Condensed Matter Physics 2006, Vol. 9, No 1(45), pp. 201–210

Information model of economy

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Received July 11, 2005, in final form February 7, 2006

A new stochastic model of economy is developed that takes into account the choice of consumers are the dependent random fields. Axioms of such a model are formulated. The existence of random fields of consumer's choice and decision making by firms are proved. New notions of conditionally independent random fields and random fields of evaluation of information by consumers are introduced. Using the above mentioned random fields the random fields of consumer choice and decision making by firms are constructed. The theory of economic equilibrium is developed.

Key words: *technological mapping, making a decision by firms, productive process*

PACS: *89.65.G*

1. Introduction

The proposed stochastic model of economy contains a new approach to the description of consumer choice and making a decision by firm. This description is based on real observations of consumer choice that is described by probability measures ensemble given on budget sets of consumer. This description differs from classical description of consumer choice and making a decision by firm because consumers and firms make their choice and decision having information about the state of economy and their choice or decision depend on the available information. Our approach permits to construct random fields of consumer choice and making a decision by firms that are dependent random fields. This is very important because as Pareto showed the distribution of wealth for several nations has a power law. This result it is impossible to obtain if we restrict ourselves to classical description of consumer choice and take into account that consumers make their choice independently [6]. We develop the theory of economic equilibrium that is as strict as in the classical approach [2]. We also develop algorithms of finding equilibrium states that are constructive and are also applicable to the classical case [4,5].

Such an approach solves the problem that the wealth in several societies is distributed according to Pareto law [7,6].

2. General notions

The paper presents some results expounded in detail in [1]. Let $S \subseteq R^n_+$ be a set of possible goods that are ordered and $\mathcal{B}(S)$ is Borell σ-algebra of subsets of S. For example,

$$S = \left\{x = \{x_i\}_{i=1}^n,\ x \in R^n_+,\ 0 \leqslant x_i \leqslant c_i,\ i = \overline{1,n}\right\}, \qquad c_i > 0, \qquad i = \overline{1,n}.$$

We assume that the set of possible prices K^n_+ is also ordered and it is a certain subcone of the cone R^n_+, $\mathcal{B}(K^n_+)$ is Borell σ-algebra of subsets of the set K^n_+. Suppose that in an economic system there are m firms that are described by technological mappings $F_i(x),\ X_i,\ i = \overline{1,m}$, where X_i is an expenditure set of the i-th firm, $F_i(x)$ is the set of plans at the expenditure vector $x \in X_i$. It is convenient to assume that the firms are ordered and for further consideration a set of productive processes $\Gamma_i = \{(x,y),\ x \in X_i \subseteq S,\ y \in F_i(x)\}$ of the i-th firm is only important. By $\Gamma^m = \prod\limits_{i=1}^{m} \Gamma_i$

we denote the direct product of the sets $\Gamma_i,\ i=\overline{1,m}$. By $[\Gamma^m]^k$ we denote the k-multiple direct product of the set Γ^m, $\mathcal{B}([\Gamma^m]^k)$ is Borell σ-algebra of subsets of the set $[\Gamma^m]^k,\ k=1,2,\ldots$. Budget set of an i-th insatiable consumer is given by the formula

$$X^i_{(p,z)}=\{x,\ x\in S,\ \langle p,x\rangle=K_i(p,z)\},\quad p\in K^n_+,\quad z\in\Gamma^m,\quad i=\overline{1,l},$$

where $K_i(p,z)$ is a profit function of the i-consumer. Further we use the following notation: $\{X^{k,s}_{(p,z)_k},\ \mathcal{B}(X^{k,s}_{(p,z)_k})\}$ is the direct product of budget spaces

$$\{X^s_{(p_i,z_i)},\ \mathcal{B}(X^s_{(p_i,z_i)})\},\qquad i=\overline{1,k},\quad k=\overline{1,\infty}$$

for every $s=\overline{1,l}$.

Definition 1 *A technological mapping $F(x),\ x\in X$, belongs to a class of CTM (compact technological mappings), if the domain $X\subseteq S$ is a closed bounded convex set such that $0\in X,\ 0\in F(0)$, and $F(x)$ is a Kakutani continuous technological mapping that takes the value in the set of the closed convex bounded subsets of the set S. Moreover, there exists a compact set $Y\subseteq S$ such that $F(x)\subseteq Y$, for all $x\in X$.*

Definition 2 *A set of functions $K^0_i(p,u),\ i=\overline{1,l}$, defined on the set $K^n_+\times\Gamma^m$, that take the values in the set R^1, we call profit prefunctions of consumers if it satisfies conditions:*

1. *$K^0_i(p,u)$ is a measurable mapping of the measurable space $\{\Gamma^m,\mathcal{B}(\Gamma^m)\}$ into the measurable space $\{R^1,\mathcal{B}(R^1)\}$ for every $p\in K^n_+,\ i=\overline{1,l}$;*

