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Entanglement production under electron phonon interaction in a quantum dot

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We consider the quantum dot entangler device introduced by Oliver et al. [1] coupled to a single phonon mode. While we take the phonon interaction to all orders, we perform a fourth order calculation in electron-dot coupling. Using the Von Neumann entropy we measure the degree of entanglement. We find that the phonon mechanism habilitates single electron processes which manifest themselves as decoherent processes.

Key words: *quantum dot, entanglement, decoherence*

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1. Introduction

Quantum entanglement is a very relevant property of quantum systems, and producing it in a controllable fashion is a central resource for quantum information manipulation [2,3]. Quantum entanglement is the centerpiece ingredient in order to implement such sophisticated tasks as quantum teleportation and dense coding [4,5], tasks that will perform as the basic "cogwheels" of a future quantum computer [2]. The most natural environment in physics for implementing quantum gates is solid state systems where fermions (electrons) are subject of numerous interactions that concern and compromise quantum coherence. In particular, electron-electron and electron-phonon interactions are of central importance when addressing quantum correlations and coherence in electron transport through, for example, a quantum dot. If we are concerned in generating quantum nonseparable states i.e. entangled quantum states, studying how the transmission properties are affected by such interactions and how quantum correlations are affected is a central issue in future devices. An interesting recent debate, still ongoing is whether electron-electron interaction is a necessary ingredient in order to entangle particles extracted from a Fermi sea. While it has been demonstrated that interactions can be used to enhance the production of entanglement, recent proposals have shown that entanglement between electrons and holes can be produced in its absence [6–8].

In this work we present a summary of the results obtained from a model for two electron cotunnelling through a quantum dot where on-site electron-electron interactions U are in place. Single particle processes are suppressed, by design [1], due to energy conservation, making it a "clean" system as compared to other recent proposals. Since we are also interested in possible dephasing and precursory decoherence effects [9–12], we extend the model of Oliver et al by coupling the dot to a phonon spectrum. As a simplifying assumption we restrict ourselves to considering one single phonon mode. We use a tight binding hamiltonian approach which is an extension of the study performed in references [1,8]. We resort to perturbation theory in order to take into account the coupling of the quantum dot to external leads. In this first approach to the problem we consider the limit of zero input/output bandwidth of the leads but one can readily include the effects of a finite bandwidth and a full phonon bath. Although simple, this naive approach has shown some

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remarkable features regarding the effects of Fano like resonances on the entangling properties of the model.

2. The model

The model system is depicted in figure 1 following Oliver et al [1]. The model is described by the tight binding hamiltonian

$$\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_0 + \widehat{V}, \tag{1}$$

where

$$\widehat{\mathcal{H}}_0 = \sum_{s,k,\sigma} \varepsilon_{s,k} \hat{a}^\dagger_{s,k,\sigma} \hat{a}_{s,k,\sigma} + \sum_{\sigma} \left(\varepsilon_d \hat{c}^\dagger_\sigma \hat{c}_\sigma - V_g \hat{c}^\dagger_\sigma \hat{c}_\sigma \left(\hat{b}^\dagger + \hat{b} \right) \right) + \hbar\omega \hat{b}^\dagger \hat{b} + U \hat{n}_\uparrow \hat{n}_\downarrow \,, \tag{2}$$

$$\widehat{V} = \sum_{s,k,\sigma} \left(V_s \hat{a}^\dagger_{s,k,\sigma} \hat{c}_\sigma + V^*_s \hat{c}^\dagger_\sigma \hat{a}_{s,k,\sigma} \right). \tag{3}$$

The four terms in $\widehat{\mathcal{H}}_0$ describe the leads (they are only external levels connected to the dot), the intradot level coupled to the phonon spectrum, the phonon energy levels, and the intradot electron-electron Coulomb interaction, respectively. The operator $\hat{a}^\dagger_{s,k,\sigma}$ ($\hat{a}_{s,k,\sigma}$) creates (annihilates) an electron with label k and spin σ in either lead according to the lead label, where $s = \{L, R_1, R_2\}$ as described in figure 1 and $\sigma = \{\uparrow, \downarrow\}$. The operator $\hat{c}^\dagger_\sigma$ ($\hat{c}_\sigma$) creates (annihilates) an electron with single energy level degenerate in spin. The operator $\hat{b}^\dagger$ ($\hat{b}$) creates (annihilates) the phonon within the dot. While V_s measures the dot lead coupling, V_g plays the role for the electron phonon interaction.

