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Measures on two-component configuration spaces*

D.L.Finkelshtein†

Institute of Mathematics of the Ukrainian National Academy of Sciences,
3 Tereshchenkivs'ka Str., Kyiv-4, 01601, Ukraine

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We study the measures on the configuration spaces of particles of two types. Gibbs measures on such spaces are described. Main properties of corresponding relative energy densities and correlation functions are considered. In particular, we show that a support set for such Gibbs measure is the set of pairs of non-intersected configurations.

Key words: *two-component configuration spaces, Gibbs measures, correlation functions, statistical mechanics in continuum, relative energies*

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1. Introduction

The study of measures and related objects on the spaces of infinite configurations in Euclidean spaces (or, more general, C^∞ manifolds) was started in the sixties. In 1979, in [13], several approaches to the description of Gibbs measures on the configuration spaces were considered. Different aspects of the corresponding measure theory were revealed in [5–9,12,14,16] and others. For the case of marked configurations, the Dobrushin–Lanford–Ruelle (DLR) approach was considered in [10,11]. Nevertheless, the description of marked Gibbs measures via integral equations (so-called Georgii–Nguyen–Zessin–Campbell–Mecke equations) was not realized.

In this work we study these equations for the simplest case of the space of marks: $\{+,-\}$. We extend the approach proposed in [2] to this marked (two-component) system. We concentrate our attention on the properties of the Gibbs type measures without studying the existence and the uniqueness problems. One may study this using Ruelle technique in the same way as in [2], which we represent in the forthcoming paper. Another approach used for the purpose of proving the existence and non-uniqueness was proposed in [4].

Let us describe the content of the work more in detail.

Preliminary constructions for the one-component case are presented in section 2. In section 3 we consider the main properties of a measure on the two-component configuration spaces which is locally absolutely continuous with respect to the (w.r.t.) product of two Poisson measures. Note that it is natural that these Poisson measures have the same intensities since they should not be orthogonal. This is impossible for different constant intensities but for non-constant ones we need some additional conditions (see, e.g., [15]). Hence, for simplicity we consider the same Poisson measures. One of the main results of this section is the connection between correlation functions of a measure and of their marginal distribution. In section 4 we describe the Gibbs measures in terms of the so-called relative energy densities, which characterize the energy between the particle of one type and configurations of the both types. The main properties of these densities allow us to show that the corresponding Gibbs measure is the locally absolutely continuous w.r.t. product of Poisson measures. As a result, we may study this measure only on the subspace of the two-component configuration space which includes only pairs of configurations that do not intersect.

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†E-mail: fdl@imath.kiev.ua

This plays an important role in studying different dynamics on the two-component configuration spaces. In particular, we have a useful support set for a big class of measures (see, e.g., [1,3]). In the end we show an example of the pair-potential Gibbs measure which coincides with the studies in [4].

In this work we do not construct specifications of the Gibbs measure and the corresponding DLR approach. This may be considered similarly to [2] as well as it possible to show the equivalence between these two approaches (which goes back to [13]). All our considerations may be extended to the case of the product of finite number of configuration spaces over different C^∞ manifolds.

2. Preliminaries

Let X be a connected C^∞ oriented manifold. The configuration space $\Gamma := \Gamma_X$ over X is defined as the set of all locally finite subsets of X,

$$\Gamma := \bigl\{\gamma \subset X \bigm| |\gamma_\Lambda| < \infty \text{ for every compact } \Lambda \subset X\bigr\}, \tag{2.1}$$

where $|\cdot|$ denotes the cardinality of a set and $\gamma_\Lambda := \gamma \cap \Lambda$. As usual we identify each $\gamma \in \Gamma$ with the non-negative Radon measure $\sum_{x\in\gamma} \delta_x \in \mathcal{M}(X)$, where δ_x is the Dirac measure with unit mass at x, $\sum_{x\in\varnothing} \delta_x$ is, by definition, the zero measure, and $\mathcal{M}(X)$ denotes the space of all non-negative Radon measures on the Borel σ-algebra $\mathcal{B}(X)$. This identification allows us to endow Γ with the topology induced by the vague topology on $\mathcal{M}(X)$, i.e., the weakest topology on Γ with respect to which all mappings

$$\Gamma \ni \gamma \longmapsto \langle f, \gamma\rangle := \int_X f(x)\mathrm{d}\gamma(x) = \sum_{x\in\gamma} f(x), \qquad f \in C_0(X),$$

are continuous. Here $C_0(X)$ denotes the set of all continuous functions on X with compact support. By $\mathcal{B}(\Gamma)$ we denote the corresponding Borel σ-algebra on Γ.

Let us consider the space of finite configurations

$$\Gamma_0 := \bigsqcup_{n=0}^{\infty} \Gamma^{(n)},$$

where $\Gamma^{(n)} := \Gamma_X^{(n)} := \{\gamma \in \Gamma : |\gamma| = n\}$ for $n \in \mathbb{N}$ and $\Gamma^{(0)} := \{\varnothing\}$. For $n \in \mathbb{N}$, there is a natural bijection between the space $\Gamma^{(n)}$ and the symmetrization $\widetilde{X^n}\big/ S_n$ of the set $\widetilde{X^n} := \{(x_1, \dots, x_n) \in X^n : x_i \neq x_j \text{ if } i \neq j\}$ under the permutation group S_n over $\{1, \dots, n\}$ acting on $\widetilde{X^n}$ by permuting the coordinate indexes. This bijection induces a metrizable topology on $\Gamma^{(n)}$, and we endow Γ_0 with the topology of disjoint union of topological spaces. By $\mathcal{B}(\Gamma^{(n)})$ and $\mathcal{B}(\Gamma_0)$ we denote the corresponding Borel σ-algebras on $\Gamma^{(n)}$ and Γ_0, respectively.

Given a non-atomic Radon measure σ on $(X, \mathcal{B}(X))$ with $\sigma(X) = \infty$, let λ_σ be the Lebesgue-Poisson measure on $\bigl(\Gamma_0, \mathcal{B}(\Gamma_0)\bigr)$, namely,

$$\lambda_\sigma := \sum_{n=0}^{\infty} \frac{1}{n!}\sigma^{(n)},$$

where each $\sigma^{(n)}$, $n \in \mathbb{N}$, is the image measure on $\Gamma^{(n)}$ of the product measure $\mathrm{d}\sigma(x_1)\dots\mathrm{d}\sigma(x_n)$ under the mapping $\widetilde{X^n} \ni (x_1, \dots, x_n) \mapsto \{x_1, \dots, x_n\} \in \Gamma^{(n)}$. For $n = 0$ we set $\sigma^{(0)}(\{\varnothing\}) := 1$.

Let $\mathcal{B}_c(X)$ denote the set of all bounded Borel sets in X, and for any $\Lambda \in \mathcal{B}_c(X)$ let $\Gamma_\Lambda := \{\eta \in \Gamma : \eta \subset \Lambda\}$. Evidently $\Gamma_\Lambda = \bigsqcup_{n=0}^{\infty} \Gamma_\Lambda^{(n)}$, where $\Gamma_\Lambda^{(n)} := \Gamma_\Lambda \cap \Gamma^{(n)}$ for each $n \in \mathbb{N}_0$, leading to a situation similar to the one for Γ_0, described above. We endow Γ_Λ with the topology of the disjoint union of topological spaces and with the corresponding Borel σ-algebra $\mathcal{B}(\Gamma_\Lambda)$. Let $\mathbf{p}_\Lambda : \Gamma \to \Gamma_\Lambda$ be a projection mapping: $\mathbf{p}_\Lambda(\gamma) = \gamma_\Lambda$. Then if we define Poisson measure on $\bigl(\Gamma_\Lambda, \mathcal{B}(\Gamma_\Lambda)\bigr)$ as $\pi_\sigma^\Lambda = \mathrm{e}^{-\sigma(\Lambda)}\lambda_\sigma$ (here we understand λ_σ as measure on Γ_Λ), it is well known that there exists a unique Poisson measure on $(\Gamma, \mathcal{B}(\Gamma))$ such that $\pi_\sigma^\Lambda = \pi_\sigma \circ \mathbf{p}_\Lambda^{-1}$ for any $\Lambda \in \mathcal{B}_c(X)$. Note that $(\Gamma, \mathcal{B}(\Gamma), \pi_\sigma)$ is a projective limit of the family $\Big\{ (\Gamma_\Lambda, \mathcal{B}(\Gamma_\Lambda), \pi_\sigma^\Lambda) \;\Big|\; \Lambda \in \mathcal{B}_c(X) \Big\}$.

From the very beginning we assume that there exists a sequence $\{\Lambda_m\}_{m\in\mathbb{N}} \subset \mathcal{B}_c(X)$ such that $\bigcup_{m\in\mathbb{N}} \Lambda_m = X$.