2. *For every $p\in K^n_+$ the set $D(p)=\bigcap\limits_{i=1}^{l}D_i(p)$ is non empty, where*

$$D_i(p)=\left\{u\in\Gamma^m,\ K^0_i(p,u)\geqslant 0\right\},\qquad i=\overline{1,l};$$

3. *$K^0_i(tp,u)=tK^0_i(p,u),\quad t>0,\quad (p,u)\in K^n_+\times\Gamma^m,\quad i=\overline{1,l}$.*

Definition 3 *Let m firms be described by technological mappings $F_i(x),\ \ x\in X_i,\ \ i=\overline{1,m}$, and an i-th consumer have the vector of property $b_i(p,z)\geqslant 0,\ \ i=\overline{1,l}$. Assume that for every $(p,z)\in K^n_+\times\Gamma^m$ there exist productive processes*

$$(X_i(p,z),Y_i(p,z)),\quad X_i(p,z)\in X_i,\quad Y_i(p,z)\in F_i(X_i(p,z)),\quad i=\overline{1,m},$$

that satisfy the conditions:

1. *$(X_i(p,z),Y_i(p,z))$ is a measurable mapping of the measurable space $\{\Gamma^m,\mathcal{B}(\Gamma^m)\}$ into the measurable space $\{\Gamma_i,\mathcal{B}(\Gamma_i)\}$ for every $p\in K^n_+,\ \ i=\overline{1,m}$, where $\mathcal{B}(\Gamma_i)$ is Borel σ-algebra of subsets of the set Γ_i.*

2. *$(X_i(tp,z),Y_i(tp,z))=(X_i(p,z),Y_i(p,z)),\quad t>0,\quad (p,z)\in K^n_+\times\Gamma^m$.*

A measurable mapping $Q(p,z)$ of the measurable space $\{\Gamma^m,\mathcal{B}(\Gamma^m)\}$ into itself for every $p\in K^n_+$, given by the formula

$$Q(p,z)=\{(X_i(p,z),Y_i(p,z))\}_{i=1}^m\,, \tag{1}$$

we call a productive economic process if for every $p\in K^n_+$ the set of values $Q(p,\Gamma^m)$ of the mapping $Q(p,z)$ belongs to the set

$$G(p)=\{z\in\Gamma^m,\ R(p,z)\in S\}\,,\quad R(p,z)=\sum_{i=1}^{m}[y_i-x_i]+\sum_{k=1}^{l}b_k(p,z),$$

where $b_i(p,z)\geqslant 0,\ i=\overline{1,l}$, is an initial vector stock of goods of the i-th consumer at the initial moment of economy functioning.

Definition 4 *A set of functions $K_i(p,z),\ i=\overline{1,l}$, given on the set $K_+^n \times \Gamma^m$, that are measurable mappings of the measurable space $\{\Gamma^m, \mathcal{B}(\Gamma^m)\}$ into the measurable space $\{R_+^1, \mathcal{B}(R_+^1)\}$ for every $p \in K_+^n$, we call profit functions of consumers, if there exist a set of profit prefunctions of consumers $K_i^0(p,z),\ i=\overline{1,l}$, a productive economic process $Q(p,z)$, given on the set $K_+^n \times \Gamma^m$, such that $Q(p,\Gamma^m)$ belongs to the set $D(p)$ from the definition 2 and for every $p \in K_+^n$ there hold equalities:*

1. $K_i(p,z) = K_i^0(p, Q(p,z)), \quad (p,z) \in K_+^n \times \Gamma^m,\ i=\overline{1,l};$
2. $\sum\limits_{i=1}^{l} K_i(p,z) =$

$$= \left\langle p, \sum_{i=1}^{m} [Y_i(p,z) - X_i(p,z)] + \sum_{k=1}^{l} b_k(p, Q(p,z)) \right\rangle, \quad (p,z) \in K_+^n \times \Gamma^m. \qquad (2)$$

Definition 5 *Let l consumers be described by profit functions $K_i(p,z),\ i=\overline{1,l}$, given on the set $K_+^n \times \Gamma^m$, and m firms be described by technological mapping $F_i(x), x \in X_i,\ i=\overline{1,m}$. The description of consumers is completely given if for every $s=\overline{1,l}$ and on every direct product $\{X_{(p,z)_k}^{k,s},\ \mathcal{B}(X_{(p,z)_k}^{k,s})\}$ of budget spaces*

$$\left\{X_{(p_i,z_i)}^s,\ \mathcal{B}(X_{(p_i,z_i)}^s)\right\}, \quad i=\overline{1,k}, \quad k=\overline{1,\infty}$$

a probability measure

$$F_{p_1,\ldots,p_k}^s(A^s | z_1,\ldots,z_k), \quad A^s \in \mathcal{B}(X_{(p,z)_k}^{k,s}), \quad s=\overline{1,l},$$

is given for all

$$\{p_1,\ldots,p_k\} \in [K_+^n]^k, \quad \{z_1,\ldots,z_k\} \in [\Gamma^m]^k, \quad k=1,2,\ldots.$$

The measure $F_{p_1,\ldots,p_k}^s(A^s|z_1,\ldots,z_k)$ is the probability that the s-th consumer chooses the collection of goods from the set $A^s \in \mathcal{B}(X_{(p,z)_k}^{k,s})$ on the assumption that in the economic system the productive processes $\{z_1,\ldots,z_k\} \in [\Gamma^m]^k$ were realized on conditions that the prices vector $\{p_1,\ldots,p_k\} \in [K_+^n]^k$ was carried out correspondingly.

