Condensed Matter Physics, 2012, Vol. 15, No 4, 43005: 1–8
DOI: 10.5488/CMP.15.43005
http://www.icmp.lviv.ua/journal

CONDENSED MATTER PHYSICS

Goldstone mode singularities in $O(n)$ models

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Received July 3, 2012, in final form August 28, 2012

Monte Carlo (MC) analysis of the Goldstone mode singularities for the transverse and the longitudinal correlation functions, behaving as $G_\perp(\mathbf{k}) \simeq ak^{-\lambda_\perp}$ and $G_\parallel(\mathbf{k}) \simeq bk^{-\lambda_\parallel}$ in the ordered phase at $k \to 0$, is performed in the three-dimensional $O(n)$ models with $n = 2, 4, 10$. Our aim is to test some challenging theoretical predictions, according to which the exponents $\lambda_\perp$ and $\lambda_\parallel$ are non-trivial ($3/2 < \lambda_\perp < 2$ and $0 < \lambda_\parallel < 1$ in three dimensions) and the ratio bM^2/a^2 (where M is a spontaneous magnetization) is universal. The trivial standard-theoretical values are $\lambda_\perp = 2$ and $\lambda_\parallel = 1$. Our earlier MC analysis gives $\lambda_\perp = 1.955 \pm 0.020$ and $\lambda_\parallel$ about 0.9 for the $O(4)$ model. A recent MC estimation of $\lambda_\parallel$, assuming corrections to scaling of the standard theory, yields $\lambda_\parallel = 0.69 \pm 0.10$ for the $O(2)$ model. Currently, we have performed a similar MC estimation for the $O(10)$ model, yielding $\lambda_\perp = 1.9723(90)$. We have observed that the plot of the effective transverse exponent for the $O(4)$ model is systematically shifted down with respect to the same plot for the $O(10)$ model by $\Delta\lambda_\perp = 0.0121(52)$. It is consistent with the idea that $2 - \lambda_\perp$ decreases for large n and tends to zero at $n \to \infty$. We have also verified and confirmed the expected universality of bM^2/a^2 for the $O(4)$ model, where simulations at two different temperatures (couplings) have been performed.

Key words: *Monte Carlo simulation, n-component vector models, correlation functions, Goldstone mode singularities*

PACS: *05.10.Ln, 75.10.Hk, 05.50.+q*

1. Introduction

Our work is devoted to the Monte Carlo (MC) investigation of the Goldstone mode effects in n-component vector-spin models ($O(n)$ models), which have $O(n)$ global rotational symmetry at zero external field $\mathbf{h}$. The Hamiltonian $\mathscr{H}$ is given by

$$\frac{\mathscr{H}}{T} = -\beta\left(\sum_{\langle ij\rangle} \mathbf{s}_i\mathbf{s}_j + \sum_i \mathbf{h}\mathbf{s}_i\right), \tag{1.1}$$

where T is temperature, $\mathbf{s}_i \equiv \mathbf{s}(\mathbf{x}_i)$ is the n–component vector of unit length, i. e., the spin variable of the i-th lattice site with coordinate $\mathbf{x}_i$, and β is the coupling constant. The summation takes place over all nearest neighbors in the lattice with periodic boundary conditions.

The Fourier-transformed longitudinal and transverse correlation functions are

$$G_\parallel(\mathbf{k}) = N^{-1}\sum_{\mathbf{x}} \tilde{G}_\parallel(\mathbf{x})\mathrm{e}^{-\mathrm{i}\mathbf{k}\mathbf{x}}, \tag{1.2}$$

$$G_\perp(\mathbf{k}) = N^{-1}\sum_{\mathbf{x}} \tilde{G}_\perp(\mathbf{x})\mathrm{e}^{-\mathrm{i}\mathbf{k}\mathbf{x}}, \tag{1.3}$$

where $\tilde{G}_\parallel(\mathbf{x})$ and $\tilde{G}_\perp(\mathbf{x})$ are the corresponding two-point correlation functions in the coordinate space.

