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THE TWO-PARTICLE TIME-ASYMMETRIC RELATIVISTIC MODEL WITH CONFINEMENT INTERACTION. QUANTIZATION

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Двочастинкова часо-асиметрична релятивістична модель з утримуючою взаємодією. Квантування

А. Дувіряк

Анотація. Розглядається релятивістична двочастинкова система з утримуючою взаємодією. На основі класичної часо-асиметричної моделі, представленої в роботі [1], будується квантово-механічний опис системи. Аналізується спектр мас і траєкторії Редже.

The Two-Particle Time-Asymmetric Relativistic Model with Confinement Interaction. Quantization

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Abstract. The relativistic two-particle system with confinement interaction is considered. The quantum-mechanical description of the system is built on the base of classical time-asymmetric model presented in Ref. [1]. The mass spectrum and the Regge trajectories are analyzed.

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1. Introduction

In the previous paper [1] the time-asymmetric model of the relativistic two-particle system with confining interaction was proposed, and the classical dynamics of this system was analyzed. Semiclassical estimates of the mass spectrum shown that the model can be a good candidate for the relativistic potential model of mesons. The present paper is devoted to a quantization of this model.

Nonrelativistic models of hadrons involve potential which consists of the short-range and long-range parts (for example, the Coulomb + linear potential) [2]. The short-range potential follows from the perturbative quantum chromodynamics (QCD) while the long-range one is built rather phenomenologically. Simplest relativistic models are constructed as single-particle wave equations [3] with the nonrelativistic potential and, possibly, spin corrections. More consistent models are based on the relativistic direct interaction theory (RDIT) [4], mainly, in various Hamiltonian versions [5–7]. Given a nonrelativistic potential, RDIT allows one to construct (although non-uniquely) the consistent relativistic model of few-body system. The examples of such potential models of hadrons are presented in Refs [8].

We are based on another approach to RDIT, the formalism of Fokker-type action integrals [9]. It permits the treatment of particle interaction in terms of the retarded and advanced solutions of classical field equations. In the considered time-asymmetric model the short-range interaction was built with the Lienard-Wiechert potentials which satisfy the Yang-Mills equations with moving point-like sources [1]. For the long-range interaction we proceeded with the classical 4th-order equations for the effective Yang–Mills field averaged over quantum fluctuations [10]. Abelian retarded and advanced solutions to these equations obtained in Ref. [1] provide the confinement of interacting particles.

Both the short- and long-range interactions were chosen as the time-asymmetric ones, i.e., as if the first particle perceives the retarded field of the second particle while the latter senses the advanced field of the first particle. From the physical point of view the time-asymmetric interaction is not too artificial for mesons which are quark-antiquark systems. Indeed, following the Dirac treatment an antiparticle can be considered as a particle moving backwards in time and thus generating the advanced field. On the other hand, this choice allowed us to reduce the Fokker-type integral to a single-time action. Then we constructed various Hamiltonian descriptions of the model which are the base for a quantization.

Here we use the three-dimensional Hamiltonian description of the model in the point form of relativistic dynamics [5]. The complexity of the total mass of the system as a function of canonical variables does not permit us (even in the case without short-range interaction) to construct a relevant Hermitian operator leading to a solvable quantum model. Instead, we modify a quantization procedure by the construction of the mass-shell equation. It has a structure of stationary Schrödinger equation with potential depending on the energy (i.e., the total mass in our case).

The solution to the mass-shell equation is search as the expansion series in the Jacobi polynomials. This allow us to reduce the eigenvalue problem to an algebraic equation in terms of continued fraction. Then we analyze the asymptotic behaviour of the mass spectrum and present numerical results.

The paper is organized as follows. In Sec.2 we consider the classical time-asymmetric model with purely confining (long-range) interaction. Following Refs [1,11] we reformulate the Fokker-type version of the model into the canonical formalism with constraints and then into the three-dimensional Hamiltonian formalism (see also Appendix A). The canonical quantization is performed in Sec.3. It leads to the mass-shell equation analyzed in Sec.4. The eigenvalue problem is reduced to an algebraic equation expressed via the continued fraction (some remarks on continued fractions are collected in Appendix B). The spectrum mass for $\ell = 0$ and $\ell \gg 0$ is analyzed in Subsec. 4.1 and 4.2, respectively. In Sec.5 we include the short-range interaction in the model. Numerical results are presented in Sec.6.

2. Classical time–asymmetric model with confining interaction

We begin our consideration with the time-asymmetric two-particle model with purely confining (i.e., long-range) interaction. It is based on the time-asymmetric Fokker-type action of the following form [1]:

$$I = I_{\rm free} + I_{\rm conf}, \tag{1}$$

where

$$I_{\rm free} = -\sum_{a=1}^{2} m_a \int d\tau_a \sqrt{\dot{z}_a^2} \tag{2}$$

is a free-particle term, and

$$I_{\rm conf} = -\beta \int\!\!\int d\tau_1 d\tau_2 (z\cdot\dot{z}_1)(z\cdot\dot{z}_2) D_\eta(z) \tag{3}$$

describes the confining interaction with the coupling constant β. Here m_a ($a = 1, 2$) is the rest mass of ath particle; $z_a^\mu(\tau_a)$ ($\mu = 0, ..., 3$) are the covariant coordinates of ath particle in the Minkowski space $\mathbb{M}_4$; τ_a is an arbitrary evolution parameter on the ath world line; $\dot{z}_a^\mu(\tau_a) \equiv dz_a^\mu/d\tau_a$; $z^\mu \equiv z_1^\mu - z_2^\mu$; $D_\eta(z) = 2\Theta(\eta z^0)\delta(z^2)$ is the retarded (for $\eta = +1$) or advanced ($\eta = -1$) Green's function of d'Alembert equation. The time-like Minkowski metrics, i.e., $\|\eta_{\mu\nu}\| = \mathrm{diag}(+, -, -, -)$, is chosen, and the light speed is put to be unit.

The model makes the sense if particle world lines are timelike, i.e., $\dot{z}_a^2 > 0$. In the nonrelativistic limit this model gives the linear potential $U_{\mathrm{conf}} = \beta r$.

