can immediately write down P^0, because of $cP^0 = H$ in the instant form of dynamics. Similarly, in a given approximation we find

$$M^{k0} = \sum_{a=1}^{N} q_a^k\left(m_a + \frac{\mathbf{k}_a^2}{2m_ac^2}\right) + \frac{1}{2c^2}\sum_{a,b=1}^{N}{}' q_a^k\frac{e_ae_b}{|\mathbf{q}_{ab}|} - tP^k. \tag{6.6}$$

We can check directly that

$$\{\Lambda, P^k\} = 0, \quad \{\Lambda, M^{ik}\} = 0, \quad \{\Lambda, M^{k0}\} = 0. \tag{6.7}$$

Therefore, generators, which are found by means of elimination of the field in the first-order in the coupling constant, in weak-relativistic approximation give us well-known expressions. Moreover, we note that in a given approximation the covariant positions x_a^i coincide with canonical variables q_a^i.

7. Conclusions

Using the chronometrical invariance of the action of relativistic system of charged particles plus electromagnetic field, the Hamiltonian description and canonical realization of the Poincaré algebra are considered in the Dirac's instant and front form of dynamics with the break of a manifest Lorentz covariance. Besides, we have eliminated the proper gauge freedom of the 4–potential A_μ. By means of reduction of the gauge degrees of freedom the Hamiltonian formulation of dynamics is obtained in the gauge-invariant manner which is not manifestly Lorentz covariant. At this step the particle and field variables are treated on equal level. Another kind of reduction consists in elimination of the physical field degrees of freedom with the help of the Dirac theory of constraints. In our approach such a reduction is performed *after* transition to the Hamiltonian formulation. The suggested procedure of elimination of the field has three steps: (i) finding a solution to the field equation of motion, (ii) canonization of the free-field variables ϕ_α, χ^α by means of suitable transformation, (iii) fixation of the free field. In our problem the free field is equal to zero. However, we may fix another value of the field variables by means of constraints.

Here we have limited ourselves by study of the first-order approximation in the coupling constant. As a result, we find the canonical realization of the Poincaré algebra in the both forms of dynamics in the terms of particle variables. Similar approach is studied in Ref. 4, where electric charges are described by Grassman variables. As distinct from the results in Ref. 4, our expressions of Poincaré generators are written in a non-manifestly Lorentz covariant way for space-like (instant) and isotropic form of relativistic dynamics. Note that we suggest a canonical transformation which comes to the form of generators without differential projectors like (2.25). Obtained expressions are easy analyzed and simply applied. Moreover, the canonical transformation of the second step of our procedure, which is not observed in literature, allows us to trace the exclusion of interaction between particles and fields. It is demonstrated that the Poincaré generators in the instant and front form are related by a canonical transformation. Also we show that the instant form Hamiltonian in the c^{-2} approximation leads to the Darwin Hamiltonian. Although, the problem of canonical exclusion of the second class field constraints in the front form of dynamics is still remained. Although, the problem of canonical exclusion of the second class field constraints in the front form of dynamics is still remained.

Perspective research is the reduction of the field in the case of the higher order approximation in the coupling constant. Then, by using retarded Green's function, we may study radiation effects in the terms of particle variables. The planning task is to apply the field elimination to the gravity and Yang–Mills theory. The obtained description may be the base of relativistic statistical and quantum mechanics of the system of charged particles.