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In the thermodynamic limit below the critical temperature (at $\beta > \beta_c$), the magnetization $M(h)$ and the correlation functions exhibit Goldstone mode power-law singularities:

$$M(h) - M(+0) \propto h^{\rho} \quad \text{at} \quad h \to 0\,, \tag{1.4}$$

$$G_{\perp}(\mathbf{k}) = a\,k^{-\lambda_{\perp}} \quad \text{at} \quad h = +0 \text{ and } k \to 0\,, \tag{1.5}$$

$$G_{\parallel}(\mathbf{k}) = b\,k^{-\lambda_{\parallel}} \quad \text{at} \quad h = +0 \text{ and } k \to 0\,, \tag{1.6}$$

where a and b are the amplitudes.

There exist different theoretical predictions for the values of the exponents in these expressions. In a series of theoretical works (e. g., [1–7]), it has been claimed that these exponents are exactly $\rho = 1/2$ at $d = 3$, $\lambda_{\perp} = 2$ and $\lambda_{\parallel} = 4 - d$. Here, d is the spatial dimensionality $2 < d < 4$. These theoretical approaches are further referred to as the standard theory.

More non-trivial universal values are expected according to [8], such that

$$d/2 < \lambda_{\perp} < 2\,, \tag{1.7}$$

$$\lambda_{\parallel} = 2\lambda_{\perp} - d\,, \tag{1.8}$$

$$\rho = (d/\lambda_{\perp}) - 1 \tag{1.9}$$

hold for $2 < d < 4$. These relations were obtained in [8] by analyzing self-consistent diagram equations for correlation functions without cutting the perturbation series. As introduced in [9, 10], we will call this approach the GFD (grouping of Feynman diagrams) theory. Apart from the mathematical analysis, certain physical arguments were also provided [8] to show that $\lambda_{\perp} = 2$ could not be the correct result for the XY model ($n = 2$) within $2 < d < 4$.

Several MC simulations were performed in the past [11–14] to verify the compatibility of MC data with some standard–theoretical expressions, where the exponents are fixed. In recent years, we performed a series of accurate MC simulations [10, 15, 16] for remarkably larger lattices than previously with an aim to reexamine the theoretical predictions by evaluating the exponents in (1.7)–(1.9). In particular, lattices of the linear sizes $L \leqslant 512$ for $n = 2$ and $L \leqslant 350$ for $n = 4$ were simulated in our papers [10, 15] and [16], respectively. These L values remarkably exceed the largest sizes simulated by other authors, i. e., $L = 160$ for $n = 2$ in [13] and $L = 120$ for $n = 4$ in [12, 14]. In the current work, the $O(10)$ model is simulated up to $L = 384$.

The relations (1.7) and (1.8) are consistent with MC simulation results for the 3D $O(4)$ model [16], where an estimate $\lambda_{\perp} = 1.955 \pm 0.020$ was found. It was also stated that the behavior of the longitudinal correlation function is well consistent with $\lambda_{\parallel}$ about 0.9 rather than with the standard-theoretical value $\lambda_{\parallel} = 1$. According to (1.9), we have $1/2 < \rho < 1$ in three dimensions. It is consistent with the MC estimate $\rho = 0.555(17)$ for the 3D XY model [15], which corresponds to $\lambda_{\perp} = 1.929(21)$ according to (1.9). A clear MC evidence that the behavior of $G_{\parallel}(\mathbf{k})$ is not quite consistent with the standard–theoretical predictions has been recently provided [10], where an estimate $\lambda_{\parallel} = 0.69 \pm 0.10$ has been obtained for the 3D XY (i. e., 3D $O(2)$) model (at $\beta = 0.55$), assuming corrections to scaling of the standard theory.

In the actual study, we have extended our MC simulations and analysis to include the $n = 10$ case and to test the n-dependence of the exponents. Apart from the exponents, we have performed here an extended analysis of the $O(4)$ model to verify the expected universality of the ratio bM^2/a^2 [8], where $M \equiv M(+0)$ is the spontaneous magnetization, a and b are the amplitudes in (1.5) and (1.6).