Following Refs [1,11] we reformulate the model into the single-time manifestly covariant Hamiltonian formalism on the 16-dimensional phase space $\mathrm{T}^*\mathbb{M}_4^2$ parameterized by the particle positions z_a^μ and conjugated momenta $p_{a\mu}$. The canonical generators of Poincaré group acting onto this space have the standard kinematic form:

$$P_\mu = \sum_{a=1}^{2} p_{a\mu}, \qquad J_{\mu\nu} = \sum_{a=1}^{2} (z_{a\mu} p_{a\nu} - z_{a\nu} p_{a\mu}). \tag{4}$$

Due to the parametric invariance of the description the canonical Hamiltonian vanishes while there are two first class constraints. One of them, the *light-cone constraint*, is holonomic:

$$z^2 = 0, \quad \eta z^0 > 0, \qquad \text{i.e.,} \qquad \eta z^0 = |\mathbf{z}|. \tag{5}$$

Here $\mathbf{z} \equiv (z_i = -z^i)$ ($i = 1, 2, 3$). Another, the *mass-shell constraint*, determines the dynamics of the model. It has the following form:

$$\phi(P^2,\ v^2,\ P\cdot z,\ v\cdot z) \equiv \phi_{\mathrm{free}} + \phi_{\mathrm{conf}} = 0, \tag{6}$$

where $v_\mu \equiv w_\mu - z_\mu\, P{\cdot}w/P{\cdot}z$, $w_\mu \equiv (p_{1\mu} - p_{2\mu})/2$,

$$\phi_{\mathrm{free}} = \tfrac{1}{4}P^2 - \tfrac{1}{2}(m_1^2 + m_2^2) + (m_1^2 - m_2^2)\frac{v{\cdot}z}{P{\cdot}z} + v^2, \tag{7}$$

$$\phi_{\mathrm{conf}} = -2\beta\frac{b_1 b_2}{\eta P{\cdot}z}, \tag{8}$$

$$b_a \equiv \eta\left(\tfrac{1}{2}P{\cdot}z - (-)^a v{\cdot}z\right), \qquad a = 1, 2. \tag{9}$$

The Poincaré-invariance of these constraints guarantees that the generators (4) are conserved.

The Hamiltonian description is equivalent to the Fokker one provided the following conditions hold:

$$P^2 > 0, \qquad P_0 > 0, \qquad b_a > 0, \qquad a = 1, 2. \tag{10}$$

They determine a physical domain in the phase space $\mathrm{T}^*\mathbb{M}_4^2$.

The next step towards quantization is the construction of the three-dimensional Hamiltonian description of the model. The reformulation of the time-asymmetric models into the three-dimensional Hamiltonian formalism was described in Ref [11]. It consists in the elimination of four timelike canonical variables by means of the pair of first-class constraints and the pair of gauge-fixing constraints. The choice of one of the gauge-fixing constraints determines the evolution parameter t as well as the form of relativistic dynamics [5–7] in the reduced 12-dimensional phase space $\mathbb{P}$. In Refs [1,11] the instant form of dynamics was built. Here we fix the evolution parameter by the following constraint:

$$\sum_{a=1}^{2} \frac{z_a \cdot p_a}{|P|} - t = 0, \tag{11}$$

where $|P| \equiv \sqrt{P^2}$. This choice leads to the description which is very close to the point form of dynamics [5].

The technique of the reduction procedure uses two successive canonical transformations in $\mathrm{T}^*\mathbb{M}_4^2$ which are concerted with the structure of constraints and provide the parameterization of $\mathbb{P}$ with canonical variables. Transformations relevant to the choice (11) are written down in Appendix A.

We use the external Q^i, P_i and the internal ρ^i, π_i centre-of-mass canonical variables ($i = 1, 2, 3$) satisfying standard Poisson-bracket (PB) relations: $\{Q^i, P_j\} = \delta^i_j$, $\{\rho^i, \pi_j\} = \delta^i_j$ (other PB vanish). In these terms the dynamics of the model in $\mathbb{P}$ is determined by canonical generators of Poincaré group:

$$\begin{aligned} P_\mu &= MG_\mu, \quad G_0 \equiv \sqrt{1 + \mathbf{G}^2}, \\ J_i &\equiv \varepsilon_i^{\ jk} J_{jk} = \varepsilon_i^{\ jk} Q_j G_k + S_i, \\ K_i &\equiv J_{i0} = G_0 Q_i + \frac{(\mathbf{G} \times \mathbf{S})_i}{1 + G_0}. \end{aligned} \tag{12}$$

Here $\mathbf{S} \equiv \boldsymbol{\rho} \times \boldsymbol{\pi}$ is the total spin (the internal angular momentum) of the system, and $M(\boldsymbol{\rho}, \boldsymbol{\pi})$ is the total mass of the system which takes the role of Hamiltonian, i.e., it generates the evolution in t.

Hereafter we consider the system of equal rest masses, $m_1 = m_2 \equiv m$. Then the function $M(\boldsymbol{\rho}, \boldsymbol{\pi})$ is determined by the following mass-shell equation:

$$\Gamma(M, \boldsymbol{\rho}, \boldsymbol{\pi}) \equiv (1 - \rho)\left(\frac{M^4}{16\beta^2} - \pi_\rho^2\right) - \frac{M^2 m^2}{4\beta^2} - \frac{S^2}{\rho^2} = 0, \tag{13}$$

where $\rho \equiv |\boldsymbol{\rho}|$, $S \equiv |\mathbf{S}|$, $\pi_\rho \equiv \boldsymbol{\pi}\cdot\mathbf{n}$ and $\mathbf{n} \equiv \boldsymbol{\rho}/\rho$. The equation (13) must be complemented by the condition $|\pi_\rho| < \frac{1}{4}M^2/\beta$ which follows from (10). Then the only positive solution of this equation,

$$M = \sqrt{2\left(\frac{m^2}{1-\rho} + \sqrt{4\beta^2\left(\pi_\rho^2 + \frac{S^2}{(1-\rho)\rho^2}\right) + \frac{m^4}{(1-\rho)^2}}\right)}, \qquad (14)$$

satisfies the inequality $M \geq 2m$. It exists in the domain

$$0 < \rho < 1. \qquad (15)$$

Particle trajectories possessing nonrelativistic and free-particle limits entirely belong to this domain.