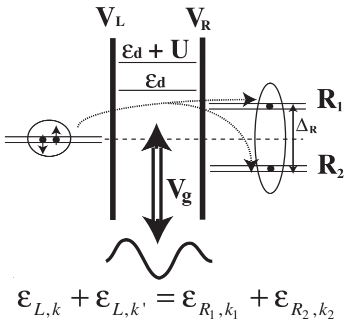

Figure 1. Energy diagram for the three port quantum dot as described in the text. By construction, single electron processes are forbidden by energy conservation. Electrons couple to the double well through the couplings V_L and V_R while coupling to phonons is described by the canonical electron phonon interaction through V_g.

In figure 1 the energy levels are disposed so that single electron processes are virtual, hence they do not conserve energy. In addition, double occupancy incurs in an on site virtual Coulomb interaction U. The phonon mode also couples virtually to the electrons present within the dot. We note that two electron co-tunnelling might occur for $E_i = E_f$, where $E_i = \varepsilon_{L,k} + \varepsilon_{L,k'}$ and $E_f = \varepsilon_{R_1,k_1} + \varepsilon_{R_2,k_2}$ are the input and output energies, respectively. We assume that $\varepsilon_{L,k} \neq \varepsilon_{R_1,k_1} \neq \varepsilon_{R_2,k_2}$ eliminating single electron tunnelling. For later convenience, let us define the (half) energy differences $\Delta_L = 1/2(\varepsilon_{L,k} - \varepsilon_{L,k'})$ and $\Delta_R = 1/2(\varepsilon_{R_1} - \varepsilon_{R_2})$. The intradot energy level is given by the term ε_d.

In order to compute the transition amplitudes from an initial state consisting of an antisymmetrized electron pair extracted from a Fermi sea at zero temperature, $|\phi_i\rangle = \hat{a}^\dagger_{L,k,\sigma} \hat{a}^\dagger_{L,k',\sigma'} |0\rangle$, to a non separable (entangled) final singlet or triplet state $|\phi_f\rangle = |s\rangle, |t\rangle = 1/\sqrt{2}(\hat{a}^\dagger_{R_1\uparrow} \hat{a}^\dagger_{R_2\downarrow} \mp$

$\hat{a}^\dagger_{R_1\downarrow}\hat{a}^\dagger_{R_2\uparrow})|0\rangle$ we use the T matrix defined by the recursive relation

$$\widehat{T}(\varepsilon) = \widehat{V} + \widehat{V}\frac{1}{\varepsilon - \widehat{H}_0}\widehat{T}\,. \tag{4}$$

Although it might seem that the T matrix perturbation theory is rather sophisticated to address this model it is worth noting that it is an appropriate tool to address off shell scattering. This is a necessary ingredient for the case at hand since single electron transfer is off shell while two electron transfer is on shell.

Since we are interested in the effect of the phonon terms on the entangling properties of the device, we shall make use of the following scenario when phonons only couple to the electrons at the dot. In the present model there will only be virtual excitations, meaning that the number of phonons in the dot does not change after both electrons have been transmitted. Thus, we only consider globally (two particle) elastic processes. In a more general situation we can consider that the electron can absorb or emit phonons that remain after the electrons have left the dot. We shall then have both elastic and inelastic components [11,12]. As a consequence of the simplifications, one would expect our model not to exhibit decoherence but to show antiresonances due to two electron paths within the dot analogous to the Fano effect [13]. However, we have found that the phonon field habilitates sequential single electron tunnelling, which in turn can be interpreted as a manifestation of decoherence.