Definition 6 *Let l consumers be described by profit functions $K_i(p,z),\ i=\overline{1,l}$, defined on the set $K_+^n \times \Gamma^m$, and m firms be described by technological mapping $F_i(x), x \in X_i,\ i=\overline{1,m}$.*

If there exists a probability space $\{\Omega, \mathcal{F}, P\}$, l random fields $\xi_i(p),\ p \in K_+^n,\ i=\overline{1,l}$, defined on it, that take the values in the set of possible goods S and m random fields $\zeta(p) = \{\eta_1^0(p),\ldots,\eta_m^0(p)\},\ p \in K_+^n$, on the same probability space that take the values in the set of possible productive processes Γ^m such that

$$P\left(\{\xi_i(p_1),\ldots,\xi_i(p_k)\} \in A^i | \zeta(p_1)=z_1,\ldots,\zeta(p_k)=z_k\right) = F_{p_1,\ldots,p_k}^i(A^i|z_1,\ldots,z_k),$$
$$A^i \in \mathcal{B}(X_{(p,z)_k}^{k,i}), \qquad i=\overline{1,l},$$

then the random field $\xi_i(p)$ is called the random field of choice of the i-th consumer that is described by the probability measures ensemble

$$F_{p_1,\ldots,p_k}^i(A^i|z_1,\ldots,z_k), \quad A^i \in \mathcal{B}(X_{(p,z)_k}^{k,i}), \quad i=\overline{1,l},$$

the random field $\eta_s^0(p),\ s=\overline{1,m}$, is called the random field of decision making by s-th firm relative to productive processes.

Theorem 1 *Let X be a bounded closed convex set every point of which is internal for a set X_1 and $F(x)$ is down convex technological mapping from the CTM class, given on the convex compact set X_1. For every sufficiently small $\varepsilon > 0$ there exists a continuous strategy of firm behaviour*

$$(x^0(p), y^0(p)), \qquad y^0(p) \in F(x^0(p))$$

such that

$$\sup_{p \in P} |\varphi(p) - \langle y^0(p) - x^0(p), p\rangle| < \varepsilon,$$

where

$$\varphi(p) = \sup_{x \in X} \sup_{y \in F(x)} \langle y - x, p\rangle.$$

3. Axioms of random consumer's choice and decision making by firms

Under uncertainty conditions the description of economy is given if for every fixed $s,\ s = \overline{1,l}$, a family of finite dimensional conditional distributions

$$F^s_{p_1,\dots,p_k}(A^k|z_1,\dots,z_k)$$

satisfies the conditions:

1) $F^s_{p_1,\dots,p_k}(A^k|z_1,\dots,z_k)$ is a probability measure on the σ-algebra of Borell subset $A^k \in \mathcal{B}(X^{k,s}_{(p,z)_k})$ for every fixed values of variables

$$\{p_1,\dots,p_k\} \in K^{nk}_+, \qquad \{z_1,\dots,z_k\} \in [\Gamma^m]^k, \qquad k = 1,2,\dots,$$

and for every fixed $A_s \in \mathcal{B}(S^k)$ $F^s_{p_1,\dots,p_k}(A_s \cap X^{k,s}_{(p,z)_k}|z_1,\dots,z_k)$ is a measurable mapping of the measurable space $\{[\Gamma^m]^k, \mathcal{B}([\Gamma^m]^k)\}$ into measurable space $\{[0,1], \mathcal{B}([0,1])\}$;

2) for every permutation π of indexes $\{1,\dots,k\}$ there holds the equality

$$F^s_{p_{\pi(1)},\dots,p_{\pi(k)}}\left(\Pi^0_k A^k|z_{\pi(1)},\dots,z_{\pi(k)}\right) = F^s_{p_1,\dots,p_k}(A^k|z_1,\dots,z_k), \qquad k = 1,2,\dots,$$

where $\Pi^0_k A^k$ is the image of the set $A^k \in \mathcal{B}(X^{k,s}_{(p,z)_k})$ under transformation Π^0_k of the set S^k into itself: $\Pi^0_k x = \{x_{\pi(1)},\dots,x_{\pi(k)}\}$, $x = \{x_1,\dots,x_k\} \in S^k$, where π is a permutation of indexes $\{1,\dots,k\}$;