Besides the canonical realization of the Poincaré group, the equations (A.4), (A.7) permit to obtain the particle positions z_a^μ as functions of the canonical variables, what makes it possible to build particle world lines in the Minkowski space $\mathbb{M}_4$. They was analyzed in Ref [1]. Here we note that in the centre-of-mass reference frame (where $\mathbf{P} = \mathbf{0}$) the relative distance between particles $r = \frac{1}{2}M\rho/\beta$. Then it follows from (15) that the classical system does not exceeds the extension $\frac{1}{2}M/\beta$.

3. Quantization

Let us put $\hbar = 1$ and introduce quantum operators $\hat{Q}^i$, $\hat{P}_i$, $\hat{\rho}^i$, $\hat{\pi}_i$ satisfying commutational relations: $[\hat{Q}^i, \hat{P}_j] = \mathrm{i}\delta^i_j$, $[\hat{\rho}^i, \hat{\pi}_j] = \mathrm{i}\delta^i_j$ (other commutators vanish). Using $(\mathbf{G}, \boldsymbol{\rho})$-representation with the basis $|\mathbf{G}, \boldsymbol{\rho}\rangle = |\mathbf{G}\rangle \otimes |\boldsymbol{\rho}\rangle$,

$$\hat{\mathbf{G}}|\mathbf{G}\rangle = \mathbf{G}|\mathbf{G}\rangle, \qquad \langle\mathbf{G}'|\mathbf{G}\rangle = G_0\delta^3(\mathbf{G}' - \mathbf{G}), \qquad (16)$$

$$\hat{\boldsymbol{\rho}}|\boldsymbol{\rho}\rangle = \boldsymbol{\rho}|\boldsymbol{\rho}\rangle, \qquad \langle\boldsymbol{\rho}'|\boldsymbol{\rho}\rangle = \delta^3(\boldsymbol{\rho}' - \boldsymbol{\rho}), \qquad (17)$$

splits the Hilbert space $\mathcal{H}$,

$$|\Psi\rangle = \int \frac{d^3\mathbf{G}}{G_0} \int d^3\boldsymbol{\rho}\ \Psi(\mathbf{G}, \boldsymbol{\rho})|\mathbf{G}, \boldsymbol{\rho}\rangle, \qquad |\Psi\rangle \in \mathcal{H}, \qquad (18)$$

with the Poincaré-invariant inner product

$$\langle\Phi|\Psi\rangle = \int \frac{d^3\mathbf{G}}{G_0} \int d^3\boldsymbol{\rho}\ \Phi^*(\mathbf{G}, \boldsymbol{\rho})\Psi(\mathbf{G}, \boldsymbol{\rho}), \qquad |\langle\Phi|\Psi\rangle| < \infty \qquad (19)$$

into the external and internal spaces, $\mathcal{H} = \mathcal{H}_{\rm ext} \otimes \mathcal{H}_{\rm int}$.

The unitary representation of Poincaré group is generated by the Hermitian operators:

$$\hat{P}_0 = \hat{M}G_0, \quad \hat{\mathbf{P}} = \hat{M}\mathbf{G},$$
$$\hat{\mathbf{J}} = \mathrm{i}\,\frac{\partial}{\partial\mathbf{G}} \times \mathbf{G} + \hat{\mathbf{S}},$$
$$\hat{\mathbf{K}} = \mathrm{i}\,G_0\frac{\partial}{\partial\mathbf{G}} + \frac{\mathbf{G}\times\hat{\mathbf{S}}}{1 + G_0}, \qquad (20)$$

where $\hat{\mathbf{S}} = -\mathrm{i}\,\boldsymbol{\rho}\times\partial/\partial\boldsymbol{\rho}$.

The mass operator $\hat{M}$ is supposed to be the positively defined Poincaré-invariant Hermitian operator acting in $\mathcal{H}_{\rm int}$:

$$[\hat{M}, (\hat{S}^2, \hat{S}_3, \hat{\mathbf{G}})] = 0. \qquad (21)$$

Thus any eigenfunction of $\hat{M}$ can be split as follows: $\Psi(\mathbf{G}, \boldsymbol{\rho}) = \Psi_{\rm ext}(\mathbf{G})\Psi_{\rm int}(\boldsymbol{\rho})$, where $\Psi_{\rm ext}(\mathbf{G}) \in \mathcal{H}_{\rm ext}$ is arbitrary, and $\Psi_{\rm int}(\boldsymbol{\rho}) \in \mathcal{H}_{\rm int}$ satisfies the stationary Schrödinger equation:

$$\hat{M}\Psi_{\rm int}(\boldsymbol{\rho}) = M\Psi_{\rm int}(\boldsymbol{\rho}). \qquad (22)$$

Then separating variables $\Psi_{\rm int}(\boldsymbol{\rho}) = \psi(\rho)Y_{\ell\sigma}(\mathbf{n})$, where $Y_{\ell\sigma}(\mathbf{n})$ are the spherical harmonics:

$$\hat{S}^2 Y_{\ell\sigma}(\mathbf{n}) = \ell(\ell+1)Y_{\ell\sigma}(\mathbf{n}), \qquad \ell = 0, 1, ...,$$
$$\hat{S}_3 Y_{\ell\sigma}(\mathbf{n}) = \sigma Y_{\ell\sigma}(\mathbf{n}), \qquad \sigma = -\ell, ..., \ell, \qquad (23)$$

reduces equation (22) to a one-dimensional problem for radial eigenfunction $\psi(\rho)$ in the Hilbert space $L^2(\mathbb{R}_+)$ with the inner product

$$\langle\varphi|\psi\rangle_{\rm rad} = \int_0^\infty \rho^2 d\rho\varphi^*(\rho)\psi(\rho), \qquad |\langle\varphi|\psi\rangle_{\rm rad}| < \infty. \qquad (24)$$

The cumbersome classical expression (14) for the mass function complicates the construction of relevant Hermitian operator. Moreover, the eigenvalue problem obtained in this way is not expected to be solvable. Thus we use another quantization procedure which is inspired by quasipotential approach [12].