The phonon term in the hamiltonian $\widehat{\mathcal{H}}_0$ in principle couples to a infinite number of excitations. We can nevertheless truncate the coupling to a finite number of phonons and systematically increase such number until the results do not change within a certain tolerance. This procedure is easy to do following the decimation process described in reference [12]. The resulting $\widehat{\mathcal{H}}_0 \to \widehat{\mathcal{H}}_{\text{eff}}$ is given by

$$\sum_{s,k,\sigma} \varepsilon_{s,k}\hat{a}^\dagger_{s,k,\sigma}\hat{a}_{s,k,\sigma} + \sum_{\sigma}(\varepsilon_d + \Sigma(\varepsilon))\hat{c}^\dagger_\sigma \hat{c}_\sigma + U\hat{n}_\uparrow\hat{n}_\downarrow\,,$$

where for the one phonon corrections $\Sigma(\varepsilon) = V_g^2/(\varepsilon - (\varepsilon_d + \hbar\omega))$, is an energy dependent quantity that will describe a second resonance within the dot. Adding phonon excitation is straightforward using this decimation scheme and will only result in additional algebraic corrections. Systematically following this approach, one is able to exactly describe the phonon contribution and the procedure is not restricted to small electron phonon coupling. As described earlier one can also include further phonon excitations systematically and better consider the reservoir character of the phonon bath. It is also worth noting that we are implicitly making the assumption $V_g \gg V_s$, since otherwise the self-energy correction associated to the leads should also be included. Due to this restriction, the range of parameters for V_g is somewhat restricted to such large values as compared to those for V_s.

In this manner, the expression (4) becomes

$$\widehat{T}(\varepsilon) = \widehat{V} + \widehat{V}\frac{1}{\varepsilon - \widehat{H}_{\text{eff}}(\varepsilon)}\widehat{T}\,.$$

From this point of view, one can compute the contribution due to the coupling to the leads using perturbation theory up to the lowest nontrivial term of the transition matrix, in this case fourth order. We perform the transition amplitude for both final singlet and triplet states i.e.

$$T_s = \langle s|T|\phi_i\rangle, \tag{5}$$
$$T_t = \langle t|T|\phi_i\rangle. \tag{6}$$

3. Quantifying entanglement

Since, in general, the two particle output from our model will be in an arbitrary (pure) state we need a general way to quantify the amount of entanglement obtained at the output. We consider the Von Neumann entropy E [5] for the single particle subspace of a two particle density matrix. We briefly discuss the interpretation of this entropy. Consider a pure bipartite quantum nonseparable state $|\Psi\rangle$ composed of two subsystems A and B. The density matrix describing the whole system is

$$\rho(\Psi) = |\Psi\rangle\langle\Psi|, \tag{7}$$

then, E is given by

$$E(\Psi) = -\mathrm{Tr}\rho_A \log_2 \rho_A \,, \tag{8}$$

where ρ_A is obtained from (7) by summing over the degrees of freedom of particle B as

$$\rho_A = \mathrm{Tr}_B \left(|\Psi\rangle\langle\Psi|\right).$$

The resulting entropy does not depend on whether we do the partial trace over the states of A or B so one can also use $\mathrm{Tr}_A(|\Psi\rangle\langle\Psi|)$. We build the state $|\Psi\rangle$ from $|s\rangle$ and $|t\rangle$ through the following expression

$$|\Psi\rangle = \frac{1}{\sqrt{1+r^2}}\left(|s\rangle + r|t\rangle\right),$$

where, $r = T_t/T_s$ gives the relation between both transition amplitudes. With such a state one gets a Von Neumann entropy given by the expression

$$E = -p\log_2 p - (1-p)\log_2(1-p),$$

with

$$p = \frac{(1+r)^2}{2(1+r^2)}.$$

Thus E is practically indistinguishable from another quantification of entanglement called the concurrence given by $C = (1-r^2)/(r^2+1)$. Hence, the Von Neumann entropy give us a measure of the total system's quantum correlation in terms of the uncertainty in the information on the state of one of its components. For this reason the reduced density matrix will be in a mixed state meaning that its wave function is only statistically known. This can be seen by considering a maximally entangled state such as the singlet state for which $E = 1$. When we trace over one of the particles (say B) in the bipartite system the density matrix of particle A represents a mixed state, which tells us that we have less information on the state of particle A.