2. Simulation results

We simulated the 3D $O(10)$ model by a modified Wolff cluster algorithm, used also in [15, 16], and evaluated the Fourier-transformed correlation functions by techniques described in [16]. The standard Wolff cluster algorithm [17] was modified to enable simulations at nonzero external field $\mathbf{h}$. Simple cubic lattices of the linear size up to $L = 384$ were simulated at $\beta = 3$ and $h = |\,\mathbf{h}\,| = h_{\min},\ 2h_{\min},\ 4h_{\min}$, where $h_{\min} = 0.00021875$. The coupling constant $\beta = 3$ corresponds to the ordered phase, since the spontaneous magnetization $M(+0)$ is about 0.467 in this case — see section 5 for details. This value of $M(+0)$ is comparable with those for the $O(2)$ and $O(4)$ models in our previous MC simulations [15, 16]. The simulation

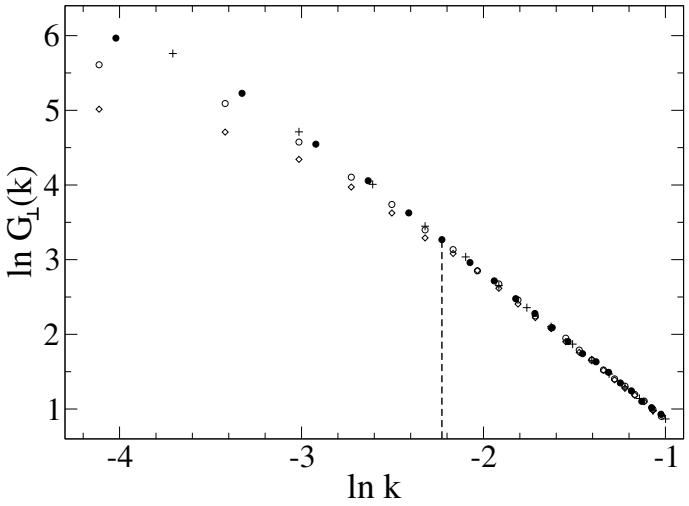

Figure 1. Log-log plots of the transverse correlation function $G_\perp(\mathbf{k})$ at $h = h_{\min} = 0.00021875$ and $L = 350$ (solid circles), $h = h_{\min}$ and $L = 256$ (pluses), $h = 2h_{\min}$ and $L = 384$ (empty circles), as well as at $h = 4h_{\min}$ and $L = 384$ (empty diamonds). Statistical errors are about the symbol size or smaller. The lower value k^* of the wave vector magnitude, used in estimations of the exponent $\lambda_\perp$, is indicated by a vertical dashed line.

results for the correlation functions $G_\perp(\mathbf{k})$ and $G_\parallel(\mathbf{k})$ in the $\langle 100 \rangle$ crystallographic direction at the three values of h and different sizes L are illustrated in figures 1 and 2. It is important for an estimation of the exponents $\lambda_\perp$ and $\lambda_\parallel$ to ensure that the finite-size as well as finite-h effects are small. This condition is satisfied for $k > k^*$, where the values of k^* are indicated in the figures by vertical dashed lines.

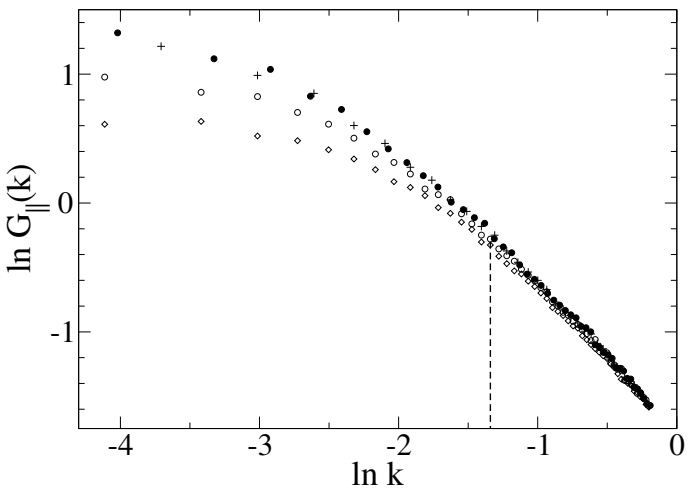

Figure 2. Log-log plots of the longitudinal correlation function $G_\parallel(\mathbf{k})$ at $h = h_{\min} = 0.00021875$ and $L = 350$ (solid circles), $h = h_{\min}$ and $L = 256$ (pluses), $h = 2h_{\min}$ and $L = 384$ (empty circles), as well as at $h = 4h_{\min}$ and $L = 384$ (empty diamonds). Statistical errors are about the symbol size. The lower value k^* of the wave vector magnitude, used in estimations of the exponent $\lambda_\parallel$, is indicated by a vertical dashed line.