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Appendix A

Deriving the field equation in a given form of relativistic dynamics, we need use the d'Alambertian $\Box \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu$ in the terms of t and $\mathbf{x}$. If equation of hypersurface is given by $x^0 = t + \mathbf{n}\mathbf{x}$, where the components of vector $\mathbf{n}$ are constants, we have

$$\Box = (1-\mathbf{n}^2)\partial_t^2 + 2n^i\partial_t\partial_i - \Delta. \tag{A.1}$$

In our paper we use symmetric Green's function

$$G = \frac{1}{4\pi}\delta[(x^0)^2 - \mathbf{x}^2] \tag{A.2}$$

which satisfies equation

$$\Box\, G[(x^0)^2 - \mathbf{x}^2] = \delta(x^0)\delta^3(\mathbf{x}). \tag{A.3}$$

Appendix B

In the instant and front form of dynamics we have presented solutions of inhomogeneous field equations by means of a number of functions depending on particle variables. Performing canonical transformation to the free field variables ϕ_α, χ^α, it is necessary to know commutation relations between these functions and the Poincaré generators. Here we write down these relations. Let particle variables be x_a^i and p_{ai} with the Poisson bracket $\{x_a^i, p_{bj}\} = -\delta_{ab}\delta_j^i$. Then the instant form functions $\mathcal{A}_i$, $\mathcal{E}^i$ and $\mathcal{A}_0$ are transformed as

$$\{\mathcal{A}^l(t,\mathbf{x}), x^ip^k - x^kp^i - m^{ik}\} = \delta^{il}\mathcal{A}^k(t,\mathbf{x}) - \delta^{kl}\mathcal{A}^i(t,\mathbf{x}), \tag{B.1}$$
$$\{\mathcal{A}^l(t,\mathbf{x}), x^kp^0 - m^{k0}\} = \delta^{kl}\mathcal{A}_0(t,\mathbf{x}), \tag{B.2}$$
$$\{\mathcal{E}^l(t,\mathbf{x}), x^ip^k - x^kp^i - m^{ik}\} = \delta^{il}\mathcal{E}^k(t,\mathbf{x}) - \delta^{kl}\mathcal{E}^i(t,\mathbf{x}), \tag{B.3}$$
$$\{\mathcal{E}^l(t,\mathbf{x}), x^kp^0 - m^{k0}\} = \frac{1}{4\pi}\left(\partial^l\mathcal{A}^k(t,\mathbf{x}) - \partial^k\mathcal{A}^l(t,\mathbf{x})\right), \tag{B.4}$$
$$\{\mathcal{A}_0(t,\mathbf{x}), x^ip^k - x^kp^i - m^{ik}\} = 0, \tag{B.5}$$
$$\{\mathcal{A}_0(t,\mathbf{x}), x^kp^0 - m^{k0}\} = \mathcal{A}^k(t,\mathbf{x}). \tag{B.6}$$

Here

$$p^0 = \sum_{a=1}^N \sqrt{m_a^2 + \mathbf{p}_a^2}, \quad p^i = \sum_{a=1}^N p_a^i, \tag{B.7}$$

$$m^{k0} = \sum_{a=1}^N x_a^k\sqrt{m_a^2 + \mathbf{p}_a^2}, \quad m^{ik} = \sum_{a=1}^N (x_a^ip_a^k - x_a^kp_a^i). \tag{B.8}$$

In the front form of dynamics the functions α_0 and b^α have the following properties

$$\{\mathsf{b}^\alpha(t,\mathbf{x}), \sum_{a=1}^N (x^k - x_a^k)p_a^3\} = -\eta^{\alpha k}\partial_3\alpha_0(t,\mathbf{x}) - \delta_3^k\partial^\alpha\alpha_0(t,\mathbf{x}), \tag{B.9}$$

$$\{\mathsf{b}^\alpha(t,\mathbf{x}), m^{\beta\gamma} - x^\beta p^\gamma + x^\gamma p^\beta\} = \eta^{\alpha\beta}\mathsf{b}^\gamma(t,\mathbf{x}) - \eta^{\alpha\gamma}\mathsf{b}^\beta(t,\mathbf{x}), \tag{B.10}$$

$$\{\mathsf{b}^\alpha(t,\mathbf{x}), m^{\beta 3} - x^\beta p^{\gamma 3} + x^3p^\beta - x^\beta h\} = \eta^{\alpha\beta}\partial_3\alpha_3(t,\mathbf{x}) + \partial_\beta\alpha_\alpha, \tag{B.11}$$