Let us construct the Hermitian operator $\hat{\Gamma}(M)$ corresponding to classical function $\Gamma(M, \boldsymbol{\rho}, \boldsymbol{\pi})$, where M is treated as a constant. Then one can consider, instead of the Schrödinger equation (22), the quantum analog of the mass-shell equation (13):

$$\hat{\Gamma}(M)\Psi_M = 0, \qquad (25)$$

where M is a spectral parameter. If $\hat{\Gamma}(M)$ involves M non-linearly, the eigenfunctions Ψ_M are not orthogonal. The orthogonality can be renewed by the following redefinition of inner product [12]:

$$\langle\Psi_{M'}|\Psi_M\rangle \to \langle\langle\Psi_{M'}|\Psi_M\rangle\rangle = \left\langle \Psi_{M'} \left| \frac{\hat{\Gamma}(M') - \hat{\Gamma}(M)}{M' - M} \right| \Psi_M \right\rangle. \tag{26}$$

In our case, after separating variables, the mass-shell equation can be presented in the form:

$$\hat{\Gamma}_\ell(M)\psi(\rho) = 0, \tag{27}$$

where

$$\hat{\Gamma}_\ell(M) = -\ \widehat{(1-\rho)\pi_\rho^2}\ + (1-\rho)\frac{M^4}{16\beta^2} - \frac{M^2 m^2}{4\beta^2} - \frac{\ell(\ell+1)}{\rho^2} \tag{28}$$

must be the Hermitian operator. Thus the operator $\widehat{(1-\rho)\pi_\rho^2}$ should be an ordered construction of ρ and the radial momentum $\hat{\pi}_\rho$ which are Hermitian in $L^2(\mathbb{R}_+)$ (24). We use the following symmetrization rule:

$$\widehat{(1-\rho)\pi_\rho^2} = (1-\rho)^\delta \hat{\pi}_\rho (1-\rho)^{1-2\delta}\hat{\pi}_\rho(1-\rho)^\delta, \quad \hat{\pi}_\rho = -\frac{\mathrm{i}}{\rho}\frac{d}{d\rho}\rho, \tag{29}$$

where the quantization parameter $\delta \sim 1$ can be adjusted to physical demands.

4. Analysis of the mass-shell equation

The mass-shell equation (27) has the form of ordinary differential equation:

$$\left\{(1-\rho)\frac{d^2}{d\rho^2} + \frac{2-3\rho}{\rho}\frac{d}{d\rho} + \mu^2(1-\rho) - 2\mu\nu - \frac{\delta^2}{1-\rho} - \frac{1}{\rho} - \frac{\ell(\ell+1)}{\rho^2}\right\}\psi(\rho) = 0, \tag{30}$$

where

$$\mu \equiv \frac{M^2}{4\beta}, \qquad \nu \equiv \frac{m^2}{2\beta}, \tag{31}$$

and μ is the dimensionless spectral parameter of the problem. Since equation (30) includes the quantization parameter as δ^2, one can put $\delta \geq 0$.

The equation (30) has three critical points $\rho = 0, 1, \infty$ at which the eigenfunction $\psi(\rho)$ behaves as follows:

$$\begin{aligned} \rho \to 0: &\quad \psi(\rho) \sim \rho^\ell \text{ or } \rho^{-(\ell+1)}; \\ \rho \to 1: &\quad \psi(\rho) \sim (1-\rho)^{\pm\delta}; \\ \rho \to \infty: &\quad \psi(\rho) \sim \exp(\pm \mathrm{i}\mu\rho). \end{aligned} \tag{32}$$

We require of $\psi(\rho)$ be a continuous function belonging to $L^2(\mathbb{R}_+)$. This demand does not contradict to estimates (32) if the following boundary conditions are imposed:

$$\begin{aligned} \rho \to +0: &\quad \psi(\rho) \sim \rho^\ell; \\ \rho \to 1-0: &\quad \psi(\rho) \sim (1-\rho)^\delta, \quad \delta > 0; \\ \rho \geq 1: &\quad \psi(\rho) = 0. \end{aligned} \tag{33}$$

In such a way the confinement condition (15) is naturally brought onto a quantum level. In fact, we restrict the Hilbert space $L^2(\mathbb{R}_+)$ to $L^2([0,1])$ in which the operator $\hat{\Gamma}_\ell(M)$ remains to be Hermitian provided the conditions (33) hold.

Using the substitution:

$$\psi(\rho) = \exp(\pm \mathrm{i}\mu\rho)\rho^\ell(1-\rho)^\delta\chi(\rho), \tag{34}$$

and then expanding $\chi(\rho) \in L^2([0,1])$ into the series in the Jacobi polynomials [13]:

$$\chi(\rho) = \sum_{k=0}^{\infty} C_k P_k^{(2\delta, 2\ell+1)}(2\rho - 1), \tag{35}$$

reduces the eigenfunction problem (30) to the infinite three-diagonal set of linear homogeneous equations for C_k:

$$\xi_k C_{k+1} + \eta_k C_k + \zeta_k C_{k-1} = 0, \qquad k = 0, 1, ..., \tag{36}$$

where $\zeta_0 = 0$ and

$$\begin{aligned} \xi_k &= \mp\frac{\mathrm{i}\mu(k+2\ell+2)(k+2\delta+1)}{2(k+\ell+\delta+2)}\left(1 \pm \frac{\mathrm{i}\nu}{k+\ell+\delta+3/2}\right), \\ \eta_k &= (k+\ell+1)(k+\ell+2\delta+1) \\ &\quad + \mu\nu\left(1 + \frac{(\ell+\delta+1/2)(\ell-\delta+1/2)}{(k+\ell+\delta+1/2)(k+\ell+\delta+3/2)}\right), \\ \zeta_k &= \pm\frac{\mathrm{i}\mu k(k+2\ell+2\delta+1)}{2(k+\ell+\delta)}\left(1 \mp \frac{\mathrm{i}\nu}{k+\ell+\delta+1/2}\right). \end{aligned} \tag{37}$$

It is known [14] that the determinant of three-diagonal linear set can be expressed in terms of continued fractions (see [15]; basic notions are collected in Appendix B). The secular equation for the set (36) reads:

$$\Delta_\ell(\mu) \equiv b_0 + \mathop{\mathrm{K}}_{k=1}^{\infty} (a_k/b_k) = 0, \tag{38}$$

where $b_k = \eta_k$ and

$$a_k = -\xi_{k-1}\zeta_k = -\frac{\mu^2 k(k+2\delta)(k+2\ell+1)(k+2\ell+2\delta+1)}{4(k+\ell+\delta)(k+\ell+\delta+1)} \times \left(1 + \frac{\nu^2}{(k+\ell+\delta+1/2)^2}\right) \tag{39}$$

are real functions of the spectral parameter μ (in despite of the coefficients ξ_k, ζ_k be complex). Since the continued fraction $\Delta_\ell(\mu)$ meets the criterium of conventional convergence (B.5), it represents a real function of μ. Then positive solutions of the algebraic equation (38) form the spectrum of the system.