3) $F^s_{p_1,\dots,p_k}\left(A^j \times \prod_{i=j+1}^{k} \hat{X}^s_{(p_i,z^i)}|z_1,\dots,z_k\right) = F^s_{p_1,\dots,p_j}\left(A^j|z_1,\dots,z_j\right), \qquad A^j \in \mathcal{B}\left(X^{j,s}_{(p,z)_j}\right);$

4) $F^s_{tp_1,\dots,tp_k}\left(A^k|z_1,\dots,z_k\right) = F^s_{p_1,\dots,p_k}\left(A^k|z_1,\dots,z_k\right),\ \forall t > 0$

and a family of unconditional finite dimensional distributions

$$\psi_{p_1,\dots,p_k}(B^k), \quad B^k \in \mathcal{B}([\Gamma^m]^k), \quad p_i \in K^n_+, \quad i = \overline{1,k}, \quad k = 1,2,\dots,$$

satisfies the conditions:

1) $\psi_{p_1,\dots,p_j}(B^j) = \psi_{p_1,\dots,p_k}\left(B^j \times [\Gamma^m]^{k-j}\right), \quad B^j \in \mathcal{B}\left([\Gamma^m]^j\right),$
$k = j+1, j+2, \dots, \quad \psi_{p_1}(\Gamma^m) = 1;$

2) for every permutation π of indexes $\{1,\dots,k\}$

$$\psi_{p_{\pi(1)},\dots,p_{\pi(k)}}(\Pi_k^1 B^k) = \psi_{p_1,\dots,p_k}(B^k), \quad k=1,2,\dots,$$

where $\Pi_k^1 B^k$ is the image of the set B^k under transformation Π_k^1 of the set $[\Gamma^m]^k$ into itself: $\Pi_k^1 z = \{z_{\pi(1)},\dots,z_{\pi(k)}\}$, $z=\{z_1,\dots,z_k\} \in [\Gamma^m]^k$, where π is a permutation of indexes $\{1,\dots,k\}$;

3) $\psi_{tp_1,\dots,tp_k}(B^k) = \psi_{p_1,\dots,p_k}(B^k), \quad \forall t>0, \quad k=1,2,\dots;$

4) If $b_i(p,z)$ is a stock vector of goods of the i-th consumer, $i=\overline{1,l}$, at the initial moment of the economy functioning that is a measurable mapping of the measurable space $\{\Gamma^m, \mathcal{B}(\Gamma^m)\}$ into the measurable space $\{S,\mathcal{B}(S)\}$ for every $p\in K_+^n$, and

$$G(p) = \{z\in\Gamma^m,\ R(p,z)\in S\} \ \in\ \mathcal{B}(\Gamma^m),$$

where

$$R(p,z) = \sum_{i=1}^m [y_i - x_i] + \sum_{k=1}^l b_k(p,z), \quad z^i=(x_i,y_i)\ \in\ \Gamma_i\,,$$

then for all $p\in K_+^n$

$$\int\limits_{G(p)} \psi_p(\mathrm{d}z) = 1.$$

Definition 7 *A family of functions of sets*

$$\Phi_{p_1,\dots,p_k}(\mathcal{D}\times A_1\times\ldots\times A_l) = \int\limits_{\mathcal{D}} \prod_{i=1}^l F^i_{p_1,\dots,p_k}\left(A_i\cap X^{k,i}_{(p,z)_k} | z_1,\dots,z_k\right) \mathrm{d}\psi_{p_1,\dots,p_k}(z_1,\dots,z_k),$$

where $\mathcal{D}\in[\mathcal{B}(\Gamma^m)]^k$, $A_i\in\mathcal{B}(S^k)$, $i=\overline{1,l}$, we call the finite dimensional distributions of l consumers choice and decision making by m firms, where

$$A_i = \prod_{s=1}^k A_s^i, \quad \mathcal{D} = \prod_{i=1}^k \mathcal{D}_i, \quad A_s^i\in\mathcal{B}(S), \quad i=\overline{1,l}, \quad s=\overline{1,k}.$$

The economic sense of $\Phi_{p_1,\dots,p_k}(\mathcal{D}\times A_1\times\ldots\times A_l)$ is the probability that the i-th consumer chooses a set of goods from the set $A_s^i\in\mathcal{B}(S)$ on the assumption that the firms have made a decision as to the productive processes that belong to the set $\mathcal{D}_s\in\mathcal{B}(\Gamma^m)$ and the price vector in the economic system is $p_s\in K_+^n$, $i=\overline{1,l}$, $s=\overline{1,k}$.