4. Results

In the model of Oliver et al the triplet transition is identically vanishing and singlet transition is mediated by Coulomb interaction U i.e. it is only in the presence of electron-electron interaction that we have a finite singlet transition amplitude. In figure 2 we show the amplitude for the singlet and triplet as a function of the entry (half) energy difference Δ_L. As shown in [8] the latter energy difference modifies the destructive interference that kills the triplet component. Nevertheless, in the absence of U there is no new effect unless the electron phonon coupling is turned on. In the figure we show both amplitudes for $U = 0$ and we see that the phonon field induces both finite triplet and singlet transitions. While at $\Delta_L = 0$ the system only produces the singlet, at finite values of this parameter the triplet is produced, reducing the entanglement. Notice nevertheless that for small degeneracy, an increasing coupling to the phonon generates entanglement in the absence of direct electron-electron interaction.

In order to display the aforementioned observation directly we depict in figure 3 the one particle subspace entropy, as a function of Δ_L, at finite electron phonon coupling and for zero (panel a) and finite (panel b) electron-electron interaction. The entropy does not distinguish the amplitude of the entangled signal but only the degree of entanglement i.e. as long as the singlet alone is produced then $E = 1$. Panel a depicts the subspace entropy for U = 0. The $V_g = 0$ curve is absent, since for this value of the coupling we have both T_s and T_t vanishing, hence the entropy expression is not well defined. As concluded in the analysis of figure 2, the appearance of the triplet destroys the entanglement as the input energy difference grows. This reduction is rather insensitive to the magnitude of the electron-phonon coupling.

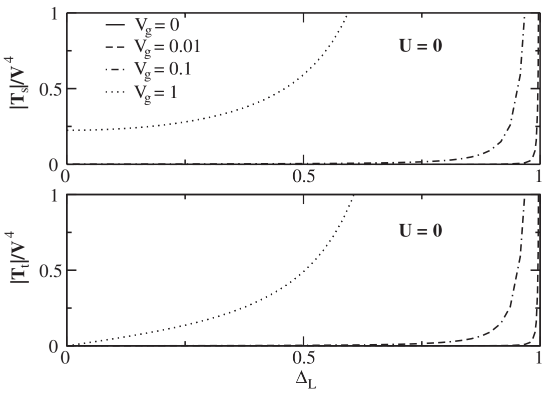

Figure 2. Singlet and triplet transition amplitudes versus Δ_L for vanishing U, parameterized by electron-phonon coupling V_g. The parameters are chosen as $E_L = -2$, $\Delta_R = 1$, $\varepsilon_d = 0$ and phonon energy $\hbar\omega = 0.5$.

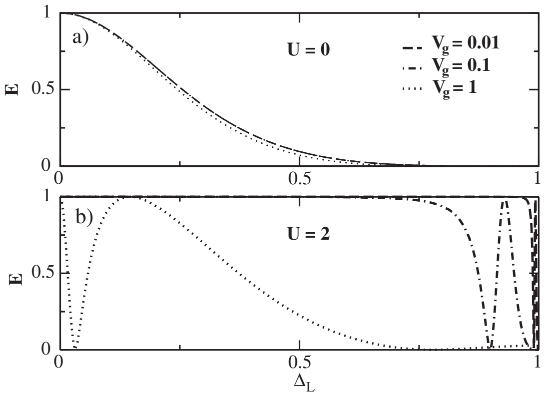

Figure 3. Entropy E as a function of Δ_L for both vanishing (upper panel) and finite $U = 2$ Coulomb interaction (lower panel) parameterized by electron-phonon coupling V_g. The other parameters are chosen as in figure 2.