4.1. Spectrum of S-states

In the case $\ell = 0$ the continued fraction $\Delta_0(\mu)$ simplifies by means of the equivalence transformation (B.6) with some r_k:

$$\begin{aligned}\Delta_0(\mu) &= \frac{C}{\mu}\left(\beta_{\delta+1/2} + \mathop{\mathrm{K}}_{k=1}^{\infty} (\alpha_{k+\delta+1/2}/\beta_{k+\delta+1/2})\right) \\ &= \frac{C\alpha_{\delta+1/2}}{\mu}\left(\mathop{\mathrm{K}}_{k=0}^{\infty} (\alpha_{k+\delta+1/2}/\beta_{k+\delta+1/2})\right)^{-1};\end{aligned} \tag{40}$$

here C is a constant, and

$$\alpha_\lambda = -\frac{(\lambda+1)\sqrt{\lambda^2+\nu^2}}{\lambda\sqrt{(\lambda+1)^2+\nu^2}}, \quad \beta_\lambda = \frac{(2\lambda+1)[\nu+\lambda(\lambda+1)/\mu]}{\lambda\sqrt{(\lambda+1)^2+\nu^2}}. \tag{41}$$

Then using the continued fraction representation of Coulomb wave functions [15]:

$$\frac{F_\lambda(\nu,\mu)}{F_{\lambda-1}(\nu,\mu)} = -\mathop{\mathrm{K}}_{k=0}^{\infty} (\alpha_{k+\lambda}/\beta_{k+\lambda}), \tag{42}$$

reduces equation (38) to the form:

$$F_{\delta-1/2}(\nu,\mu) = 0, \quad \mu > 0. \tag{43}$$

This equation has an infinite number of positive roots $\mu = \mu_{n_\rho}(\nu)$ labelled by the radial quantum number $n_\rho = 0, 1, \dots$.

Let us consider the spectrum of S-states in two limiting cases.

Nonrelativistic spectrum. In the chosen units of measurement the coupling constant β has the dimension of mass (or energy) squared. Then in the nonrelativistic case the binding energy $E = M - 2m$ is comparable to $\sqrt{\beta}$ while the rest mass is large: $m \gg \sqrt{\beta}$. In dimensionless terms this corresponds to $\mu \sim 2\nu \gg 0$. Using for this case the asymptotic representation of the Coulomb wave function [13]:

$$F_\lambda(\nu,\mu) \sim \mathrm{Ai}(x), \tag{44}$$

where $\mathrm{Ai}(x)$ is the Airy function, and

$$x = (1 - \mu/\mu_\lambda)[\mu^2 + \lambda(\lambda+1)]^{1/3}, \quad \mu_\lambda = \nu + \sqrt{\nu^2 + \lambda(\lambda+1)}, \tag{45}$$

and solving the equation (43) gives

$$\sqrt{\mu} = \sqrt{2\nu} + \tfrac{1}{2}(2\nu)^{-1/6} x_{n_\rho+1} + O(\nu^{-5/6}), \tag{46}$$

or (in dimensional terms),

$$E = M - 2m \approx (\beta^2/m)^{1/3} x_{n_\rho+1}, \tag{47}$$

where $(-x_n)$ are zeros of the Airy function. Equation (47) describes the well-known nonrelativistic S-spectrum of two-particle system with a linear potential. We note that the dependence on the quantization parameter appears as $O(\nu^{-3/2})$, i.e., far from the area of nonrelativistic approximation.

Ultrarelativistic spectrum: $M \to \infty$. In the case $\mu \to \infty$ we have [13]:

$$F_\lambda(\nu,\mu) \sim \sin(\mu - \nu\ln 2\mu - \lambda\pi/2 + \sigma_\lambda), \tag{48}$$

where $\sigma_\lambda = \arg\Gamma(\lambda + 1 + \mathrm{i}\nu)$. Then equation (43) yields:

$$M^2 = \pi\beta(4n_\rho + 2\delta + 3) + 2m^2\ln[2\pi(n_\rho+1)] - \sigma_{\delta-1/2} + O(n_\rho^{-1}). \tag{49}$$

The dependence of M^2 on $n_\rho \gg 0$ becomes asymptotically linear.

4.2. Regge trajectories in the oscillator approximation

Here we consider the asymptotic behaviour of the mass spectrum for $\ell \gg 0$. For this purpose we use the substitution

$$\psi(\rho) = \frac{\varphi(\rho)}{\rho\sqrt{1-\rho}}, \quad \varphi(\rho) \overset{\rho\to 0}{\sim} \rho^{\ell+1}, \quad \varphi(\rho) \overset{\rho\to 1}{\sim} (1-\rho)^{\delta+1/2} \tag{50}$$

reducing the mass-shell equation to a two-term form:

$$\frac{d^2\varphi(\rho)}{d\rho^2} + \left\{\mu^2 - \frac{2\mu\nu}{1-\rho} - \frac{\delta^2 - 1/4}{(1-\rho)^2} - \frac{\ell(\ell+1)}{(1-\rho)\rho^2}\right\}\varphi(\rho) = 0. \qquad (51)$$