3. Measures on two-component spaces

Let $\Gamma^+ = \Gamma^- = \Gamma_X$ and $\Gamma^2 = \Gamma^+ \times \Gamma^-$. We consider a topology of direct product on Γ^2. Then $\mathcal{B}(\Gamma^2) := \mathcal{B}(\Gamma^+) \times \mathcal{B}(\Gamma^-)$ is the corresponding Borel σ-algebra. We denote a class of probability measures on $(\Gamma^2, \mathcal{B}(\Gamma^2))$ by $\mathcal{M}^1(\Gamma^2)$.

Let us consider a projection mapping $p_{\Lambda^+,\Lambda^-} : \Gamma^2 \to \Gamma^+_{\Lambda^+} \times \Gamma^-_{\Lambda^-}$ such that

$$p_{\Lambda^+,\Lambda^-}(\gamma^+, \gamma^-) = \big(\gamma^+_{\Lambda^+}, \gamma^-_{\Lambda^-}\big).$$

Definition 1. We call a measure $\mu \in \mathcal{M}^1(\Gamma^2)$ *locally absolutely continuous* w.r.t. $\pi_\sigma \times \pi_\sigma$ if $\mu^{\Lambda^+,\Lambda^-} := \mu \circ p^{-1}_{\Lambda^+,\Lambda^-}$ is absolutely continuous w.r.t. product of the Poisson measures $\pi_\sigma^{\Lambda^+} \times \pi_\sigma^{\Lambda^-}$ on $\big(\Gamma^+_{\Lambda^+} \times \Gamma^-_{\Lambda^-}, \mathcal{B}(\Gamma^+_{\Lambda^+}) \times \mathcal{B}(\Gamma^-_{\Lambda^-})\big)$.

In the case when $\Lambda^+ = \Lambda^- = \Lambda$ we will write $p_\Lambda, \mu^\Lambda, \Gamma^2_\Lambda$ instead of $p_{\Lambda,\Lambda}, \mu^{\Lambda,\Lambda}, \Gamma^+_\Lambda \times \Gamma^-_\Lambda$ correspondingly.

Proposition 3.1. *For any $\mu \in \mathcal{M}^1(\Gamma^2)$ which is locally absolutely continuous w.r.t. $\pi_\sigma \times \pi_\sigma$ the set*

$$\widetilde{\Gamma}^2 := \Big\{ (\gamma^+, \gamma^-) \in \Gamma^2 \;\big|\; \gamma^+ \cap \gamma^- = \emptyset \Big\} \tag{3.1}$$

has full μ-measure.

Proof. Take $\{\Lambda_m\}_{m\in\mathbb{N}} \subset \mathcal{B}_c(X)$ such that $\bigcup_{m\in\mathbb{N}} \Lambda_m = X$. Then we can decompose the set $\Gamma^2 \setminus \widetilde{\Gamma}^2$ as

$$\Gamma^2 \setminus \widetilde{\Gamma}^2 = \bigcup_{m\in\mathbb{N}} p^{-1}_{\Lambda_m} \{ (\gamma^+, \gamma^-) \in \Gamma^2_{\Lambda_m} \mid \gamma^+ \cap \gamma^- \neq \emptyset \},$$

hence,

$$\mu\big(\Gamma^2 \setminus \widetilde{\Gamma}^2\big) \leqslant \sum_{m\in\mathbb{N}} \mu^{\Lambda_m} \Big(\big\{ (\gamma^+, \gamma^-) \in \Gamma^2_{\Lambda_m} \mid \gamma^+ \cap \gamma^- \neq \emptyset \big\} \Big).$$

Since μ^{Λ_m} is absolutely continuous w.r.t. $\lambda_\sigma \times \lambda_\sigma$ it is sufficient to prove that

$$(\lambda_\sigma \times \lambda_\sigma)\Big(\big\{ (\gamma^+, \gamma^-) \in \Gamma^2_{\Lambda_m} \mid \gamma^+ \cap \gamma^- \neq \emptyset \big\} \Big) = 0.$$

But if we denote for any fixed $\gamma^+ \in \Gamma^+_{\Lambda_m}$

$$A_{\gamma^+} := \big\{ \gamma^- \in \Gamma^-_{\Lambda_m} \mid \gamma^+ \cap \gamma^- \neq \emptyset \big\}$$

then one has

$$\lambda_\sigma(A_{\gamma^+}) \leqslant \sum_{x\in\gamma^+} \lambda_\sigma\Big(\big\{ \gamma^- \in \Gamma^-_{\Lambda_m} \mid x \in \gamma^- \big\} \Big) = 0.$$

The remark that

$$(\lambda_\sigma \times \lambda_\sigma)\Big(\big\{ (\gamma^+, \gamma^-) \in \Gamma^2_{\Lambda_m} \mid \gamma^+ \cap \gamma^- \neq \emptyset \big\} \Big) = \int_{\Gamma^+_{\Lambda_m}} A_{\gamma^+} \mathrm{d}\lambda_\sigma(\gamma^+)$$

fulfills the proof. $\square$

Proposition 3.2. *Let $\mu \in \mathcal{M}^1(\Gamma^2)$ be a locally absolutely continuous measure w.r.t. $\pi_\sigma \times \pi_\sigma$ and let A be a $\mathcal{B}(X)$-measurable set such that $\sigma(A) = 0$. Then the following set*

$$B := \bigl\{(\gamma^+, \gamma^-) \in \Gamma^2 \bigm| \gamma^- \cap A \neq \emptyset\bigr\}$$

has zero μ-measure.

Proof. Using the same trick as in the previous Proposition one can show that it is sufficient to prove that for any $m \in \mathbb{N}$

$$(\lambda_\sigma \times \lambda_\sigma)\Bigl(\bigl\{(\gamma^+, \gamma^-) \in \Gamma^2_{\Lambda_m} \bigm| x \in A \text{ for some } x \in \gamma^-\bigr\}\Bigr) = 0.$$

But the left hand side is equal to

$$\begin{aligned} &\lambda_\sigma(\Gamma^+_{\Lambda_m})\lambda_\sigma\Bigl(\bigl\{\gamma^- \in \Gamma^-_{\Lambda_m} \bigm| x \in A \text{ for some } x \in \gamma^-\bigr\}\Bigr) \\ &\quad = \mathrm{e}^{\sigma(\Lambda_m)} \sum_{n=0}^{\infty} \frac{1}{n!} \sigma^{\otimes}\Bigl(\bigl\{(x_1, \ldots, x_n) \in (\Lambda_m)^n \bigm| x_i \in A \text{ for some } i\bigr\}\Bigr) = 0. \end{aligned}$$

The statement is proven. □

Corollary 3.3. *Let $\mu \in \mathcal{M}^1(\Gamma^2)$ be a locally absolutely continuous measure w.r.t. $\pi_\sigma \times \pi_\sigma$. Then the set*

$$\bigl\{(\gamma^+, \gamma^-, x) \in \Gamma^2 \times X \bigm| x \in \gamma^+\bigr\}$$

has $\mu \times \sigma$-measure 0.

We define the *marginal distribution* of μ in a usual way, namely,

$$\mathrm{d}\mu^\pm(\gamma^\pm) := \int_{\Gamma^\mp} \mathrm{d}\mu(\gamma^+, \gamma^-). \tag{3.2}$$

Hence, for example, μ^+ is a probability measure on $\bigl(\Gamma^+, \mathcal{B}(\Gamma^+)\bigr)$. Then one can consider the projection of μ^+ on Γ^+_Λ: $(\mu^+)^\Lambda = \mu^+ \circ \mathbf{p}_\Lambda^{-1}$. On the other hand, we may consider marginal distribution of μ^Λ which we denote by $(\mu^\Lambda)^+$.

It is easy to see that

$$(\mu^+)^\Lambda = (\mu^\Lambda)^+. \tag{3.3}$$

Indeed, let $F : \Gamma^2 \to \mathbb{R}$ be a measurable function such that there exists a measurable function $F^+ : \Gamma^+ \to \mathbb{R}$ such that $F(\gamma^+, \gamma^-) = F^+\left(\gamma^+_\Lambda\right)$. Then

$$\begin{aligned} \int_{\Gamma^2} F\left(\gamma^+, \gamma^-\right) \mathrm{d}\mu\left(\gamma^+, \gamma^-\right) &= \int_{\Gamma^+_\Lambda \times \Gamma^-_\Lambda} F\left(\gamma^+_\Lambda, \gamma^-_\Lambda\right) \mathrm{d}\mu^\Lambda\left(\gamma^+_\Lambda, \gamma^-_\Lambda\right) \\ &= \int_{\Gamma^+_\Lambda} F^+\left(\gamma^+_\Lambda\right) \int_{\Gamma^-_\Lambda} \mathrm{d}\mu^\Lambda\left(\gamma^+_\Lambda, \gamma^-_\Lambda\right) = \int_{\Gamma^+_\Lambda} F^+\left(\gamma^+_\Lambda\right) \mathrm{d}\left(\mu^\Lambda\right)^+\left(\gamma^+_\Lambda\right). \end{aligned}$$

On the other hand

$$\begin{aligned} \int_{\Gamma^2} F\left(\gamma^+, \gamma^-\right) \mathrm{d}\mu\left(\gamma^+, \gamma^-\right) &= \int_{\Gamma^+ \times \Gamma^-} F^+\left(\gamma^+\right) \mathrm{d}\mu\left(\gamma^+, \gamma^-\right) \\ &= \int_{\Gamma^+} F^+\left(\gamma^+\right) \mathrm{d}\mu^+\left(\gamma^+\right) = \int_{\Gamma^+_\Lambda} F^+\left(\gamma^+_\Lambda\right) \mathrm{d}\left(\mu^+\right)^\Lambda\left(\gamma^+_\Lambda\right). \end{aligned}$$

Remark 3.4. Using (3.3) it is clear that if μ is locally absolutely continuous w.r.t. $\pi_\sigma \times \pi_\sigma$ then $\mu^\pm$ are locally absolutely continuous w.r.t. π_σ.