Theorem 2 *Let a set of conditional finite dimensional distributions*

$$F^i_{p_1,\dots,p_k}\left(A^i | z_1,\dots,z_k\right), \qquad A^i\in\mathcal{B}\left(X^{k,i}_{(p,z)_k}\right), \qquad i=\overline{1,l}, \qquad s=\overline{1,k},$$

and a set of unconditional finite dimensional distributions $\psi_{p_1,\dots,p_k}(D)$, $D\in\mathcal{B}([\Gamma^m]^k)$, $k=\overline{1,\infty}$, satisfy the above formulated axioms. The function of sets given by the formula

$$\Phi_{p_1,\dots,p_k}(\mathcal{D}\times A_1\times\ldots\times A_l) = \int\limits_{\mathcal{D}} \prod_{i=1}^l F^i_{p_1,\dots,p_k}\left(A_i\cap X^{k,i}_{(p,z)_k} | z_1,\dots,z_k\right) \mathrm{d}\psi_{p_1,\dots,p_k}(z_1,\dots,z_k)$$

on the sets of the kind

$$\mathcal{D}_1\times\ldots\times\mathcal{D}_k\times A_1^1\times\ldots\times A_1^k\times\ldots\times A_l^1\times\ldots\times A_l^k, \qquad A_i^s\in\mathcal{B}(S), \qquad \mathcal{D}_i\in\mathcal{B}(\Gamma^m),$$

where the set A_i *has the form* $A_i = A_i^1 \times \ldots \times A_i^k$, $i = \overline{1,l}$, *and the set* $\mathcal{D}$ *is of the kind* $\mathcal{D} = \mathcal{D}_1 \times \ldots \mathcal{D}_k$, *admits an extension on the measurable space*

$$V_1 = \left\{[\Gamma^m \times S^l]^k, \mathcal{B}(\Gamma^m) \times \mathcal{B}(S^l)]^k\right\},$$

that is, there exists a family of measures $\bar{\mu}_{z_1,\ldots,z_k}^{p_1,\ldots,p_k}(E)$, *given on the measurable space* V_1, *such that for every fixed* $\{p_1,\ldots,p_k\} \in K_+^{nk}$ *and* $E \in [\mathcal{B}(\Gamma^m) \times \mathcal{B}(S^l)]^k$ *every measure of the family is a measurable mapping of the space* $L = \{[\Gamma^m]^k, [\mathcal{B}(\Gamma^m)]^k$ *into the space* $\{R^1, \mathcal{B}(R^1)\}$ *and the extension is given by the formula*

$$\bar{\Phi}_{p_1,\ldots,p_k}(E) = \int\limits_{[\Gamma^m]^k} \bar{\mu}_{z_1,\ldots,z_k}^{p_1,\ldots,p_k}(E) \mathrm{d}\psi_{p_1,\ldots,p_k}(z_1,\ldots,z_k).$$

The extension satisfies the conditions:

$$\bar{\Phi}_{p_{\pi(1)},\ldots,p_{\pi(k)}}\left(\Pi_k^2 E\right) = \bar{\Phi}_{p_1,\ldots,p_k}(E), \qquad E \in \left[\mathcal{B}(\Gamma^m) \times \mathcal{B}(S^l)\right]^k, \tag{3}$$

$$\bar{\Phi}_{p_1,\ldots,p_k}\left(A \times (\Gamma^m \times S^l)^{k-r}\right) = \bar{\Phi}_{p_1,\ldots,p_r}(A), \qquad A \in \left[\mathcal{B}(\Gamma^m) \times \mathcal{B}(S^l)\right]^r, \tag{4}$$

where $\Pi_k^2 E$ *is the image of the set* E *under transformation* Π_k^2 *of the set* $[\Gamma^m \times S^l]^k$ *into itself:* $\Pi_k^2\{w_1,\ldots,w_k\} = \{w_{\pi(1)},\ldots,w_{\pi(k)}\}$, $w_i = \{z_i, x_1^i, \ldots, x_l^i\} \in \Gamma^m \times S^l$, *and* π *is a permutation of indexes* $\{1,\ldots,k\}$.

Theorem 3 *The family of finite dimensional distributions* $\bar{\Phi}_{p_1,\ldots,p_k}(E)$, *where* $\{p_1,\ldots,p_k\} \in [K_+^n]^k$, *and* $E \in [\mathcal{B}(\Gamma^m) \times \mathcal{B}(S^l)]^k$, *that was constructed in the theorem 2, satisfies the conditions of the Kolmogorov theorem with the full separable metric space of state* $X = \Gamma^m \times S^l$ *and the* σ*-algebra subsets* $\Sigma = \mathcal{B}(\Gamma^m) \times \mathcal{B}(S^l)$ *and, thus, the family generates a unique measure* P *on the measurable space* $\{X^T, \Sigma^T\}$ *such that the family of finite dimensional distributions of a random field*

$$\nu_p(\omega) = \{\zeta_0(p), \xi_1(p), \ldots, \xi_l(p)\} = \omega(p), \quad \omega(p) \in X^T, \quad p \in K_+^n,$$

coincides with the family $\bar{\Phi}_{p_1,\ldots,p_k}(E)$, *that is,*

$$P\left(\omega \in X^T, \{\nu_{p_1}(\omega), \ldots, \nu_{p_k}(\omega)\} \in E\right) = \bar{\Phi}^{p_1,\ldots,p_k}(E), \qquad E \in \left[\mathcal{B}(\Gamma^m) \times \mathcal{B}(S^l)\right]^k.$$