Let us put for simplicity $\delta = 1/2$. Then equation (51) has a Schrödinger-like form with the potential

$$U(\rho) = \frac{2\mu\nu + \gamma/\rho^2}{1-\rho}, \qquad \gamma \equiv \ell(\ell+1) \qquad (52)$$

depending on the spectral parameter μ. If $\ell > 0$, this potential has a local minimum at some point $\rho_0 \in (0, 2/3)$ depending on μ and satisfying the condition:

$$U'(\rho_0) \propto 2\mu\nu\rho_0^3 + \gamma(3\rho_0 - 2) = 0. \qquad (53)$$

Thus one can expand the potential (52) at the minimum,

$$U(\rho) \approx U(\rho_0) + \frac{1}{2}U''(\rho_0)(\rho-\rho_0)^2 = \frac{2\gamma}{\rho_0^3} + \frac{3\gamma}{(1-\rho_0)\rho_0^4}(\rho-\rho_0)^2, \quad (54)$$

and search a solution of this oscillator problem. A quantization condition then reads:

$$\rho_0^2\sqrt{\frac{1-\rho_0}{3\gamma}}\left(\mu^2 - \frac{2\gamma}{\rho_0^3}\right) = 2n_\rho + 1. \qquad (55)$$

If $n_\rho \sim 1$, the approximate solution differs exponentially little from the exact solutions of eq. (51).

Eqs (53) and (55) form the set of algebraic equations with μ and ρ_0 to be found. Solving this set with the usage of power series in ℓ leads to the asymptotic formula:

$$M^2 = 6\sqrt{3}\beta(\ell + n_\rho + 1) + 6m^2 + O(\ell^{-1}) \qquad (56)$$

which represents the spectrum of the system at $\ell \gg n_\rho$. This spectrum falls on the family of the leading (for $n_\rho = 0$) and daughter's ($n_\rho > 0$) linear Regge trajectories. In the case of arbitrary quantization parameter one obtains the same result since δ-dependent terms occur as $O(\ell^{-1})$.

5. Accounting a short-range interaction

The short-range interaction is introduced in the model by adding to the Fokker-type action (1) the following term [1]:

$$I_{\rm vec} = \alpha\iint d\tau_1 d\tau_2 (\dot z_1 \cdot \dot z_2) D_\eta(z). \qquad (57)$$

It corresponds to the vector-type interaction which in the nonrelativistic limit is described by the Coulomb potential $U_{\rm Coul} = \alpha/r$ with a coupling constant α. Then the mass-shell constraint takes the form:

$$\phi_{\rm free} + \phi_{\rm conf} + \phi_{\rm vec} - 2\alpha\beta = 0, \qquad (58)$$

where $\phi_{\rm vec}$ is rather complicated function. Here we restrict ourselves by linear approximation in α:

$$\phi_{\rm vec} = \frac{\alpha(P^2 - m_1^2 - m_2^2)}{\eta P\cdot z} + O(\alpha^2). \qquad (59)$$

Then the analysis of the model is quite similar to the purely confinement case.

The operator $\hat\Gamma_\ell(M)$ (28) involved in the quantum mass-shell equation (27) should be modified as follows:

$$\hat\Gamma_\ell(M) \to \hat\Gamma_\ell(M;\alpha) = \hat\Gamma_\ell(M) + \alpha\frac{(1-\rho)M^2 - 2m^2}{2\beta\rho}. \qquad (60)$$

This leads to the secular equation of the form (38) where the modified continued fraction $\Delta_\ell(\mu;\alpha)$ is defined by the elements:

$$\begin{aligned} a_k &= -\frac{\mu^2 k(k+2\delta)(k+2\ell+1)(k+2\ell+2\delta+1)}{4(k+\ell+\delta)(k+\ell+\delta+1)} \\ &\quad\times\left(1 + \frac{(\nu+\alpha)^2}{(k+\ell+\delta+1/2)^2}\right), \\ b_k &= (k+\ell+1)(k+\ell+2\delta+1) + 2\alpha\nu \\ &\quad+\mu\left(\nu - \alpha + \frac{(\nu+\alpha)(\ell+\delta+1/2)(\ell-\delta+1/2)}{(k+\ell+\delta+1/2)(k+\ell+\delta+3/2)}\right). \qquad (61) \end{aligned}$$

In the oscillator approximation one obtains the asymptotic mass spectrum:

$$M^2 = 6\sqrt{3}\beta(\ell + n_\rho + 1) + 6(m^2 - \alpha\beta) + O(\ell^{-1}), \quad \ell \gg n_\rho. \qquad (62)$$

It coincides with the semiclassical result of Ref. [1][1].

6. Numerical results and discussion

The model with long- and short-range interactions represents the relativistic generalization of Coulomb + linear potential model. Both the

[1]The author asks pardon for errors made in eqs.(77) and (93) of Ref.[1] where the expression $\ldots - 3\alpha\beta)\ldots$ is to be read as $\ldots - \alpha\beta)\ldots$

relativistic and nonrelativistic problems are not exactly solvable. But the former has that advantage it admits of the algebraic secular equation $\Delta_\ell(\mu;\alpha) = 0$. Since the function $\Delta_\ell(\mu;\alpha)$ is expressed exactly in terms of continued fraction, standard numerical algorithms can be applied to solve the secular equation with arbitrary accuracy [14,15]. As a result one obtains the mass spectrum or (if to let $\ell \in \mathbb{R}_+$) the family of leading and daughter's Regge trajectories.

In the Figs 1–4 we present few examples of Regge trajectories. They are obtained by adjusting the parameters m, α and β to the masses of some unflavoured mesons. All these parameters (especially, m) are flavour-dependent (see Tabl. 1) while the quantization parameter δ weakly influences spectra. Thus here we do not use δ as an adjustable parameter and put $\delta = 1/2$ (in this case the operator (29) simplifies).

Up to spin effects the obtained masses of mesons and quarks correlate well with the experimental data [16]. This is concerned especially with the Regge trajectories for heavy $b\bar{b}$ and $c\bar{c}$ mesons (Figs 1 and 2, respectively). For light mesons spin effects are essential, but they cannot be described by the present model. In this case the validity of the model can be estimated by the consideration of Regge trajectories for mesons with a similar spin structure. Here we choose those light mesons which can be treated as ${}^3\ell_{\ell+1}$ states. Corresponding Regge trajectories for $s\bar{s}$ and $l\bar{l}$ mesons (l means u or d) are presented in Figs 3 and 4, respectively. They agree well with the linear asymptotic formula (62) even at $\ell \sim 1$.