Definition 2. We will say that locally absolutely continuous w.r.t. $\pi_\sigma \times \pi_\sigma$ probability measure μ satisfies *local Ruelle bound* if for any $\Lambda^\pm \in \mathcal{B}_c(X)$ there exists $C_{\Lambda^\pm} > 0$ such that for $\lambda_\sigma \times \lambda_\sigma$-a.a. $(\eta^+, \eta^-) \in \Gamma^+_{\Lambda^+} \times \Gamma^-_{\Lambda^-}$

$$\frac{\mathrm{d}\mu^{\Lambda^+,\Lambda^-}}{\mathrm{d}(\lambda_\sigma \times \lambda_\sigma)}(\eta^+, \eta^-) \leqslant (C_{\Lambda^+})^{|\eta^+|}(C_{\Lambda^-})^{|\eta^-|}. \tag{3.4}$$

For the measure μ from Definition 2 one can define a correlation function k_μ, namely, for $\lambda_\sigma \times \lambda_\sigma$-a.a. $(\eta^+, \eta^-) \in \Gamma^+_{\Lambda^+} \times \Gamma^-_{\Lambda^-}$, $\Lambda^\pm \in \mathcal{B}_c(X)$ we set

$$k_\mu(\eta^+, \eta^-) = \int_{\Gamma^+_{\Lambda^+}} \int_{\Gamma^-_{\Lambda^-}} \frac{\mathrm{d}\mu^{\Lambda^+,\Lambda^-}}{\mathrm{d}(\lambda_\sigma \times \lambda_\sigma)}(\eta^+ \cup \xi^+, \eta^- \cup \xi^-)\mathrm{d}\lambda_\sigma(\xi^+)\mathrm{d}\lambda_\sigma(\xi^-). \tag{3.5}$$

Clearly,

$$k_\mu(\emptyset, \emptyset) = 1.$$

It follows from infinitely-divisible property of λ_σ that r.h.s. of (3.5) does not depend on $\Lambda^\pm$. Also, from definition of λ_σ and (3.4) it follows that

$$k_\mu(\eta^+, \eta^-) \leqslant \mathrm{e}^{C_{\Lambda^+}\sigma(\Lambda^+)}\mathrm{e}^{C_{\Lambda^-}\sigma(\Lambda^-)}(C_{\Lambda^+})^{|\eta^+|}(C_{\Lambda^-})^{|\eta^-|}. \tag{3.6}$$

We will denote the correlation function of the marginal distribution μ^+ by k^+_μ and define it as

$$k^+_\mu(\eta^+) = \int_{\Gamma^+_\Lambda} \frac{\mathrm{d}(\mu^+)^\Lambda}{\mathrm{d}\lambda^\Lambda_\sigma}(\eta^+ \cup \xi^+)\mathrm{d}\lambda_\sigma(\xi^+) \tag{3.7}$$

for λ_σ-a.a. $\eta^+ \in \Gamma^+_\Lambda$, $\Lambda \in \mathcal{B}_c(X)$. Similarly, one can define k^-_μ.

Putting in (3.5) $\eta^- = \emptyset$, $\Lambda^+ = \Lambda^- = \Lambda$ and using (3.3) we obtain

$$\begin{aligned} k_\mu(\eta^+, \emptyset) &= \int_{\Gamma^+_\Lambda} \left(\int_{\Gamma^-_\Lambda} \frac{\mathrm{d}\mu^\Lambda}{\mathrm{d}(\lambda_\sigma \times \lambda_\sigma)}(\eta^+ \cup \xi^+, \xi^-)\mathrm{d}\lambda_\sigma(\xi^-) \right) \mathrm{d}\lambda_\sigma(\xi^+) \\ &= \int_{\Gamma^+_\Lambda} \frac{\mathrm{d}(\mu^\Lambda)^+}{\mathrm{d}\lambda_\sigma}(\eta^+ \cup \xi^+)\mathrm{d}\lambda_\sigma(\xi^+) = k^+_\mu(\eta^+). \end{aligned} \tag{3.8}$$

Similarly,

$$k^-_\mu(\eta^-) = k_\mu(\emptyset, \eta^-). \tag{3.9}$$

4. Two-component Gibbs measures

Definition 3. The measure $\mu \in \mathcal{M}^1\left(\Gamma^2\right)$ is called a *Gibbs measure* if there exist non-negative measurable functions $r^\pm : \Gamma^2 \times X \to [0; +\infty)$ such that for all non-negative measurable functions $h_{1,2} : \Gamma^2 \times X \to [0; +\infty)$ the following *partial Campbell–Mecke identities* hold

$$\begin{aligned} &\int_{\Gamma^2} \sum_{x \in \gamma^+} h_1\left(\gamma^+, \gamma^-, x\right) \mathrm{d}\mu\left(\gamma^+, \gamma^-\right) \\ &\qquad = \int_{\Gamma^2} \int_X h_1\left(\gamma^+ \cup x, \gamma^-, x\right) r^+\left(\gamma^+, \gamma^-, x\right) \mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+, \gamma^-\right), \\ &\int_{\Gamma^2} \sum_{y \in \gamma^-} h_2\left(\gamma^+, \gamma^-, y\right) \mathrm{d}\mu\left(\gamma^+, \gamma^-\right) \\ &\qquad = \int_{\Gamma^2} \int_X h_2\left(\gamma^+, \gamma^- \cup y, y\right) r^-\left(\gamma^+, \gamma^-, y\right) d\sigma\left(y\right) d\mu\left(\gamma^+, \gamma^-\right). \end{aligned}$$

We denote the class of such measures by $\mathcal{G}(r^+, r^-, \sigma)$.

We will call the functions $r^\pm$ *partial relative energy densities* of the measure μ. As required, these functions have the following properties.

Lemma 4.1. *For μ-a.a. $(\gamma^+,\gamma^-)\in\Gamma^2$ and for σ-a.a. $x,y\in X$* the partial co-cycle identities *hold*

$$\begin{aligned} r^+\left(\gamma^+\cup x',\gamma^-,x\right)r^+\left(\gamma^+,\gamma^-,x'\right) &= r^+\left(\gamma^+\cup x,\gamma^-,x'\right)r^+\left(\gamma^+,\gamma^-,x\right), \qquad (4.1)\\ r^-\left(\gamma^+,\gamma^-\cup y',y\right)r^-\left(\gamma^+,\gamma^-,y'\right) &= r^-\left(\gamma^+,\gamma^-\cup y,y'\right)r^-\left(\gamma^+,\gamma^-,y\right), \qquad (4.2)\end{aligned}$$

as well as the balance identity *holds*

$$r^+\left(\gamma^+,\gamma^-\cup y,x\right)r^-\left(\gamma^+,\gamma^-,y\right)=r^-\left(\gamma^+\cup x,\gamma^-,y\right)r^+\left(\gamma^+,\gamma^-,x\right). \qquad (4.3)$$

Proof. 1. Using (4.1) for any measurable $h_{1,2}:\Gamma\times X\to[0;+\infty)$ we have

$$\begin{aligned} I &:= \int_{\Gamma^2}\sum_{x\in\gamma^+}h_1\left(\gamma^+,\gamma^-,x\right)\sum_{x'\in\gamma^+}h_2\left(\gamma^+,\gamma^-,x'\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\ &= \int_{\Gamma^2}\int_X h_1\left(\gamma^+\cup x,\gamma^-,x\right)\sum_{x'\in\gamma^+\cup x}h_2\left(\gamma^+\cup x,\gamma^-,x'\right)r^+\left(\gamma^+,\gamma^-,x\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\ &= \int_{\Gamma^2}\int_X h_1\left(\gamma^+\cup x,\gamma^-,x\right)\sum_{x'\in\gamma^+}h_2\left(\gamma^+\cup x,\gamma^-,x'\right)r^+\left(\gamma^+,\gamma^-,x\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\ &\quad+\int_{\Gamma^2}\int_X h_1\left(\gamma^+\cup x,\gamma^-,x\right)h_2\left(\gamma^+\cup x,\gamma^-,x\right)r^+\left(\gamma^+,\gamma^-,x\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\ &= \int_{\Gamma^2}\int_X\int_X h_1\left(\gamma^+\cup x\cup x',\gamma^-,x\right)h_2\left(\gamma^+\cup x\cup x',\gamma^-,x'\right)\\ &\qquad\times r^+\left(\gamma^+\cup x',\gamma^-,x\right)r^+\left(\gamma^+,\gamma^-,x'\right)\mathrm{d}\sigma\left(x'\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\ &\quad+\int_{\Gamma^2}\int_X h_1\left(\gamma^+\cup x,\gamma^-,x\right)h_2\left(\gamma^+\cup x,\gamma^-,x\right)r^+\left(\gamma^+,\gamma^-,x\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right),\end{aligned}$$

and, similarly,

$$\begin{aligned} I &= \int_{\Gamma^2}\int_X\int_X h_1\left(\gamma^+\cup x\cup x',\gamma^-,x\right)h_2\left(\gamma^+\cup x\cup x',\gamma^-,x'\right)\\ &\qquad\times r^+\left(\gamma^+\cup x,\gamma^-,x'\right)r^+\left(\gamma^+,\gamma^-,x\right)\mathrm{d}\sigma\left(x'\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\ &+ \int_{\Gamma^2}\int_X h_1\left(\gamma^+\cup x,\gamma^-,x\right)h_2\left(\gamma^+\cup x,\gamma^-,x\right)r^+\left(\gamma^+,\gamma^-,x\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right).\end{aligned}$$