3. Estimation of the exponents

Here we estimate the exponents $\lambda_\perp$ and $\lambda_\parallel$, describing the behavior of the correlation functions in the limit $k \to 0$, $h \to 0$, $L \to \infty$, taking the limit $L \to \infty$ at first, followed by $h \to 0$. For this purpose, first we find good approximations of the effective exponents at $h \to 0$, $L \to \infty$, and then fit these k-dependent effective exponents to evaluate their asymptotic values at $k \to 0$. By comparing the simulation results for different L and h, we conclude that the largest-L and smallest-h data for $k > k^*$ with a good enough accuracy correspond to the thermodynamic limit at $h = +0$, i. e., $h \to 0$, $L \to \infty$. We have tested this precisely by looking how the estimates of the effective exponents depend on L and h. This method of analysis was applied in [10, 16]. The effective transverse exponent $\lambda_{\rm eff}(k)$ for the $O(4)$ model was evaluated in [16] from the slope of the $\ln G_\perp(\mathbf{k})$ vs $\ln k$ plot within $[k, 4k]$. Here we use a wider interval — $[k, 6k]$, because we have found that the $\lambda_{\rm eff}(k)$ data in this case can be perfectly fit by a parabola

$$\lambda_{\rm eff}(k) = \lambda_\perp + a_1 k + a_2 k^2 \,, \tag{3.1}$$

the finite-size and finite-h effects being very small. The ansatz (3.1) is consistent with the general statement $\lim_{k\to 0} \lambda_{\rm eff}(k) = \lambda_\perp$ (in the thermodynamic limit at $h = +0$) and with corrections to scaling of the standard theory, where the correlation functions are supplied with correction factors in the form of an expansion in powers of k^{4-d} and k^{d-2} [1, 5]. Some of the fit results are shown in figure 3. We have performed a series of fits at different sizes L for the smallest-h value $h = h_{\rm min} = 0.00021875$. At the largest size $L = 350$ for this h, the effective exponent $\lambda_{\rm eff}(k)$ was fit within $k \in [k_5, k_{25}]$, where $k_\ell = 2\pi\ell/350$ are the possible discrete values of k. Similar fit intervals were chosen for all L. These fits to (3.1) give us $\lambda_\perp = 1.9680(84)$ at $L = 128$, $\lambda_\perp = 1.9840(98)$ at $L = 192$, $\lambda_\perp = 1.9727(86)$ at $L = 256$ and $\lambda_\perp = 1.9723(90)$ at $L = 350$. As we can see, the finite-size effects are smaller than the statistical error bars. The $\lambda_{\rm eff}(k)$ data for $L = 350$ and $L = 256$ at $h = h_{\rm min}$ are shown in figure 3 by solid circles and exes, respectively. The corresponding fit curves lie practically on top of each other. Therefore, only that one for $L = 350$ is shown by solid line. The fit curves for $h = 2h_{\rm min}$ and $h = 4h_{\rm min}$ at $L = 384$ (the largest size) are also depicted here to see the finite-h effects. These fits give us $\lambda_\perp = 1.9251(86)$ at $h = 4h_{\rm min}$ and $\lambda_\perp = 1.9666(91)$ at $h = 2h_{\rm min}$. A rather fast convergence to the $h = +0$ limit is evident. According to this discussion, the fit result $\lambda_\perp = 1.9723(90)$, obtained at $h = h_{\rm min}$ and $L = 350$, with a good accuracy corresponds to the thermodynamic limit at $h = +0$. Besides, the systematical errors due to finite-size and finite-h effects are probably smaller than the statistical error bars.