The presented here mathematics and numerical results suggest the further elaboration of the model. The generalization to the case of different rest masses is straightforward. More important task is the accounting spin effects. It implies, firstly, the construction of the Fokker-type action for confining system of spinning particles and, secondly, the hamiltonization and quantization of this model. As a guideline to this task one can take the description of relativistic spinning particles within the framework of Wheeler-Feynman dynamics [17].

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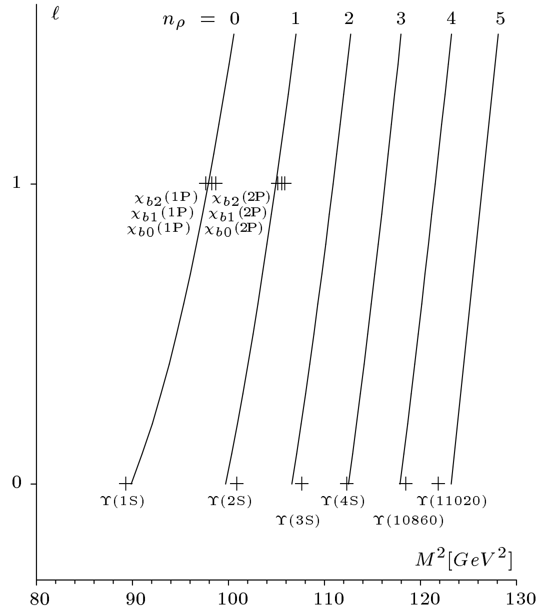

Figure 1

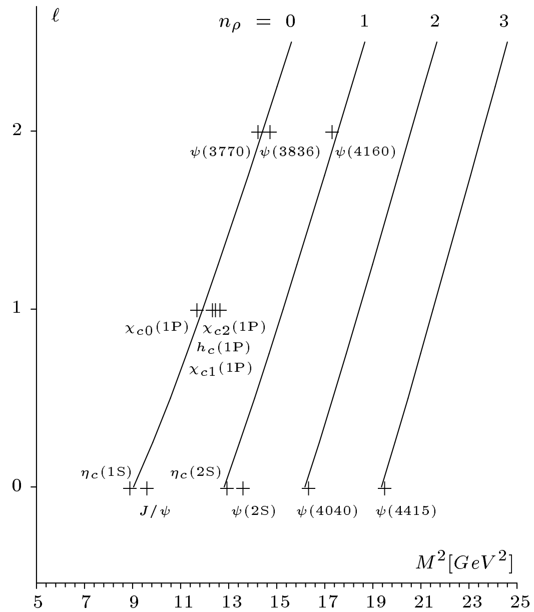

Figure 2

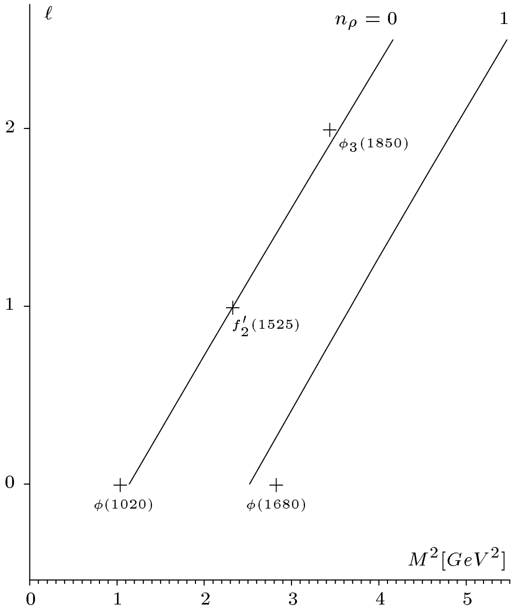

Figure 3

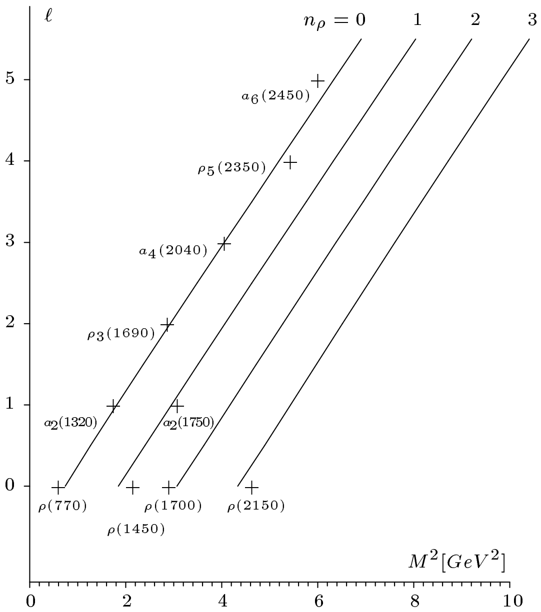

Figure 4

Table 1

Figure No	Mesons	$q\bar{q}$	m $[GeV]$	α	β $[GeV^2]$
Figure 1	$\Upsilon,\ \chi_b$	$b\bar{b}$	4.73	0.45	0.20
Figure 2	$\eta_c,\ h_c,\ \psi,\ \chi_c$	$c\bar{c}$	1.25	0.37	0.21
Figure 3	$\phi,\ f'$	$s\bar{s}$	0.10	0.40	0.12
Figure 4	$\rho,\ a$	$l\bar{l}$	0.005	0.80	0.11

Appendix

A. Transition into the three-dimensional Hamiltonian formalism in the point form of relativistic dynamics

The first canonical transformation

$$(y^\mu,\ P_\mu,\ x^\mu,\ w_\mu) \mapsto (Q^\mu,\ M,\ G_i,\ \rho^\mu,\ \omega_\mu), \tag{A.1}$$

is determined by means of the generating function:

$$W(y, M, \mathbf{G}, x, \omega) = MG_\mu\, y^\mu + (2\beta/M)\omega_\nu\, \Lambda(G)^\nu{}_\mu\, x^\mu, \tag{A.2}$$

where $y^\mu \equiv (z_1^\mu + z_2^\mu)/2$, $G_0 \equiv \sqrt{1+\mathbf{G}^2}$, and the matrix $\|\Lambda(G)^\nu{}_\mu\| \in \mathrm{SO}(1,3)$ has the form:

$$\|\Lambda^\mu{}_\nu\| \;=\; \left\| \begin{array}{c|c} G_0 & G_j \\ \hline G_i & \delta_{ij} + \dfrac{G_iG_j}{1+G_0} \end{array} \right\|. \tag{A.3}$$