Comparing the right hand sides of these equalities we obtain (4.1). (4.2) is obtained in the same way.

2. Using (4.1) and (4.1) for any measurable $h:\Gamma^2\times X\times X\to[0;+\infty)$ we have

$$\begin{aligned} J &:= \int_{\Gamma^2}\sum_{x\in\gamma^+}\sum_{y\in\gamma^-}h\left(\gamma^+,\gamma^-,x,y\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\ &= \int_{\Gamma^2}\int_X\sum_{y\in\gamma^-}h\left(\gamma^+\cup x,\gamma^-,x,y\right)r^+\left(\gamma^+,\gamma^-,x\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\ &= \int_{\Gamma^2}\int_X\int_X h\left(\gamma^+\cup x,\gamma^-\cup y,x,y\right)r^+\left(\gamma^+,\gamma^-\cup y,x\right)\\ &\qquad\times r^-\left(\gamma^+,\gamma^-,y\right)\mathrm{d}\sigma\left(y\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right),\end{aligned}$$

on the other hand,

$$J=\int_{\Gamma^2}\int_X\int_X h\left(\gamma^+\cup x,\gamma^-\cup y,x,y\right)r^-\left(\gamma^+\cup x,\gamma^-,y\right)r^+\left(\gamma^+,\gamma^-,x\right)\mathrm{d}\sigma\left(y\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right).$$

Comparing the right hand sides of these equalities we obtain (4.3). $\square$

Corollary 4.2. *As a result, we can define the* relative energy density *of the measure μ as*

$$r\left(\gamma^{+},\gamma^{-},x,y\right):=r^{+}\left(\gamma^{+},\gamma^{-}\cup y,x\right)r^{-}\left(\gamma^{+},\gamma^{-},y\right)=r^{-}\left(\gamma^{+}\cup x,\gamma^{-},y\right)r^{+}\left(\gamma^{+},\gamma^{-},x\right), \tag{4.4}$$

and the following Campbell–Mecke identity *holds*

$$\int_{\Gamma^2}\sum_{x\in\gamma^+}\sum_{y\in\gamma^-}h\left(\gamma^{+},\gamma^{-},x,y\right)\mathrm{d}\mu\left(\gamma^{+},\gamma^{-}\right)$$
$$=\int_{\Gamma^2}\int_X\int_X h\left(\gamma^{+}\cup x,\gamma^{-}\cup y,x,y\right)r\left(\gamma^{+},\gamma^{-},x,y\right)\mathrm{d}\sigma\left(y\right)\mathrm{d}\sigma(x)\mathrm{d}\mu\left(\gamma^{+},\gamma^{-}\right). \tag{4.5}$$

The next Lemma shows that the function r also satisfies the co-cycle identity.

Lemma 4.3. *For μ-a.a. $(\gamma^+,\gamma^-)\in\Gamma^2$ and for σ-a.a. $x,x',y,y'\in X$*

$$r\left(\gamma^{+}\cup x',\gamma^{-}\cup y',x,y\right)r\left(\gamma^{+},\gamma^{-},x',y'\right)=r\left(\gamma^{+}\cup x,\gamma^{-}\cup y,x',y'\right)r\left(\gamma^{+},\gamma^{-},x,y\right). \tag{4.6}$$

Proof. First of all let us prove that for μ-a.a. $(\gamma^+,\gamma^-)\in\Gamma^2$ and for σ-a.a. $x,x',y,y'\in X$

$$\begin{aligned}r^{+}\left(\gamma^{+}\cup x,\gamma^{-}\cup y,x'\right)r\left(\gamma^{+},\gamma^{-},x,y\right) &= r\left(\gamma^{+}\cup x,\gamma^{-},x',y\right)r^{+}\left(\gamma^{+},\gamma^{-},x\right)\\ &= r\left(\gamma^{+}\cup x',\gamma^{-},x,y\right)r^{+}\left(\gamma^{+},\gamma^{-},x'\right)\\ &= r^{+}\left(\gamma^{+}\cup x',\gamma^{-}\cup y,x\right)r\left(\gamma^{+},\gamma^{-},x',y\right).\end{aligned} \tag{4.7}$$

Really, using (4.3), one has

$$\begin{aligned}&r^{+}\left(\gamma^{+}\cup x,\gamma^{-}\cup y,x'\right)r\left(\gamma^{+},\gamma^{-},x,y\right)\\ &\quad= r^{+}\left(\gamma^{+}\cup x,\gamma^{-}\cup y,x'\right)r^{+}\left(\gamma^{+},\gamma^{-}\cup y,x\right)r^{-}\left(\gamma^{+},\gamma^{-},y\right)\\ &\quad= r^{+}\left(\gamma^{+}\cup x,\gamma^{-}\cup y,x'\right)r^{-}\left(\gamma^{+}\cup x,\gamma^{-},y\right)r^{+}\left(\gamma^{+},\gamma^{-},x\right)\\ &\quad= r\left(\gamma^{+}\cup x,\gamma^{-},x',y\right)r^{+}\left(\gamma^{+},\gamma^{-},x\right);\end{aligned}$$

similarly,

$$r^{+}\left(\gamma^{+}\cup x',\gamma^{-}\cup y,x\right)r\left(\gamma^{+},\gamma^{-},x',y\right)=r\left(\gamma^{+}\cup x',\gamma^{-},x,y\right)r^{+}\left(\gamma^{+},\gamma^{-},x'\right);$$

then, using (4.3) and (4.1),we obtain

$$\begin{aligned}&r\left(\gamma^{+}\cup x',\gamma^{-},x,y\right)r^{+}\left(\gamma^{+},\gamma^{-},x'\right)\\ &\quad= r^{-}\left(\gamma^{+}\cup x'\cup x,\gamma^{-},y\right)r^{+}\left(\gamma^{+}\cup x',\gamma^{-},x\right)r^{+}\left(\gamma^{+},\gamma^{-},x'\right)\\ &\quad= r^{-}\left(\gamma^{+}\cup x'\cup x,\gamma^{-},y\right)r^{+}\left(\gamma^{+}\cup x,\gamma^{-},x'\right)r^{+}\left(\gamma^{+},\gamma^{-},x\right)\\ &\quad= r\left(\gamma^{+}\cup x,\gamma^{-},x',y\right)r^{+}\left(\gamma^{+},\gamma^{-},x\right),\end{aligned}$$

which fulfills (4.7).

In the same way we obtain

$$\begin{aligned}r^{-}\left(\gamma^{+}\cup x,\gamma^{-}\cup y,y'\right)r\left(\gamma^{+},\gamma^{-},x,y\right) &= r\left(\gamma^{+},\gamma^{-}\cup y,x,y'\right)r^{-}\left(\gamma^{+},\gamma^{-},y\right)\\ &= r\left(\gamma^{+},\gamma^{-}\cup y',x,y\right)r^{-}\left(\gamma^{+},\gamma^{-},y'\right)\\ &= r^{-}\left(\gamma^{+}\cup x,\gamma^{-}\cup y',y\right)r\left(\gamma^{+},\gamma^{-},x,y'\right).\end{aligned} \tag{4.8}$$

As a result, using (4.3), (4.7), (4.8), one has

$$\begin{aligned}&r\left(\gamma^{+}\cup x',\gamma^{-}\cup y',x,y\right)r\left(\gamma^{+},\gamma^{-},x',y'\right)\\ &\quad= r^{-}\left(\gamma^{+}\cup x'\cup x,\gamma^{-}\cup y',y\right)r^{+}\left(\gamma^{+}\cup x',\gamma^{-}\cup y',x\right)r\left(\gamma^{+},\gamma^{-},x',y'\right)\\ &\quad= r^{-}\left(\gamma^{+}\cup x'\cup x,\gamma^{-}\cup y',y\right)r\left(\gamma^{+}\cup x,\gamma^{-},x',y'\right)r^{+}\left(\gamma^{+},\gamma^{-},x\right)\\ &\quad= r\left(\gamma^{+}\cup x,\gamma^{-}\cup y,x',y'\right)r^{-}\left(\gamma^{+}\cup x,\gamma^{-},y\right)r^{+}\left(\gamma^{+},\gamma^{-},x\right)\\ &\quad= r\left(\gamma^{+}\cup x,\gamma^{-}\cup y,x',y'\right)r\left(\gamma^{+},\gamma^{-},x,y\right)\end{aligned}$$

which proves the statement. □

Co-cycle and balance identities allow us to construct more complicated objects that characterize the energies between finite and infinite configurations.