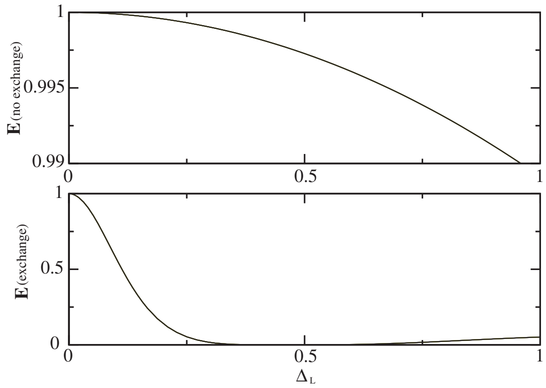

Figure 4. Entropy behavior for processes with no phonon exchange (upper panel) and those including exchange (lower panel) in terms of input energy degeneracy Δ_L. From these, it is evident that phonon exchange is the mechanism inducing dephasing. We take $U = 0$, $V_g = 0.01$ and the same value for the other parameters as in previous figures.

When one turns on the interaction term U, the subspace entropy exhibits very interesting behavior. For small electron-phonon coupling the entanglement is very robust against breaking electron entry energy degeneracy Δ_L. Nevertheless, as V_g increases there is an interplay between the electron couplings which generates oscillations in the entanglement magnitude. Stronger electron-phonon coupling makes for complete entanglement elimination even for small values of the entry degeneracy breaking.

To better understand the role of the phonon coupling we performed a second order perturbative expansion in V_g (restricted to the fourth order in the lead coupling) evaluating separately those contributions with phonon exchange (panel a in figure 4), from those with no phonon exchange (panel b). By phonon exchange we mean Feynman paths for which the first electron excites a phonon in the well and the second electron absorbs it. One such a process is illustrated in figure 5. Note that the phonon exchange donates a mechanism for sequential electron tunnelling i.e. single electron processes that conserve energy. With this mechanism in place an electron with energy ε_L can tunnel to the dot, emit a phonon $\hbar\omega$ so that it reduces its energy to the output channel ε_{R_2}. The following electron absorbs the phonon excitation in the dot and now accesses the output energy ε_{R_1}, so that the two electrons tunnel independently conserving energy. Put another way, these

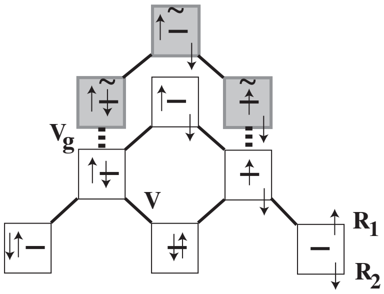

Figure 5. Feynman paths connecting the initial state on the left to the final state on the right. Arrows represent electron spin states. The horizontal segment in the boxes represent a state in the dot and the diagonal segments between boxes represent couplings V from the external states to the dot, while the vertical dashed lines represent coupling to the phonons. The path involving a phonon in grey is an alternate path assisting single electron tunnelling to the final states and thus enabling independent electron events that destroy the entangling properties of the model.

processes are both real processes and not virtual processes that must be summed coherently. It is evident then that in the case interaction between electrons is mediated by the phonon field there is a gain in information on one of the particle's state that results in the reduction of the entropy. This is a very elementary form of decoherence that destroys the possibility of entanglement.

5. Conclusions

We have shown that coupling a quantum dot entangler device to a truncated phonon field induces nonvanishing triplet transitions and the entangling properties of the device are degraded due to phonon exchange between the electrons within the dot. We should also notice that this can be seen as a lowest order environment decoherence [14,15] process in which the phonon field acts as a bath habilitating independent electron events. We must mention that a possible extension of the present work might include dynamical aspects in order to determine decoherence time scales. Another interesting approach would be to make an extension to mixed states. A recent paper by Yu and Eberly [16] discusses the possibility that the decay rates of the coherence of individual qubit states, composing a qubit pair entangled state, might be different from those of the entangled state as a whole.

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Questions and answers

$\mathcal{Q}$ *(Imre Mikos)*: Is the coherence due to conservation laws?