By X^T *we denote the set of all functions, given on the set* $T = K_+^n$, *with the values in the set* $X = \Gamma^m \times S^l$, Σ^T *is the minimal* σ*-algebra, generated by cylindrical sets of the kind*

$$\left\{\omega(p) \in X^T, \{\nu_{p_1}(\omega), \ldots, \nu_{p_1}(\omega)\} \in E\right\}, \qquad E \in \left[\mathcal{B}(\Gamma^m) \times \mathcal{B}(S^l)\right]^k.$$

4. Conditionally independent random fields

Definition 8 *Let* $\{\Omega, \mathcal{F}, P\}$ *be a measurable space. A family of sub* σ*-algebras* $\{B_i, i \in I\}$ *of the* σ*-algebra* $\mathcal{F}$ *is called conditionally independent with respect to a sub* σ*-algebra* $B \subseteq \mathcal{F}$, *if*

$$M\left\{\prod_{j \in I_s} X_j | B\right\} = \prod_{j \in I_s} M\{X_j | B\}$$

for every finite subset $I_s \subseteq I$ *and a family of random values* $\{X_j, j \in I_s\}$, *where* X_j *is* B_j*-measurable positive random value.*

Definition 9 *Let* l *random fields of consumers choice* $\xi_1(p), \ldots, \xi_l(p)$, $p \in K_+^n$, *be given on the probability space* $\{\Omega, \mathcal{F}, P\}$ *and take the values in the set of possible goods* S, *that is, they are measurable mappings of the measurable space* $\{\Omega, \mathcal{F}\}$ *into the measurable space* $\{S, B(S)\}$ *for every fixed* $p \in K_+^n$, *and let random fields of decision making by* m *firms* $\zeta(p) = \{\eta_1^0(p), \ldots, \eta_m^0(p)\}$ *be* *measurable mappings of* $\{\Omega, \mathcal{F}\}$ *into* $\{\Gamma^m, \mathcal{B}(\Gamma^m)\}$ *for every fixed* $p \in K_+^n$*. The random fields of consumers choice are conditionally independent relative to random fields of decision making if* σ*-algebras*

$$\mathcal{F}_i = \mathcal{F}\{\xi_i(p),\ p \in K_+^n\}, \qquad i = \overline{1,l},$$

conditionally independent relative to σ*-algebra* $\mathcal{F}_0 = \mathcal{F}\{\zeta(p),\ p \in K_+^n\}$*, where* σ*-algebras* $\mathcal{F}_i,\ i = \overline{0,l}$*, are minimal* σ*-algebras generated by the family of random values* $\{\zeta(p),\ p \in K_+^n\}$ *for* $i = 0$ *and the family of random values* $\{\xi_i(p),\ p \in K_+^n\}$ *for* $i = \overline{1,l}$*.*

In the next theorems, we assume that the productive economic process $Q(p,z)$ and vectors of initial stock of goods $b_k(p,z),\ k = \overline{1,l}$, are continuous functions of variables $(p,z) \in K_+^n \times \Gamma^m$ with the values correspondingly in the sets $\Gamma^m,\ S$. The next theorem is very important for a construction theory of economic equilibrium.

Theorem 4 *Let a random field* $\eta_i^0(p,z,\omega_i), (p,z) \in K_+^n \times \Gamma^m$*, given on the probability space* $\{\Omega_i, \mathcal{F}_i, P_i\}$*, be a continuous function of* $(p,z) \in K_+^n \times \Gamma^m$ *for every* $\omega_i \in \Omega_i$*, take values in* $S,\ i = \overline{1,l}$*, and a random field* $\zeta_0(p,\omega_0),\ p \in K_+^n$*, given on the probability space* $\{\Omega_0, \mathcal{F}_0, P_0\}$*, take values in the set* Γ^m *and every realization of the field be continuous function of* $p \in K_+^n$*. Moreover, let* $\eta_i^0(tp,z,\omega) = \eta_i^0(p,z,\omega),\ i = \overline{1,l},\ t > 0,\ \zeta_0(tp,\omega_0) = \zeta_0(p,\omega_0),\ p \in K_+^n,\ t > 0,$ *and* $K_i(p,z), i = \overline{1,l}$*, be profit functions of consumers that satisfy all the conditions of definition 4 and be continuous functions of variables* $(p,z) \in K_+^n \times \Gamma^m$*. If*