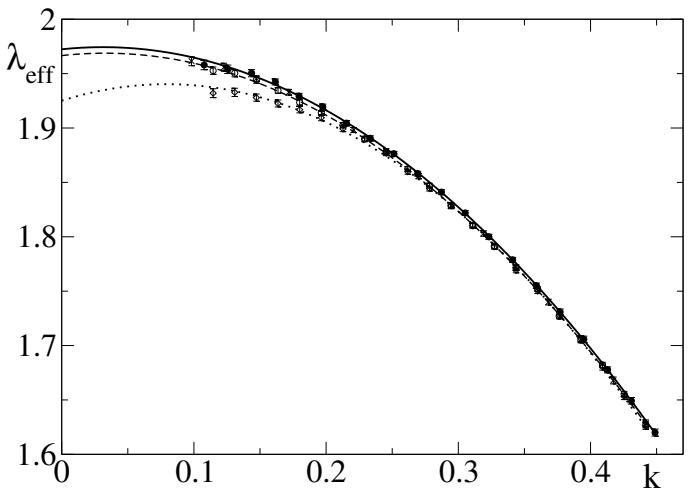

Figure 3. The transverse effective exponent $\lambda_{\rm eff}(k)$, evaluated from the $\ln G_\perp(\mathbf{k})$ vs $\ln k$ fits within $[k, 6k]$ at $h = h_{\rm min} = 0.00021875$ and $L = 350$ (solid circles), $h = h_{\rm min}$ and $L = 256$ (exes), $h = 2h_{\rm min}$ and $L = 384$ (empty circles), as well as at $h = 4h_{\rm min}$ and $L = 384$ (empty diamonds). The fits to (3.1) of the largest-L data at $h = h_{\rm min}$, $h = 2h_{\rm min}$ and $h = 4h_{\rm min}$ are shown by solid, dashed and dotted lines, respectively.

Another possible source of systematical errors is the existence of non-trivial corrections to scaling, which are not included in (3.1). These are corrections to scaling of the GFD theory [8], represented by an expansion in powers of $k^{2-\lambda_\perp}$, $k^{\lambda_\perp-\lambda_\parallel}$ and $k^{\lambda_\parallel}$. Nevertheless, the actual estimation, where only the standard-theoretical corrections have been included, is well justified as a test of consistency of the standard theory. The existence of a small correction-to-scaling exponent $2-\lambda_\perp$ can make the extrapolation of the $\lambda_{\rm eff}(k)$ plots unreliable. However, since the $\lambda_{\rm eff}(k)$ data are really well described by a parabola, it might be true that the amplitude of such a correction term is small and the estimate $\lambda_\perp = 1.9723(90)$ is quite reasonable. In any case, this estimation shows a small deviation from the standard-theoretical picture, where (3.1) should hold at small enough k with $\lambda_\perp = 2$. This deviation can be indeed small at $n = 10$, since $\lambda_\perp \to 2$ is expected in the limit $n \to \infty$, corresponding to the known behavior of the spherical model [18].

We have also attempted to evaluate the longitudinal exponent $\lambda_\parallel$ from the $G_\parallel(\mathbf{k})$ data within $k > k^*$, where k^* is indicated in figure 2 by a vertical dashed line. We have found that the longitudinal effective exponent, extracted from the data within $[k, 4k]$, can be perfectly approximated by a parabola. It leads to an estimate $\lambda_\parallel = 0.85 \pm 0.06$. The error bars indicated here include a statistical standard error as well as a systematical error due to finite-h effects. However, due to a rather large extrapolation gap (from ≈ 1.17 to ≈ 0.85), we consider this estimation as a preliminary one.

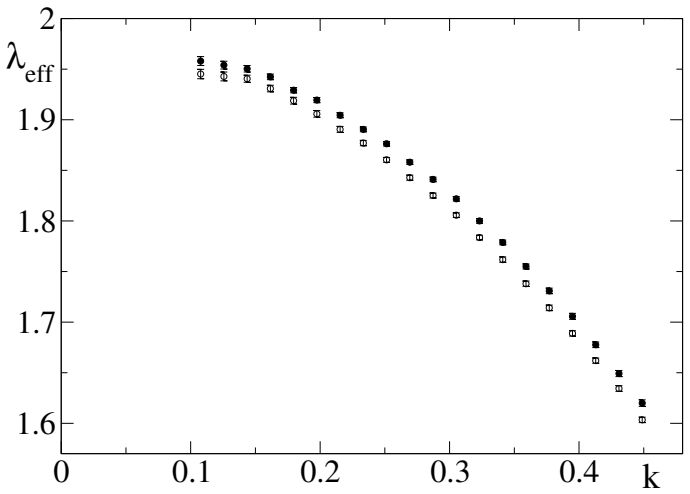

Figure 4. The plots of the transverse effective exponent $\lambda_{\rm eff}(k)$, evaluated from the $\ln G_\perp(\mathbf{k})$ vs $\ln k$ fits within $[k, 6k]$. The results of the $O(10)$ model at $\beta = 3$ are shown by solid circles, whereas those of the $O(4)$ model at $\beta = 1.1$ — by empty circles.