Definition 4. Let us fix some order of finite "+"-configuration $\eta^+ = \{x_1, x_2, \ldots, x_n\}$ and set

$$\begin{aligned} R^+\left(\gamma^+,\gamma^-,\eta^+\right) &= R^+\left(\gamma^+,\gamma^-,\{x_1,x_2,\ldots,x_n\}\right) \\ &:= r^+\left(\gamma^+,\gamma^-,x_1\right) r^+\left(\gamma^+\cup x_1,\gamma^-,x_2\right) r^+\left(\gamma^+\cup\{x_1,x_2\},\gamma^-,x_3\right)\ldots \\ &\quad \times r^+\left(\gamma^+\cup\{x_1,x_2,\ldots,x_{n-1}\},\gamma^-,x_n\right). \end{aligned}$$

In [2, Lemma 2.3], it was shown, in fact, that this definition is correct (it does not depend on the order of points in η^+) and, moreover, for any η_1^+, η_2^+ :

$$R^+\left(\gamma^+,\gamma^-,\eta_1^+\cup\eta_2^+\right) = R^+\left(\gamma^+,\gamma^-,\eta_1^+\right) R^+\left(\gamma^+\cup\eta_1^+,\gamma^-,\eta_2^+\right) \tag{4.9}$$

(note that this fact does not depend on γ^-). Let us set, by definition,

$$R^+(\gamma^+,\gamma^-,\emptyset) := 1. \tag{4.10}$$

Note that (4.10) is consistent with (4.9) if we put $\eta_1^+ = \emptyset$.

In the same way we may define function $R^-\left(\gamma^+,\gamma^-,\eta^-\right)$ fixing the order $\eta^- = \{y_1, y_2, \ldots, y_m\}$ and setting

$$R^-\left(\gamma^+,\gamma^-,\eta^-\right) := r^-\left(\gamma^+,\gamma^-,y_1\right) r^-\left(\gamma^+,\gamma^-\cup y_1,y_2\right)\ldots r^-\left(\gamma^+,\gamma^-\cup\{y_1,\ldots,y_{n-1}\},y_n\right),$$

$$R^-(\gamma^+,\gamma^-,\emptyset) := 1. \tag{4.11}$$

And again

$$R^-\left(\gamma^+,\gamma^-,\eta_1^-\cup\eta_2^-\right) = R^-\left(\gamma^+,\gamma^-,\eta_1^-\right) R^-\left(\gamma^+\cup\eta_1^-,\gamma^-,\eta_2^-\right). \tag{4.12}$$

Functions $R^\pm$ also satisfy the balance identities:

Lemma 4.4. *For μ-a.a. $(\gamma^+,\gamma^-)\in\Gamma^2$ and for $\lambda_\sigma\times\lambda_\sigma$-a.a. $(\eta^+,\eta^-)\in\Gamma_0^2$*

$$R^+\left(\gamma^+,\gamma^-\cup\eta^-,\eta^+\right) R^-\left(\gamma^+,\gamma^-,\eta^-\right) = R^-\left(\gamma^+\cup\eta^+,\gamma^-,\eta^-\right) R^+\left(\gamma^+,\gamma^-,\eta^+\right). \tag{4.13}$$

Proof. Let $|\eta^-| = 1, \eta^- = \{y\}$. Then we want to prove that

$$R^+\left(\gamma^+,\gamma^-\cup y,\eta^+\right) r^-\left(\gamma^+,\gamma^-,y\right) = r^-\left(\gamma^+\cup\eta^+,\gamma^-,y\right) R^+\left(\gamma^+,\gamma^-,\eta^+\right). \tag{4.14}$$

If $|\eta^+| = 1$, then (4.14) holds due to (4.3). Suppose that (4.14) is true for any η^+, such that $|\eta^+| = n$. Then by (4.9), (4.3)

$$\begin{aligned} &R^+\left(\gamma^+,\gamma^-\cup y,\eta^+\cup x\right) r^-\left(\gamma^+,\gamma^-,y\right) \\ &\quad= r^+\left(\gamma^+\cup\eta^+,\gamma^-\cup y,x\right) R^+\left(\gamma^+,\gamma^-\cup y,\eta^+\right) r^-\left(\gamma^+,\gamma^-,y\right) \\ &\quad= r^+\left(\gamma^+\cup\eta^+,\gamma^-\cup y,x\right) r^-\left(\gamma^+\cup\eta^+,\gamma^-,y\right) R^+\left(\gamma^+,\gamma^-,\eta^+\right) \\ &\quad= r^-\left(\gamma^+\cup\eta^+\cup x,\gamma^-,y\right) r^+\left(\gamma^+\cup\eta^+,\gamma^-,x\right) R^+\left(\gamma^+,\gamma^-,\eta^+\right) \\ &\quad= r^-\left(\gamma^+\cup\eta^+\cup x,\gamma^-,y\right) R^+\left(\gamma^+,\gamma^-,\eta^+\cup x\right), \end{aligned}$$

hence, (4.14) holds.

Suppose that we prove (4.13) for any η^-, s.t. $|\eta^-| = n$ and consider

$$\begin{aligned} &R^+\left(\gamma^+,\gamma^-\cup\eta^-\cup y,\eta^+\right) R^-\left(\gamma^+,\gamma^-,\eta^-\cup y\right) \\ &\quad= R^+\left(\gamma^+,\gamma^-\cup\eta^-\cup y,\eta^+\right) r^-\left(\gamma^+,\gamma^-\cup\eta^-,y\right) R^-\left(\gamma^+,\gamma^-,\eta^-\right) \\ &\quad= r^-\left(\gamma^+\cup\eta^+,\gamma^-\cup\eta^-,y\right) R^+\left(\gamma^+,\gamma^-\cup\eta^-,\eta^+\right) R^-\left(\gamma^+,\gamma^-,\eta^-\right) \\ &\quad= r^-\left(\gamma^+\cup\eta^+,\gamma^-\cup\eta^-,y\right) R^-\left(\gamma^+\cup\eta^+,\gamma^-,\eta^-\right) R^+\left(\gamma^+,\gamma^-,\eta^+\right) \\ &\quad= R^-\left(\gamma^+\cup\eta^+,\gamma^-,\eta^-\cup y\right) R^+\left(\gamma^+,\gamma^-,\eta^+\right). \end{aligned}$$

Hence, the statement of lemma is proved. ∎

Corollary 4.5. *As a result, we can define*

$$\begin{aligned} R\left(\gamma^+,\gamma^-,\eta^+,\eta^-\right) &:= R^+\left(\gamma^+,\gamma^-\cup\eta^-,\eta^+\right)R^-\left(\gamma^+,\gamma^-,\eta^-\right) \\ &= R^-\left(\gamma^+\cup\eta^+,\gamma^-,\eta^-\right)R^+\left(\gamma^+,\gamma^-,\eta^+\right). \end{aligned} \tag{4.15}$$

The next statement is similar to the properties (4.9), (4.12) for the function R.

Lemma 4.6. *For μ-a.a. $(\gamma^+,\gamma^-)\in\Gamma^2$ and for $\lambda_\sigma\times\lambda_\sigma$-a.a. $\left(\eta_1^+,\eta_1^-\right),\left(\eta_2^+,\eta_2^-\right)\in\Gamma_0^2$ the following equalities hold*

$$\begin{aligned} R\left(\gamma^+,\gamma^-,\eta_1^+\cup\eta_2^+,\eta^-\right) &= R\left(\gamma^+\cup\eta_2^+,\gamma^-,\eta_1^+,\eta^-\right)R^+\left(\gamma^+,\gamma^-,\eta_2^+\right), \\ R\left(\gamma^+,\gamma^-,\eta^+,\eta_1^-\cup\eta_2^-\right) &= R\left(\gamma^+,\gamma^-\cup\eta_2^-,\eta^+,\eta_1^-\right)R^-\left(\gamma^+,\gamma^-,\eta_2^-\right), \\ R\left(\gamma^+,\gamma^-,\eta_1^+\cup\eta_2^+,\eta_1^-\cup\eta_2^-\right) &= R\left(\gamma^+\cup\eta_2^+,\gamma^-\cup\eta_2^-,\eta_1^+,\eta_1^-\right)R\left(\gamma^+,\gamma^-,\eta_2^+,\eta_2^-\right). \end{aligned}$$

Proof. By (4.15), (4.9) we obtain

$$\begin{aligned} R\left(\gamma^+,\gamma^-,\eta_1^+\cup\eta_2^+,\eta^-\right) &= R^-\left(\gamma^+\cup\eta_2^+\cup\eta_1^+,\gamma^-,\eta^-\right)R^+\left(\gamma^+,\gamma^-,\eta_1^+\cup\eta_2^+\right) \\ &= R^-\left(\gamma^+\cup\eta_2^+\cup\eta_1^+,\gamma^-,\eta^-\right)R^+\left(\gamma^+\cup\eta_2^+,\gamma^-,\eta_1^+\right)R^+\left(\gamma^+,\gamma^-,\eta_2^+\right) \\ &= R\left(\gamma^+\cup\eta_2^+,\gamma^-,\eta_1^+,\eta^-\right)R^+\left(\gamma^+,\gamma^-,\eta_2^+\right). \end{aligned}$$

The second identity may be obtained in the same way.

