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Solliton-like order parameter distributions in the critical region

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Some exact one-component order parameter distributions for the Michelson thermodynamic potential are obtained. The phase transition of second kind in Ginzburg-Landau type model is investigated. The exact partial distribution of the order parameter in the form of Jakobi elliptic function is obtained. The energy of this distribution is lower at some temperature interval than for the best known models.

Key words: *symmetry, phase transition, order parameter, soliton, modulated structure*

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1. Introduction

The problem of spontaneous symmetry breaking (SSB) at the second kind phase transition (PT), which is the basis of many theories and processes in physics is notable for the fact that even in an 0-approximation it cannot be reduced to the model of free field [1]. This is conditioned by the strong interacion and correlation in the system of anomalously growing strong critical fluctuations – the system with infinite number of degrees of freedom.

In most physical systems, the number of order parameter (OP) components in the critical region decreases, and many PT can be described by a scalar one-component OP (as in ferroelectrics, numerous magnetics, superconductors without magnetic fields, etc.). In this case SSB for the discrete group of reflection symmetry ($\varphi \to -\varphi$, where $\varphi(x)$-OP field) takes plase in the system [2,3].

We consider the following model which describes the PT to the commensurate phase through the phases with modulated structures:

$$\Phi_0 = \widetilde{\Phi}_0 \int \left[\widetilde{\varphi_0}''^2 + g_0^2 \widetilde{\varphi_0}^2 \widetilde{\varphi_0}'^2 - \gamma_0 \widetilde{\varphi_0}'^2 + r_0 \widetilde{\varphi_0}^2 + \frac{s_0}{2} \widetilde{\varphi_0}^4 + \frac{h_0}{3} \widetilde{\varphi_0}^6 \right] \mathrm{d}\widetilde{x}, \tag{1}$$

where $\widetilde{\varphi_0}(x)$ is the scalar OP field ($\widetilde{\varphi_0}' = \mathrm{d}\widetilde{\varphi_0}/\mathrm{d}\widetilde{x}$); $\widetilde{\Phi_0}, g_0, \gamma_0, r_0, s_0, h_0$ are material constants [4–6].

Without the gradient terms this model describes the second kind PT from phase $\widetilde{\varphi_0} = 0$ to phase $\widetilde{\varphi_0} = \sqrt{-(r_0/s_0)}$, occuring at r_0 sign inversion. In this case the system has one degree of freedom. The inclusion into Φ_0 of only one gradient term $\sim (\nabla\widetilde{\varphi_0})^2$ (Ginzburg-Landau-Wilson model) adds to $\widetilde{\varphi_0}$ the continuum of freedom degrees, interconnected by shot-range interaction, and that allows us to take into account both the correlation and the interaction of fluctuation $\widetilde{\varphi_0}(x)$. However, the stable $\widetilde{\varphi_0}(x)$ structure can arise only as a result of the competition between several gradient terms or due to the specific boundary condition.

Using the scale transform of $\widetilde{\varphi_0}, \widetilde{x}$ as well as by relationship:

$$\widetilde{\varphi_0}\left(\widetilde{x}\right) = \sqrt{|\gamma_0|}\,\varphi\left(\widetilde{x}\sqrt{|\gamma_0|}\right), \qquad r_0 = q\gamma_0^2, \qquad s_0 = p\gamma_0, \widetilde{x}\sqrt{|\gamma_0|} = x, \tag{2}$$

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we reduce the number of parameters by 3, clearing coefficient under $(\varphi^{''2}), (\varphi^6)$ and $\varphi^{'2}$:

$$\Phi = \Phi_0 \int \left[\varphi^{''2} + g^2\varphi^2\varphi^{'2} - \varphi^{'2} + q\varphi^2 + \frac{p}{2}\varphi^4 + \frac{1}{3}\varphi^6\right] \mathrm{d}x. \tag{3}$$

The Euler-Poisson variational equation for functional (1) has the following form:

$$\varphi^{\mathrm{IV}} + g\left(\varphi^2\varphi^{''} + \varphi\varphi^{'2}\right) + \varphi^{''} + q\varphi + p\varphi^3 + \varphi^5 = 0. \tag{4}$$

2. Analysis of order parameter asymptotic behavior

The first integral of equation (4) has the form:

$$2\varphi^{'}\varphi^{'''} - \varphi^{''2} + g^2\varphi^2\varphi^{'2} - \varphi^{'2} + q\varphi^2 + \frac{p}{2}\varphi^4 + \frac{1}{3}\varphi^6 = D, \tag{5}$$

where D – integration constant.