$$\begin{aligned}
&\langle \eta_i^0(p,z,\omega_i), p\rangle > 0, && \eta_i(p,z,\omega_i) = \eta_i^0(p, Q(p,z), \omega_i),\\
&(p,z,\omega_i) \in K_+^n \times \Gamma^m \times \Omega_i, && i = \overline{1,l},\\
&P_i\left(\langle \eta_i^0(p,z,\omega_i), p\rangle < \infty\right) = 1, && (p,z) \in K_+^n \times \Gamma^m, \quad i = \overline{1,l},
\end{aligned}$$

then the random fields

$$\xi_i(p,\omega) = \frac{K_i(p, \zeta_0(p,\omega_0))\eta_i(p, \zeta_0(p,\omega_0), \omega_i)}{\langle \eta_i(p, \zeta_0(p,\omega_0), \omega_i), p\rangle}, \qquad i = \overline{1,l}, \tag{5}$$

are continuous on the probability space $\{\Omega, \mathcal{F}, P\}$ *for each realization, where*

$$\Omega = \prod_{i=0}^{l} \Omega_i, \quad \mathcal{F} = \prod_{i=0}^{l} \mathcal{F}_i, \quad P = \prod_{i=0}^{l} P_i,$$

that can be identified with random fields of choice of insatiable consumers on the same probability space under the condition that $\zeta(p,\omega_0) = Q(p, \zeta_0(p,\omega_0))$ *are identified with random fields of decision making by firms as to the productive processes.*

The random field $\eta_i^0(p,z,\omega_i)$ is called the random field of evaluation of information by an i-th consumer, $i = \overline{1,l}$.

Definition 10 *Let*

$$\eta_i(p, \zeta_0(p,\omega_0), \omega_i) = \eta_i^0(p, \zeta(p,\omega_0), \omega_i) = \{\eta_{ik}^0(p, \zeta(p,\omega_0), \omega_i)\}_{k=1}^n, \quad i = \overline{1,l},$$

and $\zeta(p,\omega_0) = Q(p, \zeta_0(p,\omega_0))$ *be the random field as in the Theorem 4. By demand vector of an* i*-th insatiable consumer we denote a random field*

$$\gamma_i(p) = \gamma_i(p, \omega_0, \omega_i) = \{\gamma_{ik}(p)\}_{k=1}^n, \quad i = \overline{1,l},$$

where

$$\gamma_{ik}(p) = \gamma_{ik}(p,\omega_0,\omega_i) = \frac{p_k \eta_{ik}^0(p, \zeta(p,\omega_0), \omega_i)}{\sum_{s=1}^n \eta_{is}^0(p, \zeta(p,\omega_0), \omega_i) p_s}, \quad k = \overline{1,n}, \quad i = \overline{1,l}.$$

A random field of choice of the i-th insatiable consumer is connected with the demand vector of the i-th insatiable consumer by the formula

$$\xi_i(p) = \left\{ \frac{D_i(p)\gamma_{ik}(p)}{p_k} \right\}_{k=1}^{n}, \quad D_i(p) = K_i(p, \zeta_0(p, \omega_0)), \quad i = \overline{1, l}. \tag{6}$$

$\gamma_{ik}(p)$ has the following economic sense: the part of the profit of the i-th consumer he spends to buy the k-th good.

A demand of society is described by a demand matrix $||\gamma_{ij}(p)||_{i,j=1}^{l,n}$. All the statements proved for the case of insatiable consumers are valid if not all consumers are insatiable.

Definition 11 *An economic system is in the Walras equilibrium state if there exists a price vector p^*, m productive processes*

$$(x_i^*(p^*),\ y_i^*(p^*)), \quad x_i^*(p^*) \in X_i, \qquad y_i^*(p^*) \in F_i(x_i^*(p^*)), \quad i = \overline{1, m},$$

such that the following inequalities

$$\phi(p^*) \leqslant \psi(p^*), \tag{7}$$

$$\langle \phi(p^*), p^* \rangle = \langle \psi(p^*), p^* \rangle, \tag{8}$$

hold, where $p^* = (p_1^*, \ldots, p_n^*)$ is an equilibrium price vector, $F_i(x)$ is a technological mapping of the i-th firm, $i = \overline{1, m}$.

Inequality (7) means that in the equilibrium state there exists a price vector p^* while the demand of society does not exceed proposition and the equality (8) means that the value of goods that society wants to buy is equal to the value of goods offered for consumption. The price vector p^*, that guarantees the fulfilment of (7) and (8), is called equilibrium price vector.

Definition 12 *Walras equilibrium state of economy is called optimal if the following conditions*

$$\langle y_i^*(p^*) - x_i^*(p^*), p^* \rangle = \sup_{x \in X_i} \sup_{y \in F_i(x)} \langle y - x, p^* \rangle, \qquad i = \overline{1, m},$$

are valid where X_i is the expenditure set of an i-th firm, $F_i(x)$ is its technological mapping.

5. Theory of economic equilibrium

In the next theorem we assume that the matrix $||\gamma_{ik}(p)||_{i=1,k=1}^{l,n}$ is not necessarily generated by random fields of evaluation of information by consumers and it is arbitrary which satisfies the conditions of this theorem.