Next, using first and second identities one has

$$\begin{aligned} &R\left(\gamma^+,\gamma^-,\eta_1^+\cup\eta_2^+,\eta_1^-\cup\eta_2^-\right) \\ &\quad= R\left(\gamma^+\cup\eta_2^+,\gamma^-,\eta_1^+,\eta_1^-\cup\eta_2^-\right)R^+\left(\gamma^+,\gamma^-,\eta_2^+\right) \\ &\quad= R\left(\gamma^+\cup\eta_2^+,\gamma^-\cup\eta_2^-,\eta_1^+,\eta_1^-\right)R^-\left(\gamma^+\cup\eta_2^+,\gamma^-,\eta_2^-\right)R^+\left(\gamma^+,\gamma^-,\eta_2^+\right) \\ &\quad= R\left(\gamma^+\cup\eta_2^+,\gamma^-\cup\eta_2^-,\eta_1^+,\eta_1^-\right)R\left(\gamma^+,\gamma^-,\eta_2^+,\eta_2^-\right) \end{aligned}$$

which completes the proof. □

The next lemma shows that the values of the function R on some elements may be defined directly via r.

Lemma 4.7. *For $\lambda_\sigma\times\lambda_\sigma$-a.a. $(\eta^+,\eta^-)\in\Gamma_0^2$ with $|\eta^+|=|\eta^-|$ one has*

$$\begin{aligned} R\left(\gamma^+,\gamma^-,\eta^+,\eta^-\right) &= r\left(\gamma^+,\gamma^-,x_1,y_1\right)r\left(\gamma^+\cup x_1,\gamma^-\cup y_1,x_2,y_2\right) \\ &\quad\times r\left(\gamma^+\cup\{x_1,x_2\},\gamma^-\cup\{y_1,y_2\},x_3,y_3\right)\ldots \\ &\quad\times r\left(\gamma^+\cup\{x_1,x_2,\ldots,x_{n-2}\},\gamma^-\cup\{y_1,y_2,\ldots,y_{n-2}\},x_{n-1},y_{n-1}\right) \\ &\quad\times r\left(\gamma^+\cup\{x_1,x_2,\ldots,x_{n-1}\},\gamma^-\cup\{y_1,y_2,\ldots,y_{n-1}\},x_n,y_n\right) \end{aligned}$$

for some fixed orders of points

$$\eta^+=\{x_1,x_2,\ldots,x_n\},\quad \eta^-=\{y_1,y_2,\ldots,y_n\}.$$

Proof. Let $|\eta^+|=|\eta^-|=1$, then the statement follows from (4.15), Definition 4 and (4.4).

Let us suppose that the statement is true for any η^+,η^-, s.t. $|\eta^+|=|\eta^-|=n$. Then, using

(4.15), (4.4), (4.14) and Definition 4, we obtain

$$
\begin{aligned}
& r\left(\gamma^{+} \cup \eta^{+}, \gamma^{-} \cup \eta^{-}, x, y\right) R\left(\gamma^{+}, \gamma^{-}, \eta^{+}, \eta^{-}\right) \\
&\qquad = r\left(\gamma^{+} \cup \eta^{+}, \gamma^{-} \cup \eta^{-}, x, y\right) R^{+}\left(\gamma^{+}, \gamma^{-} \cup \eta^{-}, \eta^{+}\right) R^{-}\left(\gamma^{+}, \gamma^{-}, \eta^{-}\right) \\
&\qquad = r^{+}\left(\gamma^{+} \cup \eta^{+}, \gamma^{-} \cup \eta^{-} \cup y, x\right) R^{-}\left(\gamma^{+}, \gamma^{-}, \eta^{-}\right) \\
&\qquad\quad \times r^{-}\left(\gamma^{+} \cup \eta^{+}, \gamma^{-} \cup \eta^{-}, y\right) R^{+}\left(\gamma^{+}, \gamma^{-} \cup \eta^{-}, \eta^{+}\right) \\
&\qquad = r^{+}\left(\gamma^{+} \cup \eta^{+}, \gamma^{-} \cup \eta^{-} \cup y, x\right) R^{-}\left(\gamma^{+}, \gamma^{-}, \eta^{-}\right) \\
&\qquad\quad \times R^{+}\left(\gamma^{+}, \gamma^{-} \cup \eta^{-} \cup y, \eta^{+}\right) r^{-}\left(\gamma^{+}, \gamma^{-} \cup \eta^{-}, y\right) \\
&\qquad = R^{+}\left(\gamma^{+}, \gamma^{-} \cup \eta^{-} \cup y, \eta^{+}\right) r^{+}\left(\gamma^{+} \cup \eta^{+}, \gamma^{-} \cup \eta^{-} \cup y, x\right) \\
&\qquad\quad \times R^{-}\left(\gamma^{+}, \gamma^{-}, \eta^{-}\right) r^{-}\left(\gamma^{+}, \gamma^{-} \cup \eta^{-}, y\right) \\
&\qquad = R^{+}\left(\gamma^{+}, \gamma^{-} \cup \eta^{-} \cup y, \eta^{+} \cup x\right) R^{-}\left(\gamma^{+}, \gamma^{-}, \eta^{-} \cup y\right) \\
&\qquad = R\left(\gamma^{+}, \gamma^{-}, \eta^{+} \cup x, \eta^{-} \cup y\right),
\end{aligned}
$$

which proves the assertion. □

Next theorem present Ruelle-type identity for Gibbs measure μ which also called "infinitely divisible property".

Theorem 4.8. *Let $\mu \in \mathcal{G}(r^+, r^-, \sigma)$. Then for any non-negative measurable function $F : \Gamma^2 \to [0; +\infty)$ and for any $\Lambda^\pm \in \mathcal{B}_c(X)$*

$$
\begin{aligned}
\int_{\Gamma^2} F(\gamma) \mathrm{d}\mu\left(\gamma^{+}, \gamma^{-}\right) = & \int_{\Gamma^{+}_{\Lambda^{+}}} \int_{\Gamma^{-}_{\Lambda^{-}}} \int_{\Gamma^{+}_{\Lambda^{+c}}} \int_{\Gamma^{-}_{\Lambda^{-c}}} F\left(\gamma^{+} \cup \eta^{+}, \gamma^{-} \cup \eta^{-}\right) \\
& \times R\left(\gamma^{+}, \gamma^{-}, \eta^{+}, \eta^{-}\right) \mathrm{d}\mu\left(\gamma^{+}, \gamma^{-}\right) \mathrm{d}\lambda_\sigma\left(\eta^{+}\right) \mathrm{d}\lambda_\sigma\left(\eta^{-}\right). \qquad (4.16)
\end{aligned}
$$

Proof. Set for $x \in X$, $n \in \mathbb{N}$, $A^- \in \mathcal{B}(\Gamma^-)$ and for measurable non-negative measurable F

$$
h^{+}\left(\gamma^{+}, \gamma^{-}, x\right) = 1_{A^-}\left(\gamma^{-}\right) 1\!\!1_{\{|\gamma^+ \cap \Lambda^+| = n\}} 1\!\!1_{\Lambda^+}(x) F\left(\gamma^{+}, \gamma^{-}\right).
$$