Using the relations

$$\begin{aligned}
P_\mu &= \partial W/\partial y^\mu = MG_\mu, \\
Q^0 &= \partial W/\partial M = G\cdot y - (2\beta/M^2)\omega_\nu\, \Lambda^\nu{}_\mu\, x^\mu, \\
Q^i &= \partial W/\partial G_i = M(y^i - y^0 G^i/G_0) + (2\beta/M)\omega_\nu\, x^\mu\, \partial\Lambda^\nu{}_\mu/\partial G_i, \\
w_\mu &= \partial W/\partial z^\mu = (2\beta/M)\omega_\nu\, \Lambda^\nu{}_\mu, \\
\rho^\mu &= \partial W/\partial \omega_\mu = (2\beta/M)\Lambda^\mu{}_\nu\, z^\nu
\end{aligned} \tag{A.4}$$

one can obtain this transformation in an explicit form.

The second canonical transformation

$$(Q^0,\ Q^i,\ M,\ G_i,\ \rho^0,\ \rho^i,\ \omega_0,\ \omega_i) \\ \mapsto (\bar{Q}^0,\ Q^i,\ M,\ P_i,\ \bar{\rho}^0,\ \rho^i,\ \omega_0,\ \pi_i) \tag{A.5}$$

which is determined by the generating function:

$$\tilde{W} = M(Q^0 - t) + G_i Q^i + \omega_0(\rho^0 - \eta|\boldsymbol{\rho}|) + \pi_i \rho^i. \tag{A.6}$$

has the following explicit form:

$$\bar{Q}^0 = Q^0 - t, \quad \bar{\rho}^0 = \rho^0 - \eta|\boldsymbol{\rho}|, \quad \pi_i = \omega_i - \eta\omega_0\rho_i/|\boldsymbol{\rho}| \tag{A.7}$$

(the remaining variables do not change). It reduces the light-cone constraint (5) to the canonical form $\bar{\rho}^0 = 0$. Choosing one of the gauge-fixing constraint in the form $\bar{Q}^0 = 0$ allows us to eliminate the timelike variables $\bar{\rho}^0$, $\bar{Q}^0$ together with conjugated ones ω_0, M and to obtain the description on the reduced phase space $\mathbb{P}$ with the canonical variables Q^i, P_i, ρ^i, π_i $(i = 1, 2, 3)$.

The transformation (A.5) depends explicitly on the evolution parameter t. Thus it generates the Hamiltonian $H = -\partial\tilde{W}/\partial t = M$ which is the total mass of the system. On the reduced phase space M becomes the function of canonical variables. It is the positive solution of the mass-shell equation

$$\Gamma(M, \boldsymbol{\rho}, \boldsymbol{\pi}) \equiv \phi\left(M^2, -\frac{4\beta^2\boldsymbol{\pi}^2}{M^2}, \frac{\eta M^2|\boldsymbol{\rho}|}{2\beta}, -\boldsymbol{\rho}\cdot\boldsymbol{\pi}\right) = 0, \tag{A.8}$$

where the function ϕ is defined by eqs (6)–(8).

B. Basic notions on continued fractions

Let $\{a_k \neq 0\}$ and $\{b_0, b_k\}$ $(k = 1, 2, ...)$ be two sequences of real or complex numbers. A *finite continued fraction*

$$f_n \equiv b_0 + \mathop{\mathrm{K}}_{k=1}^{n} (a_k/b_k) = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \ddots + \cfrac{a_n}{b_n}}} \tag{B.1}$$

(we follow the Ref. [15]) can be converted into an ordinary fraction

$$f_n = A_n/B_n, \tag{B.2}$$

where the numerator A_n and denominator B_n are determined by the following recurrent relations:

$$\begin{aligned} &A_{-1} = 1, \qquad A_0 = b_0, \qquad B_{-1} = 0, \quad B_0 = 1, \\ &A_k = b_k A_{k-1} + a_k A_{k-2}, \\ &B_k = b_k B_{k-1} + a_k B_{k-2}, \qquad k = 1, 2, ..., n. \end{aligned} \tag{B.3}$$

A *continued fraction* (i.e., an infinite continued fraction)

$$f = b_0 + \mathop{\mathrm{K}}_{k=1}^{\infty} (a_k/b_k) = b_0 + \lim_{n\to\infty} \mathop{\mathrm{K}}_{k=1}^{n} (a_k/b_k) \tag{B.4}$$

is the limit $f = \lim_{n\to\infty} f_n$ of the sequence $\{f_n\}$, where f_n is called the nth appropriate fraction of continued fraction, and A_n, B_n are called the nth numerator and denominator, respectively.

If the condition (Worpitski criterium)

$$\left|\frac{a_k}{b_{k-1}b_k}\right| < \frac{1}{4} \tag{B.5}$$

holds for $k \geq 1$, the continued fraction is convergent, i.e., $|\lim f_n| < \infty$. If (B.5) holds starting from some $k_0 > 1$, the continued fraction is conventionally convergent, i.e., $|\lim f_n| < \infty$ or $\lim f_n = \pm\infty$. Otherwise the fraction can be divergent, i.e, $\lim f_n$ can be not existing.

Let $\{r_0 = 1, r_k \neq 0\}$ $(k = 1, 2, ...)$ be a sequence of real or complex numbers. The *equivalency transformation*

$$b_k \to \tilde{b}_k = r_k b_k, \quad a_k \to \tilde{a}_k = r_{k-1} r_k a_k \tag{B.6}$$

does not change the continued fraction: $\tilde{f} = f$. If $r_0 \neq 1$ then $\tilde{f} = r_0 f$.

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