Theorem 5 *Let technological mappings $F_i(x)$, $x \in X_i^1$, $i = \overline{1, m}$, be down convex, belong to CTM class, a productive economic process $Q(p, z)$ and a family of profit prefunction $K_i^0(p, z)$, $i = \overline{1, l}$, be continuous mappings of variables $(p, z) \in R_+^n \times \Gamma^m$ and random fields of decision making by firms satisfy the conditions of the theorem 4. Moreover, if the productive economic process $Q(p, z)$ satisfies the condition*

$$R(p, Q(p, z)) > 0, \quad p \in R_+^n, \quad p \neq 0 \quad z \in \Gamma^m, \tag{9}$$

$$R(p, z) = \sum_{i=1}^{m} [y_i - x_i] + \sum_{j=1}^{l} b_j(p, z),$$

then for every continuous matrix $||\gamma_{ik}(p)||_{i=1,k=1}^{l,\ n}$, given on R_+^n, the rows of that satisfy the conditions

$$\sum_{k=1}^{n} \gamma_{ik}(p) = 1, \qquad i = \overline{1, l}, \tag{10}$$

and continuous realization $z(p) = \{z_i(p) = (x_i(p), y_i(p))\}_{i=1}^m$ *of random fields of decision making by firms* $\eta_i^0(p)$, $i = \overline{1,m}$, *the set of equations*

$$\sum_{i=1}^{l} \gamma_{ik}(p) D_i(p) = p_k \left[\sum_{i=1}^{m} [y_{ik}(p) - x_{ik}(p)] + \sum_{i=1}^{l} b_{ik}(p, z(p)) \right], \qquad k = \overline{1,n} \tag{11}$$

is solvable in R_+^n, *where* $D_i(p) = K_i^0(p, z(p))$, $\quad i = \overline{1,l}$.

Theorem 6 *Let technological mappings* $F_i(x)$, $x \in X_i^1$, $i = \overline{1,m}$, *be down convex, belong to CTM class, a productive economic process* $Q(p,z)$, *a family of profit prefunctions* $K_i^0(p,z)$, $i = \overline{1,l}$, *be continuous mappings of variables* $(p,z) \in R_+^n \times \Gamma^m$, *and random fields that describe the consumers and firms satisfy the conditions of the Theorem 4. Then with probability 1 there exists the Walras equilibrium state, that is, for every realization of random fields that describe consumers and firms there exists a corresponding price vector* $p^* \in R_+^n$ *such that the economic system is in Walras equilibrium state. Moreover, if realization of random fields that describe the consumers and firms is such that between them there exist realizations that are arbitrary close to optimal behaviour strategies of firms in the sense of the Theorem 1, then with probability 1 there exists an optimal Walras equilibrium state.*

Theorem 7 *Let technological mappings* $F_i(x)$, $x \in X_i^1$, $i = \overline{1,m}$, *a productive economic process* $Q(p,z)$, *a family of profit prefunctions of consumers* $K_i^0(p,z)$, $i = \overline{1,l}$, *satisfy the conditions of the Theorem 6. Random fields that describe the consumer's choice and decision making by firms are continuous with probability 1. Moreover, if a productive economic process* $Q(p,z)$ *satisfies the condition*

$$R(p, Q(p,z)) > 0, \quad p \in R_+^n, \quad p \neq 0 \quad z \in \Gamma^m, \tag{12}$$
$$R(p,z) = \sum_{i=1}^{m} [y_i - x_i] + \sum_{j=1}^{l} b_j(p,z),$$

then for every continuous on R_+^n *demand matrix* $||\gamma_{ik}(p)||_{i=1,k=1}^{l,\ n}$, *that satisfies conditions of the Theorem 6, and a realization of random fields of decision making by firms* $z(p)$, *that is, with probability 1 every Walras equilibrium price vector* $\bar{p}$ *satisfies the set of equations*

$$\sum_{i=1}^{l} \gamma_{ik}(p) D_i(p) = p_k \left[\sum_{i=1}^{m} [y_{ik}(p) - x_{ik}(p)] + \sum_{i=1}^{l} b_{ik}(p, z(p)) \right], \qquad k = \overline{1,n}. \tag{13}$$

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Інформаційна модель економіки

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Отримано 11 липня 2005 р., в остаточному вигляді – 7 лютого 2006 р.

Розвинено нову стохастичну модель економіки, яка враховує взаємозалежність випадкових полів вибору споживачів. Побудовано аксіоматику цієї моделі. Доведено існування випадкових полів вибору споживачів та прийняття рішень фірмами. Введено нове поняття умовно незалежних випадкових полів, за допомогою яких побудовано зазначені випадкові поля. Побудовано теорію економічної рівноваги в цій моделі.

Ключові слова: *технологічні відображення, прийняття рішень фірмами, виробничий процес*

PACS: *89.65.G*