Since

$$
\int_{\Gamma^2} \sum_{x \in \gamma^+} h^{+}\left(\gamma^{+}, \gamma^{-}, x\right) \mathrm{d}\mu\left(\gamma^{+}, \gamma^{-}\right) = n \int_{\Gamma^2} 1\!\!1_{\{|\gamma^+ \cap \Lambda^+| = n\}} F\left(\gamma^{+}, \gamma^{-}\right) 1_{A^-}\left(\gamma^{-}\right) \mathrm{d}\mu\left(\gamma^{+}, \gamma^{-}\right)
$$

and

$$
\begin{aligned}
& \int_{\Gamma^2} \int_X h^{+}\left(\gamma^{+} \cup x, x\right) r^{+}\left(\gamma^{+}, \gamma^{-}, x\right) \mathrm{d}\sigma(x) \mathrm{d}\mu\left(\gamma^{+}, \gamma^{-}\right) \\
&\quad = \int_{\Gamma^2} \int_{\Lambda^+} 1\!\!1_{\{|(\gamma^+ \cup x) \cap \Lambda^+| = n\}} 1_{A^-}\left(\gamma^{-}\right) F\left(\gamma^{+} \cup x, \gamma^{-}\right) r^{+}\left(\gamma^{+}, \gamma^{-}, x\right) \mathrm{d}\sigma(x) \mathrm{d}\mu\left(\gamma^{+}, \gamma^{-}\right) \\
&\quad = \int_{\Gamma^2} \int_{\Lambda^+} 1\!\!1_{\{|\gamma^+ \cap \Lambda^+| = n-1\}} 1_{A^-}\left(\gamma^{-}\right) F\left(\gamma^{+} \cup x, \gamma^{-}\right) r^{+}\left(\gamma^{+}, \gamma^{-}, x\right) \mathrm{d}\sigma(x) \mathrm{d}\mu\left(\gamma^{+}, \gamma^{-}\right),
\end{aligned}
$$

then using (4.1) we obtain

$$
\begin{aligned}
& \int_{\Gamma^2} 1\!\!1_{\{|\gamma^+ \cap \Lambda^+| = n\}} 1_{A^-}\left(\gamma^{-}\right) F\left(\gamma^{+}, \gamma^{-}\right) \mathrm{d}\mu\left(\gamma^{+}, \gamma^{-}\right) \\
&\quad = \frac{1}{n} \int_{\Lambda^+} \int_{\Gamma^2} 1\!\!1_{\{|\gamma^+ \cap \Lambda^+| = n-1\}} 1_{A^-}\left(\gamma^{-}\right) F\left(\gamma^{+} \cup x, \gamma^{-}\right) r^{+}\left(\gamma^{+}, \gamma^{-}, x\right) \mathrm{d}\mu\left(\gamma^{+}, \gamma^{-}\right) \mathrm{d}\sigma(y) \mathrm{d}\sigma(x)
\end{aligned}
$$

for any non-negative measurable F. Apply this formula for function

$$
\tilde{F}\left(\gamma^{+}, \gamma^{-}\right) = F\left(\gamma^{+} \cup x, \gamma^{-}\right) r^{+}\left(\gamma^{+}, \gamma^{-}, x\right)
$$

with fixed x, y. Then

$$\begin{aligned}
&\int_{\Gamma^2} 1\!\!1_{\{|\gamma^+\cap\Lambda^+|=n\}} 1_{A^-}\left(\gamma^-\right) F\left(\gamma^+,\gamma^-\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\
&\qquad = \frac{1}{n\,(n-1)} \int_{\Lambda^{+2}} \int_{\Gamma^2} 1_{A^-}\left(\gamma^-\right) 1\!\!1_{\{|\gamma^+\cap\Lambda^+|=n-2\}} F\left(\gamma^+\cup x_1\cup x_2,\gamma^-\right)\\
&\qquad\quad \times r^+\left(\gamma^+\cup x_2,\gamma^-,x_1\right) r^+\left(\gamma^+,\gamma^-,x_2\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right) \mathrm{d}\sigma\left(x_2\right) \mathrm{d}\sigma\left(x_1\right).
\end{aligned}$$

As a result, repeating this procedure we obtain,

$$\begin{aligned}
&\int_{\Gamma^2} 1\!\!1_{\{|\gamma^+\cap\Lambda^+|=n\}} 1_{A^-}\left(\gamma^-\right) F\left(\gamma^+,\gamma^-\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\
&\qquad = \frac{1}{n!} \int_{\Lambda^{+n}} \int_{\Gamma^2} 1_{A^-}\left(\gamma^-\right) 1\!\!1_{\{|\gamma^+\cap\Lambda^+|=0\}} F\left(\gamma^+\cup\{x_1,\ldots,x_n\},\gamma^-\right)\\
&\qquad\quad \times R^+\left(\gamma^+,\gamma^-,\{x_1,\ldots,x_n\}\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right) \mathrm{d}\sigma\left(x_1\right)\ldots\mathrm{d}\sigma\left(x_n\right).
\end{aligned}$$

Then

$$\begin{aligned}
&\int_{\Gamma^+\times A^-} F\left(\gamma^+,\gamma^-\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\
&\qquad = \int_{\Gamma^+_{\Lambda^+}} \int_{\Gamma^+_{\Lambda^{+c}}} \int_{A^-} F\left(\gamma^+\cup\eta^+,\gamma^-\right) R^+\left(\gamma^+,\gamma^-,\eta^+\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right) \mathrm{d}\lambda_\sigma\left(\eta^+\right). \qquad (4.17)
\end{aligned}$$

Similarly, for any $A^+ \in \mathcal{B}(\Gamma^+)$

$$\begin{aligned}
&\int_{A^+\times\Gamma^-} F\left(\gamma^+,\gamma^-\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\
&\qquad = \int_{A^+} \int_{\Gamma^-_{\Lambda^-}} \int_{\Gamma^-_{\Lambda^{-c}}} F\left(\gamma^+,\gamma^-\cup\eta^-\right) R^-\left(\gamma^+,\gamma^-,\eta^-\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right) \mathrm{d}\lambda_\sigma\left(\eta^-\right). \qquad (4.18)
\end{aligned}$$

Putting $A^- = \Gamma^-$ in (4.17) and applying (4.18) to the r.h.s. of (4.17) with $A^+ = \Gamma^+_{\Lambda^+} \times \Gamma^+_{\Lambda^{+c}}$ we obtain

$$\begin{aligned}
&\int_{\Gamma^+\times\Gamma^-} F\left(\gamma^+,\gamma^-\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\
&\qquad = \int_{\Gamma^+_{\Lambda^+}} \int_{\Gamma^+_{\Lambda^{+c}}} \int_{\Gamma^-_{\Lambda^-}} \int_{\Gamma^-_{\Lambda^{-c}}} F\left(\gamma^+\cup\eta^+,\gamma^-\cup\eta^-\right)\\
&\qquad\quad \times R^+\left(\gamma^+,\gamma^-\cup\eta^-,\eta^+\right) R^-\left(\gamma^+,\gamma^-,\eta^-\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right) \mathrm{d}\lambda_\sigma\left(\eta^+\right) \mathrm{d}\lambda_\sigma\left(\eta^-\right).
\end{aligned}$$

Hence, the statement follows from (4.15). □

The next proposition shows that any Gibbs measure (in the sense of Definition 3) is a locally absolutely continuous w.r.t. $\pi_\sigma \times \pi_\sigma$.

Proposition 4.9. *Let $\mu \in \mathcal{G}(r^+, r^-, \sigma)$. Then for any $\Lambda^\pm \in \mathcal{B}_c(X)$ there exist*

$$\frac{\mathrm{d}\mu^{\Lambda^+,\Lambda^-}}{\mathrm{d}(\pi_\sigma^{\Lambda^+}\times\pi_\sigma^{\Lambda^-})}(\eta^+,\eta^-) = \mathrm{e}^{\sigma(\Lambda^+)+\sigma(\Lambda^-)} \int_{\Gamma^+_{\Lambda^{+c}}} \int_{\Gamma^-_{\Lambda^{-c}}} R\left(\gamma^+,\gamma^-,\eta^+,\eta^-\right) \mathrm{d}\mu\left(\gamma^+,\gamma^-\right) \qquad (4.19)$$

for $\pi_\sigma^{\Lambda^+} \times \pi_\sigma^{\Lambda^-}$-a.a. $(\eta^+,\eta^-) \in \Gamma^+_{\Lambda^{+c}} \times \Gamma^-_{\Lambda^{-c}}$.

Proof. For any measurable non-negative function F such that $F(\gamma^+,\gamma^-) = F(\gamma^+_{\Lambda^+},\gamma^-_{\Lambda^-})$, using (4.16), we obtain

$$\begin{aligned}&\int_{\Gamma^+_{\Lambda^+}\times\Gamma^-_{\Lambda^-}} F(\gamma^+_{\Lambda^+},\gamma^-_{\Lambda^-})\mathrm{d}\mu^{\Lambda^+,\Lambda^-}\left(\gamma^+_{\Lambda^+},\gamma^-_{\Lambda^-}\right) = \int_{\Gamma^2} F\left(\gamma\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\\ &= \int_{\Gamma^+_{\Lambda^+}}\int_{\Gamma^-_{\Lambda^-}} F\left(\eta^+,\eta^-\right)\int_{\Gamma^+_{\Lambda^{+c}}}\int_{\Gamma^-_{\Lambda^{-c}}} R\left(\gamma^+,\gamma^-,\eta^+,\eta^-\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\mathrm{d}\lambda_\sigma\left(\eta^+\right)\mathrm{d}\lambda_\sigma\left(\eta^-\right),\end{aligned}$$

which fulfills the statement. □

In particular, for any $\mu\in\mathcal{G}(r^+,r^-,\sigma)$ Propositions 3.1 and 3.2 as well as Corollary 3.3 hold.