We have reexamined the largest-L ($L = 350$) and smallest-h ($h = 0.0003125$) data of the $O(4)$ model [16] at $\beta = 1.1$ with an aim to evaluate the transverse effective exponent in the same way as for the $O(10)$ model. Like in the $n = 10$ case, we have verified that the thermodynamic limit at $h = +0$ is practically reached in this estimation. Besides, we have found a surprising similarity of the $\lambda_{\rm eff}(k)$ plots, where the effective exponent in both cases was evaluated by fitting the $G_\perp(\mathbf{k})$ data within $[k, 6k]$ (fits within $[k, 4k]$ were used in [16]). As we can see from figure 4, the plot for the $O(4)$ model is systematically shifted down by an almost constant value relative to the same plot for the $O(10)$ model. This similarity might be partly caused by the fact that the values of spontaneous magnetization are rather similar in these two cases, i. e., $M \equiv M(+0) = 0.484475(48)$ for the $O(4)$ model at $\beta = 1.1$ and $M \approx 0.467$ (see section 5) for the $O(10)$ model at $\beta = 3$. The overall fit to (3.1) is less perfect for the $O(4)$ model as compared to the $O(10)$ model. However, the systematical shift between two plots is very well approximated by a constant value within the statistical error bars for the eight smallest k data points in figure 4. It yields an estimate $\Delta\lambda_\perp = (\lambda_\perp)_{n=10} - (\lambda_\perp)_{n=4} = 0.0121(52)$, where $(\lambda_\perp)_{n=10}$ and $(\lambda_\perp)_{n=4}$ are the values of $\lambda_\perp$ in $n = 10$ and $n = 4$ cases. According to the behavior of plots in figure 4 and this estimation, it is quite plausible that a transverse exponent $\lambda_\perp$ of the $O(10)$ model is somewhat larger than that of the $O(4)$ model. It is consistent

with the idea that $2-\lambda_\perp$ decreases for large n and tends to zero at $n \to \infty$. This behavior is fully consistent with the predictions of [8], but not so well consistent with the standard theory, according to which $\lambda_\perp$ is always 2 and, therefore, $\Delta\lambda_\perp = 0$ is expected. According to our estimates $(\lambda_\perp)_{n=10} = 1.9723(90)$ and $\Delta\lambda_\perp = 0.0121(52)$, we have $\lambda_\perp = 1.960(10)$ for $n = 4$. It perfectly agrees with our earlier estimate $\lambda_\perp = 1.955 \pm 0.020$ [16].

4. The ratio universality test

We have extended the MC analysis of our earlier data [16] for the $O(4)$ model at two different couplings, $\beta = 1.1$ and $\beta = 1.2$, to test the expected (according to [8]) universality of the ratio bM^2/a^2, discussed already in the end of section 1. According to (1.5), (1.6) and (1.8), the universality of bM^2/a^2 implies that the quantity

$$R(\mathbf{k}) = \frac{k^{-d} M^2 G_\parallel(\mathbf{k})}{G_\perp^2(\mathbf{k})} \tag{4.1}$$