Equation (5) by means of substitution:

$$\varphi^{'2} = 2A\left(\varphi\right), \tag{6}$$

can be easily led to the form [7]:

$$4AA^{''} - A^{'2} + 2gA\varphi^2 + 2\gamma A + q\varphi^2 + \frac{p}{2}\varphi^4 + \frac{1}{3}\varphi^6 = 0. \tag{7}$$

One can introduce the function $U\left(\varphi\right) = A\left(\varphi\right)^{\frac{3}{4}}$ and obtain the following equation:

$$\frac{16}{3}U^{\frac{5}{3}}U^{''} + 2g\varphi^2U^{\frac{4}{3}} + 2\gamma U^{\frac{4}{3}} + q\varphi^2 + \frac{p}{2}\varphi^4 + \frac{1}{3}\varphi^6 = 0. \tag{8}$$

In case $\varphi\left(x\right) \to 0$ one can eject the terms with infinitesimal order more than φ^2:

$$U^{\frac{5}{3}}U^{''} + \frac{3}{8}\gamma U^{\frac{4}{3}} + \frac{3}{16}q\varphi^2 = 0. \tag{9}$$

Equation (9) is equivalent to the linearized initial equation (4), it makes the scale invariance possible. Anomalous scale dimension of equation (9) is $\Delta = 3/2$. Due to this fact by means of substitution:

$$U(\varphi) = \varphi^{-\Delta}z(t), \qquad \varphi = \mathrm{e}^t, \tag{10}$$

it is possible to dispose of obvious dependence on the argument:

$$z^{''} + 2z^{'} + \frac{3}{4}z + \frac{3}{8}\gamma z^{-\frac{1}{3}} + \frac{3}{16}qz^{-\frac{5}{3}} = 0. \tag{11}$$

The analysis of equation (11) shows that if $\varphi(x) \to 0$ then $z(t) \to \mathrm{const}$, and therefore $A\left(\varphi\right) \sim \varphi^2$. The analysis of $\varphi\left(x\right)$ pole shows that if $\varphi\left(x\right) \to \infty$,then $A\left(\varphi\right) \sim \varphi^4$.

3. Exact partial solution of variational equation

Consider the solutions of (5), which satisfy the following relation:

$$\varphi^{'2} = a + b\varphi^2 + c\varphi^4 \cdots, \tag{12}$$

where $b \geqslant 0, c \ll 0$.

This approach allows us to investigate the set of PT from the ordered to the disordered state throught the phases with modulated OP structures.

After substituting the relationship (12) into (5) one can obtain the following set of equations for a, b, c:

$$
\begin{aligned}
&24c^2 + 3gc + 1 = 0, && b^2 + b + q + 12ac + ag = 0, \\
&10bc + gb + c + \frac{p}{2} = 0, && 2ab + a = D,
\end{aligned} \tag{13}
$$

the set (5) is overdetermined, therefore it is possible to find the relationship between g, q, p, which is the condition of compatibility (5) and (12).

Consider the case $D = p = 0$, in this case $a = 0$, and the set of equation (13) will take the following form:

$$
24c^2 + 3gc + 1 = 0, \qquad b^2 + b + q = 0, \qquad 10bc + gb + c = 0. \tag{14}
$$

In this case the condition of compatibility (5) and (12) will be:

$$
g^2(q) = \frac{(10b(q) + 1)^2}{3b(q)\,(2b(q) + 1)}, \tag{15}
$$

where b is the solution of the second equation (14). The graph of function $g^2(q)$ is the line of PT. This function has minimum $g^2(-7/16) = 32/3$ and asymptote $q \to \infty, g^2(q) \to 50/3$. The corresponding solution of (5) has the following form:

$$
\varphi\,(x) = \frac{\alpha}{\cosh\,(\beta x)}\,, \tag{16}
$$

and describes the bell-soliton state of OP field [9].