$$\{\alpha_0(t,\mathbf{x}), \sum_{a=1}^{N}(x^k - x_a^k)p_a^3\} = \delta_3^k \alpha_0(t,\mathbf{x}), \quad \text{(B.12)}$$

$$\{\alpha_0(t,\mathbf{x}), m^{\beta\gamma} - x^\beta p^\gamma + x^\gamma p^\beta\} = 0, \quad \text{(B.13)}$$

$$\{\alpha_0(t,\mathbf{x}), m^{\beta 3} - x^\beta p^{\gamma 3} + x^3 p^\beta - x^\beta h\} = \alpha_\beta(t,\mathbf{x}), \quad \text{(B.14)}$$

where

$$p^i = \sum_{a=1}^{N} p_a^i + \delta_3^i h, \quad h = \sum_{a=1}^{N} h_a = \sum_{a=1}^{N} \frac{\mathbf{p}_a^2 + m_a^2}{2p_{a3}}, \quad \text{(B.15)}$$

$$m^{ik} = \sum_{a=1}^{N}(x_a^i p_a^k - x_a^k p_a^i) + \sum_{a=1}^{N}(x_a^i \delta_3^k - x_a^k \delta_3^i) h_a. \quad \text{(B.16)}$$

References

1. P. A. M. Dirac, *Can. J. Math.* **2**, 129 (1950).
2. L. Lusanna, *Int. J. Mod. Phys.* **A12**, 645 (1997).
3. L. Lusanna, *Int. J. Mod. Phys.* **A10**, 3675 (1995).
4. H. Crater and L. Lusanna, SISSA e-preprint hep-th/0001046, 2000; D. Alba and L. Lusanna, *Int. J. Mod. Phys.* **A13**, 2791 (1998).
5. F. Hoyle and J. V. Narlikar, *Action-at-a-Distance in Physics and Cosmology* (Freeman, San Francisco, 1974); *Rev. Mod. Phys.* **67**, 113 (1995).
6. R. P. Gaida, *Sov. J. Part. Nucl.* **13**, 179 (1982).
7. J. Darewych, *Cond. Matter Phys.* **1**, 593 (1998).
8. J. A. Wheeler and R. P. Feynman, *Rev. Mod. Phys.* **21**, 425 (1949).
9. F. J. Kennedy, *J. Math. Phys.* **16**, 1844 (1975).
10. J. Llosa and J. Vives, *J. Math. Phys.* **35**, 2856 (1994).
11. A. Nazarenko, *Proceedings of Institute of Mathematics of Nat. Acad. Sci. of Ukraine* **30**, 343 (2000).
12. P. A. M. Dirac, *Rev. Mod. Phys.* **21**, 392 (1949).
13. L. D. Landau and E.M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley, New York, 1971).
14. F. Rohrlich, *Classical Charged Particles: Foundations of Their Theory* (Addison-Wesley, New York, 1990).
15. S. Parrott, *Relativistic Electrodynamics and Differential Geometry* (Springer, New York, 1987).
16. A. Nazarenko and V. Tretyak, *Cond. Matter Phys.* **1**, 463 (2000).
17. E. C. G. Sudarshan and N. Mukunda, *Classical Dynamics: A Modern Perspective.* (Wiley, New York, 1974).

18. D. G. Currie, J. F. Jordan, and E. C. G. Sudarshan, *Rev. Mod. Phys.* **35**, 350 (1963).
19. V. Tretyak and V. Shpytko, *J. Phys. A: Math. Gen.* **33**, 5719 (2000).
20. S. N. Sokolov and V. I. Tretyak, *Theor. Math. Phys.* **67**, 385 (1986).
21. A. Duviryak, V. Shpytko, and V. Tretyak, *Cond. Matter Phys.* **1**, 463 (1998).
22. C. G. Darwin, *Phil. Mag.* **39**, 537 (1920).

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