tends to some universal constant at $k \to 0$, i. e., $\lim_{k\to 0} R(\mathbf{k}) = bM^2/a^2$. We have tested this property by comparing the $R(k)$ plots at $\beta = 1.1$ and $\beta = 1.2$, where $R(k) \equiv R(|\,\mathbf{k}\,|)$ in the $\langle 100 \rangle$ direction. Note that the quantities in (4.1) are determined in the thermodynamic limit at $h = +0$. We have verified that this limit is practically (within the statistical error bars) reached within $k \geqslant k_{14}$ (where $k_\ell = 2\pi\ell/350$) for the largest lattice size $L = 350$ and the smallest external fields $h = 0.0003125$ and $h = 0.0004375$ at which simulations were performed. The estimates of spontaneous magnetization obtained in [8], i.e., $M = 0.484475(48)$ at $\beta = 1.1$ and $M = 0.560178(40)$ at $\beta = 1.2$, are used here. The calculated plots are depicted in figure 5. The results for both $h = 0.0003125$ and $h = 0.0004375$ are available at $\beta = 1.1$. As we can see from figure 5, the corresponding two plots of $R(k)$ (solid circles and diamonds) lie practically on top of each other, indicating that the finite-h effects are negligibly small. The range $h \geqslant 0.0004375$ is considered for $\beta = 1.2$ in [16]. Fortunately, the finite-h effects at $\beta = 1.2$ are similar to those at $\beta = 1.1$, so that the estimate of $R(k)$ at $h = 0.0004375$ is valid at $\beta = 1.2$. The corresponding plot (empty circles) in figure 5 slightly deviates from the two plots at $\beta = 1.1$. However, all three plots merge within the statistical error bars at the smallest wave vector magnitudes k considered here. This confirms the expected universality of the ratio bM^2/a^2. The plot of empty circles in figure 5 apparently saturates at a value about 0.166 for small wave vectors. Taking into account the two other plots, we can judge that $0.16 < R(0) < 0.18$ most probably holds for the asymptotic value $R(0) = \lim_{k\to 0} R(k)$. Thus, we have an estimate $R(0) = 0.17 \pm 0.01$.

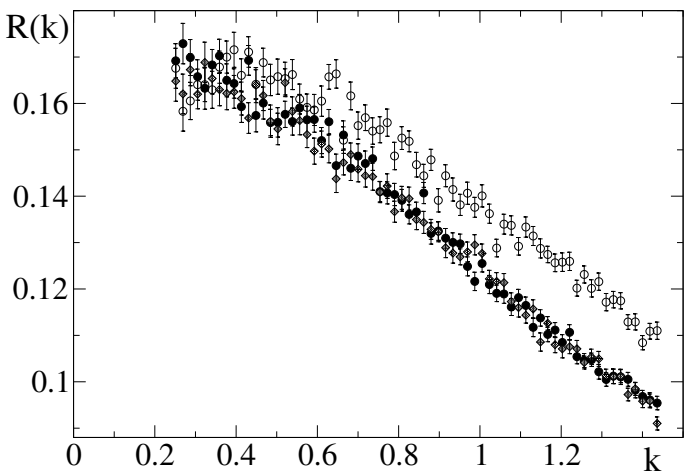

Figure 5. The $R(k)$ plots of the 3D $O(4)$ model, evaluated from the MC data for the lattice of size $L = 350$ at $\beta = 1.1$ and $h = 0.0003125$ (solid circles), $\beta = 1.1$ and $h = 0.0004375$ (diamonds), as well as at $\beta = 1.2$ and $h = 0.0004375$ (empty circles).

5. Spontaneous magnetization

We estimated the spontaneous magnetization of the 3D $O(10)$ model at $\beta = 3$ based on our magnetization data $M(h, L)$ depending on h and L. We observed a rather fast convergence to the thermodynamic limit, e. g., $M(h_{\min}, L) = 0.31658(46)$, 0.45286(23), 0.470839(90), 0.471959(38), 0.472151(23), 0.472148(16) at $L = 32, 64, 128, 192, 256$ and 350, respectively. According to this, we can take the largest-L value as a good approximation for $M(h_{\min}) = \lim_{L\to\infty} M(h_{\min}, L)$. Using this method, we obtained $M(h_{\min}) = 0.472148(16)$, $M(2h_{\min}) = 0.4742786(98)$ and $M(4h_{\min}) = 0.4772753(76)$. According to (1.4), we fit these data to the ansatz $M(h) = M(+0) + a_1 h^\rho$ with $\rho = 0.5211(69)$, evaluated from (1.9) by inserting here $\lambda_\perp = 1.9723(90)$ obtained in section 3. It yields $M \equiv M(+0) = 0.467343(99)$. Assuming the standard-theoretical value $\rho = 1/2$, we obtain $M = 0.467030(26)$. Fits to a refined ansatz $M(h) = M(+0) + a_1 h^\rho + a_2 h$ yield $M = 0.46711(14)$ at $\rho = 0.5211(69)$ and $M = 0.46696(14)$ at $\rho = 1/2$. Since we have only three data points for $M(h)$, this can be considered as a raw estimation yielding $M \approx 0.467$. However, this estimation is accurate enough to see that $\beta = 3$ corresponds to the ordered phase with $M > 0$.