Consider the solution of variational equation (4) of the following form:

$$
\varphi\,(x) = a \cdot sn(b(x - x_0), k). \tag{17}
$$

After substituting (17) into (4) we obtain the set of equations for a, b, k:

$$
\begin{aligned}
a^4 + 3g(abk)^2 + 24(bk)^4 &= 0, \\
-2(1 + k^2)(ga^2b^2 + 10k^2b^4) + 2(kb)^2 + pa^2 &= 0, \\
(1 + 14k^2 + k^4)b^4 + g(ab)^2 - (1 + k^2)b^2 + q &= 0.
\end{aligned} \tag{18}
$$

The solution of set (18) has the following form [10]:

$$
a^2 = g_s(kb)^2\,, \qquad g_s \equiv \frac{3}{2}\left(-g - \sqrt{g^2 - \frac{32}{3}}\right), \tag{19}
$$

$$
b^2 = \frac{\alpha_s}{1 + k^2}\,, \qquad \alpha_s \equiv \frac{2 + pg_s}{2(10 + gg_s)}\,, \tag{20}
$$

$$
k^2 = \frac{\left\{\left(-[(14 + gg_s)\alpha_s^2 + 2(q - \alpha_s)]\right) + (\alpha_s\sqrt{12 + gg_s})\sqrt{(16 + gg_s)\alpha_s^2 + 4(q - \alpha_s)}\right\}}{2[\alpha_s^2 + (q - \alpha_s)]}\,, \tag{21}
$$

$$
g \leqslant -\frac{10}{\sqrt{6}}, q \geqslant -\frac{1}{4}(16 + gg_s)\alpha_s^2 + \alpha_s\,. \tag{22}
$$

Consider the solution of variational equation (4) of the following form:

$$
\varphi\,(x) = a \cdot cn(b(x - x_0), k). \tag{23}
$$

After substituting (23) into (4) we obtain the set of equations for a, b, k [11,12]:

$$
\begin{aligned}
a^4 - 3g(abk)^2 + 24(bk)^4 &= 0, \\
2(1 - 2k^2)(10k^2b^4 - ga^2b^2) - 2(kb)^2 + pa^2 &= 0, \\
(1 + 16k^2 + 16k^4)b^4 + g(1 - k^2)(ab)^2 - (1 - 2k^2)b^2 + q &= 0.
\end{aligned} \tag{24}
$$

If $k^2 \neq 1/2$ then the solutions of set (24) has the following form:

$$a^2 = g_c(kb)^2, \quad b^2 = \frac{\alpha_c}{1-2k^2}, \quad k^2 = \frac{1}{2}\left\{1 \pm \sqrt{\frac{\alpha_c^2(12-gg_c)}{\alpha_c^2(16-gg_c)+4(q-\alpha_c}}\right\}, \tag{25}$$

$$g_c \equiv \frac{3}{2}\left\{g \pm \sqrt{g^2 - \frac{32}{3}}\right\}, \qquad \alpha_c \equiv \frac{2-pg_c}{2(10-gg_c)}. \tag{26}$$

In case $k^2 = 1/2$:

$$b^4 = \frac{4g}{12-gg_c}, \qquad a^2 = \frac{g_c}{2}\sqrt{\frac{4g}{12-gg_c}}. \tag{27}$$

Consider the solution of variational equation (4) of the following form:

$$\varphi(x) = a \cdot \mathrm{d}n(b(x-x_0), k). \tag{28}$$

After substituting (28) into (4) we obtain the set of equations for a, b, k:

$$\begin{aligned} a^4 - 3g(ab)^2 + 24b^4 &= 0, \\ 2(k-2^2)(10b^4 - ga^2b^2) - 2b^2 + pa^2 &= 0, \\ (16 - 16k^2 + k^4)b^4 + g(k^2-1)(ab)^2 - (2-k^2)b^2 + q &= 0. \end{aligned} \tag{29}$$

Solutions of set (29) has the following form:

$$a^2 = 2\sqrt{3}(kb)^2, \qquad b^2 = \frac{\alpha_d}{k^2-2}, \qquad \alpha_d = \frac{p\sqrt{3}-1}{2}. \tag{30}$$

4. Conclusion

In the present paper we investigate the PT in Ginzburg-Landau type model with high gradients. New OP distributions were found. These distributions make it possible to discribe the set of PT from the ordered to the disodered phase through the set of states with modulated OP structures. We also investigated soliton-like OP distributions. The presence of soliton-like solution in the model considered is determind by variational symmetry of initial Euler-Poisson equation.

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Солітонно-подібні розподіли параметра порядку в критичній області

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Отримано декілька точних розподілів параметра порядку для термодинамічного потенціалу і досліджено фазовий перехід другого роду в моделі типу Гінзбурга-Ландау. Отримано точний розподіл параметра порядку у вигляді еліптичної функції Якобі. Показано, що в певному температурному інтервалі енергія цього розподілу є нижчою ніж у випадку добре відомих моделей.

Ключові слова: *симетрія, фазовий перехід, параметр порядку, солітон, модульована структура*

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