$\mathcal{A}$ In general, conservation laws give us information on conserved quantities such as energy, linear or angular momentum, etc (Feynman R.P. Lectures on Physics 1970, **3**. Adison Wesley). Therefore, the dynamics of the system preserves such quantities. On the other hand, a quantum system initially in a coherent state (i.e. a superposition of basis states with fixed relative phase among its components) due to coupling to its environment might evolve in such a way as to lose this property. This phenomenom is called decoherence (Zurek W.H., Phys. Today, 1991 **44**, 36–44).

$\mathcal{Q}$ *(Dragi Karevski)*: You start using a one-particle state, then entropy . . . , but then you shifted to different definitions and spoke about collective state?

$\mathcal{A}$ In Information Theory, one is interested in quantifying the amount of information carried by a given system. Following the classical proposal by Shannon (Shannon C.F., Weaver W., 1949, The Mathematical Theory of Communication, (University of Illinois, Urbana, IL)) to quantify classical information, one uses the Von Neumann entropy to quantify the information content in a quantum system, and identifies the quantum entanglement with information content (there is a qualitative description of this entanglement measure in the manuscrypt; for more details see, for example, Quantum information and quantum computation, Nielsen M.A., Chuang I.L., Cambridge University Press 2001). Qualitatively speaking, the idea of Von Neumann entropy is to quantify the degree of correlation in the whole system by evaluating the degree of mixedness of any of its subsystems. However, one must be aware there have been introduced several measures for quantum information (An introduction to entanglement measures Plenio M.B., Virmani S.: quant–ph/0504163).

$\mathcal{Q}$ *(Judith Daza)*: How can you get more information about systems (entropy)?

$\mathcal{A}$ The issue of the information content carried by an arbitrary state of two qubits (quantum bit) has been settled down (Wootters W.K., PRL 1998, **80** 2245). However, to determine how much information (entanglement) there is in either bipartite, not two level, systems in a mixed state (M. Horodecki et al, Phys. Lett. A 1996, **223**, 1–8), or in pure multipartite states is still an open question.

$\mathcal{Q}$ *(Dragi Karevski)*: Is it realistic to consider the coupling with only one phonon mode?

$\mathcal{A}$ Although assuming one single mode might not look realistic, this model has been employed to describe a resonant tunneling device which might have many applications (Foá L.E.F., 2001, PRB **64**, 193304, and references therein). For example the generation of a coherent Transversal Acoustic phonon beam called a SASER in analogy to LASER (Makler S.S. et al., 2000, Solid State Commun. **116**, 191).

$\mathcal{Q}$ *(Catherine Dufour)*: How far are you from making a real experiment?

$\mathcal{A}$ So far we have not considered the experimental realization of the proposed setting. However, this question deserves attention because as has already been said, the suggestions of solid state environment as optimal for building quantum gates, should consider all possible couplings present.

$\mathcal{Q}$ *(Arnaldo Donoso)*: Are you considering only single scattering process?

$\mathcal{A}$ If I understand properly, single scattering process could mean the lowest (non-trivial) order in perturbation theory. Thus, I can certainly say that we are in fact tackling the problem in such approximation.

$\mathcal{Q}$ *(Judith Daza)*: What is the meaning of zero bandwidth?

$\mathcal{A}$ The limit of zero bandwidth for the leads means that one is assuming they are ideal wires. Otherwise, one should consider the selfenergy contributions arising from the leads-dot coupling, which is in fact more realistic, from the experimental point of view. (Leon et al, 2004, Europhys. Lett. 2004, **66**, 624).

Створення заплутаності електрон-фононною взаємодією у квантовій точці

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Ми розглядаємо пристрій, що реалізує заплутаність на базі квантової точки (запропонований Oliver et al. [1]) у взаємодії з одною фононною модою. Ми проводимо обчислення з точністю до четвертого порядку за взаємодією електрон-точка, враховуючи фононну взаємодію у всіх порядках. За ентропією фонНоймана ми вимірюємо ступінь заплутаності. Фононний механізм пояснює одноелектронні процеси, що проявляються як декогерентні процеси.

Ключові слова: *квантова точка, заплутаність, декогеренція*

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