6. Conclusions

In the actual work, the previous MC studies [10, 16] of the transverse and longitudinal correlation functions in the 3D $O(n)$ models with $n = 2$ and $n = 4$ have been extended, including the $n = 10$ case (sections 2 and 3). It gives us an important information about the behavior of the exponent $\lambda_\perp$ at large n. According to our MC analysis, a self-consistent (within the standard theory) estimation of $\lambda_\perp$ for $n = 10$ shows a small deviation from the standard-theoretical prediction $\lambda_\perp = 2$, yielding $\lambda_\perp = 1.9723(90)$ (section 3). The fact that this deviation is quite small can be well understood, since $\lambda_\perp \to 2$ is expected at $n \to \infty$ according to the known results for the spherical model, corresponding to this limit. Comparing the plots of the effective transverse exponent at $n = 10$ and $n = 4$, it has been stated that these plots are surprisingly similar, i. e., only slightly shifted with respect to each other. The estimation of this shift suggests that the transverse exponent for $n = 10$ is larger than that for $n = 4$ by an amount of $\Delta\lambda_\perp = 0.0121(52)$ (section 3). It is consistent with the idea that $2 - \lambda_\perp$ decreases for large n and tends to zero at $n \to \infty$. We have also verified and confirmed the expected universality of the ratio bM^2/a^2 for the $O(4)$ model by analyzing the correlation functions at two different couplings, i. e., $\beta = 1.1$ and $\beta = 1.2$ (section 4).

The actual MC results are fully consistent with the predictions of the GFD theory [8] (see section 1) and not so well consistent with the standard theory, according to which $\lambda_\perp$ is always 2.

Acknowledgements

This work was made possible by the facilities of the Shared Hierarchical Academic Research Computing Network (SHARCNET:www.sharcnet.ca). It has been performed within the framework of the ESF Project No. 1DP/1.1.1.2.0/09/ APIA/VIAA/142, and with the financial support of this project. R. M. acknowledges the support from the NSERC and CRC program.

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Сингулярності голдстоунівських мод в $O(n)$ моделях

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У тривимірних $O(n)$ моделях з $n = 2, 4, 10$ здійснено аналіз методом Монте Карло (МК) сингулярностей голдстоунівських мод для поперечної і поздовжньої кореляційних функцій, які поводять себе як $G_\perp(\mathbf{k}) \simeq ak^{-\lambda_\perp}$ і $G_\parallel(\mathbf{k}) \simeq bk^{-\lambda_\parallel}$ у впорядкованій фазі при $k \to 0$. Нашою метою є перевірити цікаві теоретичні передбачення, згідно яких індекси $\lambda_\perp$ і $\lambda_\parallel$ є нетривіальними ($3/2 < \lambda_\perp < 2$ і $0 < \lambda_\parallel < 1$ у трьох вимірах) і коефіцієнт bM^2/a^2 (де M є спонтанною намагніченістю) є універсальний. Тривіальні стандартні теоретичні значення є $\lambda_\perp = 2$ і $\lambda_\parallel = 1$. Наш попередній МК аналіз дає $\lambda_\perp = 1.955 \pm 0.020$ і $\lambda_\parallel$ приблизно рівне 0.9 для $O(4)$ моделі. Недавня МК оцінка $\lambda_\parallel$, яка допускає поправки для скейлінга стандартної моделі, дає $\lambda_\parallel = 0.69 \pm 0.10$ для $O(2)$ моделі. Тепер ми здійснили подібну МК оцінку для $O(10)$ моделі, яка дає $\lambda_\perp = 1.9723(90)$. Ми побачили, що графік ефективного поперечного індекса для $O(4)$ моделі є систематично зсунутий вниз по відношенню до графіка для $O(10)$ моделі на $\Delta\lambda_\perp = 0.0121(52)$. Це узгоджується з думкою, що $2 - \lambda_\perp$ зменшується для великих n і прямує до нуля при $n \to \infty$. Ми також перевірили і підтвердили очікувану універсальність bM^2/a^2 для $O(4)$ моделі, для якої було здійснено симуляції при двох різних температурах.

Ключові слова: *моделювання Монте Карло, n-компонентні векторні моделі, кореляційні функції, сингулярності голдстоунівських мод*