As we mentioned above, using (3.3), the measure μ^+ is locally absolutely continuous w.r.t. π_σ and for any $A\in\mathcal{B}(\Gamma^+_\Lambda)$, $\Lambda\in\mathcal{B}_c(X)$

$$(\mu^+)^\Lambda(A) = (\mu^\Lambda)^+(A) = \mu^\Lambda(A\times\Gamma^-_\Lambda).$$

Therefore, using (4.15) and(4.17)

$$\begin{aligned}\frac{\mathrm{d}\left(\mu^+\right)^\Lambda}{\mathrm{d}\pi^\Lambda_\sigma}\left(\eta^+\right)&\\ =\ & \mathrm{e}^{2\sigma(\Lambda)}\int_{\Gamma^-_\Lambda}\int_{\Gamma^+_{\Lambda^c}}\int_{\Gamma^-_{\Lambda^c}} R\left(\gamma^+,\gamma^-,\eta^+,\eta^-\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\mathrm{d}\pi^\Lambda_\sigma(\eta^-)\\ =\ & \mathrm{e}^{\sigma(\Lambda)}\int_{\Gamma^-_\Lambda}\int_{\Gamma^+_{\Lambda^c}}\int_{\Gamma^-_{\Lambda^c}} R^+\left(\gamma^+,\gamma^-\cup\eta^-,\eta^+\right)R^-\left(\gamma^+,\gamma^-,\eta^-\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\mathrm{d}\lambda^\Lambda_\sigma(\eta^-)\\ =\ & \mathrm{e}^{\sigma(\Lambda)}\int_{\Gamma^+_{\Lambda^c}}\int_{\Gamma^-} R^+\left(\gamma^+,\gamma^-,\eta^+\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\end{aligned}\tag{4.20}$$

for $\pi^{\Lambda^+}_\sigma$-a.a. $\eta^+\in\Gamma^+_{\Lambda^+}$.

In the next proposition we find formulas for the correlation functions of the Gibbs measures.

Proposition 4.10. *Let $\mu\in\mathcal{G}(r^+,r^-,\sigma)$ and* (3.4) *holds. Then*

$$k_\mu(\eta^+,\eta^-) = \int_{\Gamma^2} R(\gamma^+,\gamma^-,\eta^+,\eta^-)\mathrm{d}\mu(\gamma^+,\gamma^-),\tag{4.21}$$

$$k^+_\mu(\eta^+) = \int_{\Gamma^2} R^+\left(\gamma^+,\gamma^-,\eta^+\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right).\tag{4.22}$$

Proof. Using (3.5), (4.19), Lemma 4.6 and (4.16) we obtain

$$\begin{aligned}&k_\mu(\eta^+,\eta^-)\\ &= \int_{\Gamma^+_{\Lambda^+}}\int_{\Gamma^-_{\Lambda^-}}\int_{\Gamma^+_{\Lambda^{+c}}}\int_{\Gamma^-_{\Lambda^{-c}}} R\left(\gamma^+,\gamma^-,\eta^+\cup\xi^+,\eta^-\cup\xi^-\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\mathrm{d}\lambda_\sigma(\xi^+)\mathrm{d}\lambda_\sigma(\xi^-)\\ &= \int_{\Gamma^+_{\Lambda^+}}\int_{\Gamma^-_{\Lambda^-}}\int_{\Gamma^+_{\Lambda^{+c}}}\int_{\Gamma^-_{\Lambda^{-c}}} R\left(\gamma^+\cup\xi^+,\gamma^-\cup\xi^-,\eta^+,\eta^-\right)\\ &\qquad\times R\left(\gamma^+,\gamma^-,\xi^+,\xi^-\right)\mathrm{d}\mu\left(\gamma^+,\gamma^-\right)\mathrm{d}\lambda_\sigma(\xi^+)\mathrm{d}\lambda_\sigma(\xi^-)\\ &= \int_{\Gamma^2} R(\gamma^+,\gamma^-,\eta^+,\eta^-)\mathrm{d}\mu(\gamma^+,\gamma^-).\end{aligned}$$

The second formula one can obtain in the same way or just putting $\eta^- = \emptyset$ in the previous one and using (4.15), (4.11). □

At the end of article we consider examples of partial relative energies densities $r^{\pm}$ which satisfied (4.1)–(4.3).

Let $\mu_{1,2}$ be Gibbs measures on $\bigl(\Gamma,\mathcal{B}(\Gamma)\bigr)$ with relative energies densities $r_{1,2}$ in the sense of [2]. Namely, let for any measurable $h:\Gamma\times X\to[0;\infty)$

$$\int_\Gamma\sum_{x\in\gamma}h(x,\gamma)\mathrm{d}\mu_{1,2}(\gamma)=\int_\Gamma\int_X h(x,\gamma\cup x)r_{1,2}(\gamma,x)\mathrm{d}\sigma(x)\mathrm{d}\mu_{1,2}(\gamma).$$

Let $\phi:X^2\to\mathbb{R}\cup\{\infty\}$ be a symmetric function. Then on can construct an example of $r^{\pm}$ which heuristically corresponds to the following formal "pair-potential perturbation" $\mu\in\mathcal{M}^1(\Gamma^2)$ of the product $\mu_1\times\mu_2$:

$$\mathrm{d}\mu(\gamma^+,\gamma^-)=\text{“}\frac{1}{Z}\exp\Bigl\{-\sum_{\{x,y\}\subset\gamma}\phi(x,y)\Bigr\}\mathrm{d}\mu_1(\gamma^+)\mathrm{d}\mu_2(\gamma^-)\text{”}.$$

Namely, let

$$r_0(\gamma,x)=\exp\Bigl\{-\sum_{y\in\gamma}\phi(x,y)\Bigr\},$$

then one can set

$$\begin{aligned}r^+(\gamma^+,\gamma^-,x)&=r_0(\gamma^-,x)r_1(\gamma^+,x),\\ r^-(\gamma^+,\gamma^-,y)&=r_0(\gamma^+,y)r_2(\gamma^-,y).\end{aligned}$$

The partial co-cycle identities (4.1), (4.2) hold since for $r_{1,2}$ the co-cycle identities hold (see [2]). One can easily check the balance condition (4.3):

$$\begin{aligned}r^+(\gamma^+,\gamma^-\cup y,x)r^-(\gamma^+,\gamma^-,y)&=r_0(\gamma^-\cup y,x)r_1(\gamma^+,x)r_0(\gamma^+,y)r_2(\gamma^-,y)\\ &=\mathrm{e}^{-\phi(x,y)}r_0(\gamma^-,x)r_1(\gamma^+,x)r_0(\gamma^+,y)r_2(\gamma^-,y)\\ &=r_0(\gamma^-,x)r_1(\gamma^+,x)r_0(\gamma^+\cup x,y)r_2(\gamma^-,y)\\ &=r^+(\gamma^+,\gamma^-,x)r^-(\gamma^+\cup x,\gamma^-,y).\end{aligned}$$

The simplest examples of $r_{1,2}$ are also pair potential densities: let $\phi^{\pm}:X^2\to\mathbb{R}\cup\{\infty\}$ be symmetric functions and

$$r_1(\gamma^+,x)=\exp\Bigl\{-\sum_{x'\in\gamma^+}\phi^+(x,x')\Bigr\},\qquad r_2(\gamma^-,y)=\exp\Bigl\{-\sum_{y'\in\gamma^-}\phi^-(y,y')\Bigr\}.$$

Then $\mu_{1,2}$ are classical pair-potential Gibbs measures and μ is a measure of the type considered in [4]. As a result, in this case

$$\begin{aligned}r^+(\gamma^+,\gamma^-,x)&=\exp\Bigl\{-\sum_{y\in\gamma^-}\phi(x,y)-\sum_{x'\in\gamma^+}\phi^+(x,x')\Bigr\},\\ r^-(\gamma^+,\gamma^-,y)&=\exp\Bigl\{-\sum_{x\in\gamma^+}\phi(x,y)-\sum_{y'\in\gamma^-}\phi^-(y,y')\Bigr\},\end{aligned}$$

and, therefore,

$$\begin{aligned}&r(\gamma^+,\gamma^-,x,y)\\ &\quad=\exp\Bigl\{-\phi(x,y)-\sum_{x'\in\gamma^+}\phi(y,x')-\sum_{y'\in\gamma^-}\phi(x,y')-\sum_{x'\in\gamma^+}\phi^+(x,x')-\sum_{y'\in\gamma^-}\phi^-(y,y')\Bigr\}.\end{aligned}$$

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Міри на двокомпонентних просторах конфігурацій

Д.Л.Фінкельштейн

Інститут математики Національної Академії наук України,
вул. Терещенківська, 3, Київ-4, 01601, Україна

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Ми вивчаємо міри на просторах конфігурацій двох типів. Описано гібсовські міри на таких просторах. Розглянуто основні властивості відносних енергій та кореляційних функцій. Зокрема, показано, що такі гібсовські мри зосереджені на парах конфігурацій, які не перетинаються.

Ключові слова: *двокомпонентні простори конфігурацій, міри Гіббса, кореляційні функції, статистична механіка неперервних систем, відносні енергії*

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